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ScienceDirect
Procedia Engineering 199 (2017) 1374–1379
Procedia Engineering
www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

A 2.5D spectral approach to represent acoustic and elastic waveguides interaction on thin slab structures

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Abstract

In this paper, we propose a spectral element method (SEM) to study guided waves in coupled problems involving thin-walled structures and fluid-acoustic cavities. The numerical method is based on the subdomain decomposition of the fluid-structure system. Two spectral elements are developed to represent the fluid and the structure. A plate element based on a mixed Reissner-Mindlin and Kirchhoff-Love formulation is proposed to represent the thin-walled structure. This element uses C^0 approximation functions to overcome the difficulties to formulate elements with arbitrary order from C^1 functions. The proposed element uses a substitute transverse shear strain field resulting free shear locking. The fluid element is derived from the Helmholtz equation. These elements use Lagrange polynomials as shape functions at the Legendre-Gauss-Lobatto (LGL) points. The analysis is carried out by a two-and-a-half dimension (2.5D) approach in the wavenumber-frequency domain. The guided wave in a fluid cavity with a flexible side is analysed.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Fluid waveguide, solid waveguide, two-and-a-half dimension, spectral element method, acoustic, shear locking

1. Introduction

Time-harmonic wave propagation, such as fluid acoustics and solid scattering, is a common phenomenon that appears in many engineering fields. The propagation of acoustic waves triggered by static and moving pressure sources, the vibration assessment and the acoustic insulation all involve fluid and solid interaction and it should be considered rigorously. The finite element method (FEM) has been used in several works to predict the response of fluid-structure problems. For the low frequency range, the conventional finite elements with linear shape functions represent accurately the fluid and solid scattering waves. However, at high frequencies, these functions do not provide reliable results due to so-called pollution effects [1,2]: the accuracy of the numerical solution deteriorates with increasing the non-dimensional wavenumber and it is not enough the commonly employed rule of thumb of n elements per wavelength [3]. Higher element resolutions are required in order to obtain results with reasonable accuracy.

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.362

The method proposed regards with a 2.5D approach based on the SEM to study wave propagation in fluid-structure interaction (FSI) problems with invariant cross-section acting as waveguides. The 2.5 D approach can, in some cases where the geometry is invariant along one direction, be beneficial especially at high frequency since it only requires the discretization of a 2D problem. The structure is modelled as a thin-walled waveguide [4] using a plate element based on the Kirchhoff-Love and the Reissner-Mindlin formulations. In this way, the structure behaves as a thin plate in the waveguide propagation direction but the shear effect in the cross-section is properly considered. The element shape functions only require C^0 continuity which it results in a simplification regarding C^1 elements [5]. The plate element formulation employs an stabilised auxiliary shear strain field instead of reduced or selective integration procedures to avoid the shear locking [6]. The fluid spectral element is derived from the FEM based on the Helmholtz equation [7]. The proposed method is used to study the wave propagation in an acoustic waveguide with a flexible wall [9,10].

2. Numerical model

The numerical model is based on a two-and-a-half dimensional spectral element formulation for structure (Ω_s) and fluid (Ω_f) subdomains (Figure 1). The structural behaviour is governed by the equilibrium equation for a continuum solid and the fluid by the Helmholtz acoustic wave equation, accounting for Neumann and Dirichlet boundary conditions (BC). The coupling between both formulations is done by imposing appropriate boundary conditions at the fluid-structure interface Γ_q. Equilibrium of normal pressure, compatibility of normal displacement and null shear stresses are imposed at the interface Γ_q. The governing equations and the boundary conditions can be written as follows:

$$\begin{cases} \nabla\boldsymbol{\sigma} + \mathbf{b} = -\omega^2\rho_s\mathbf{u}_s & \text{at}\,\Omega_s\,, \quad \text{with BC} \\ \nabla^2 p + k_f^2 p = 0 & \text{at}\,\Omega_f\,, \quad \text{with BC} \\ \mathbf{u}_s\mathbf{n}_{qs}^T = \mathbf{u}_f\mathbf{n}_{qf}^T & \text{at}\,\Gamma_q \\ \boldsymbol{\sigma}\mathbf{n}_{qs} + p = 0 & \text{at}\,\Gamma_q \end{cases} \tag{1}$$

where the variables in the solid equation are the stress tensor $\boldsymbol{\sigma}$, the body force vector $\mathbf{b}$, the displacement vector $\mathbf{u}_s$, the solid density ρ_s and the angular frequency ω. The Helmholtz equation defines the pressure field p for a fluid wavenumber $k_f = \omega/c_f$, where c_f is the sound wave propagation velocity. Moreover, the coupling conditions include the fluid particle displacements $\mathbf{u}_f$ and the outward solid and fluid normals at Γ_q, $\mathbf{n}_{qs}$ and $\mathbf{n}_{qf}$, respectively. The 2.5D formulation is adressed as the superposition of 2D problems with different longitudinal wavenumber k_z [4,11]. Therefore, the frequency-wavenumber representation depends on (x, y, k_z, ω).

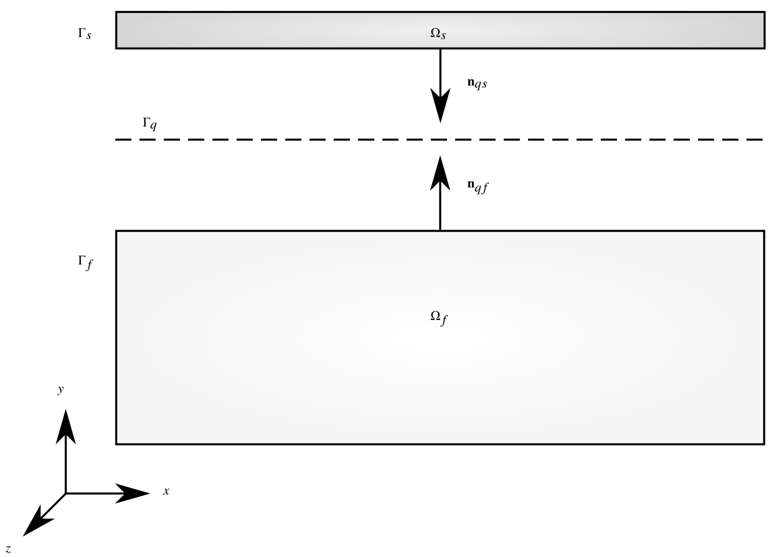
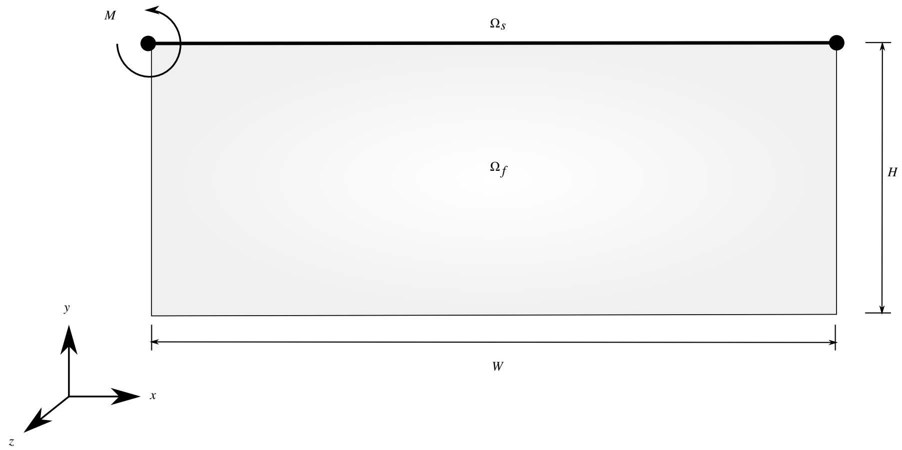


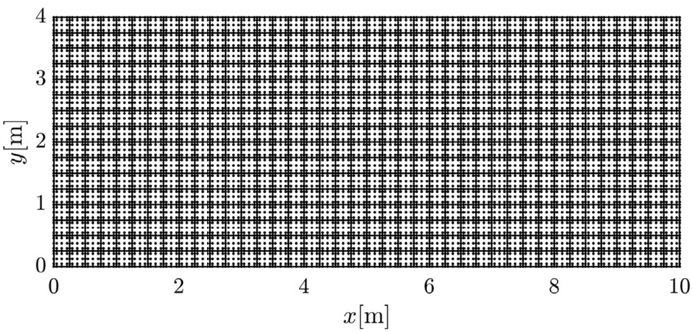
Fig. 1: Fluid-structure subdomains.

3. Numerical example

In this section, an example to show the possibilities of the proposed methodology for studying fluid–structure interaction problems was considered. The example represented an infinite cavity with one flexible side subjected to a harmonic bending moment M (Figure 2). At the remaining three sides, boundary condition $v_n = 0$ were provided. The system dimensions are width $W = 10\,\mathrm{m}$ and height $H = 4\,\mathrm{m}$.



(a)



(b)

Fig. 2: (a) Acoustic cavity with a flexible gate. (b) Discretisation.

In this work, we study the 3D response of the fluid-structure system considering that the cross-section and the problem properties remains invariant along the z direction. The structure was modelled as a simply supported slab of thickness $t = 0.1202\,\mathrm{m}$, Young modulus $E = 2.1 \times 10^{11}\,\mathrm{N/m^2}$, Poisson's ratio $\nu = 0.3$ and density $\rho = 416\,\mathrm{kg/m^3}$. These properties give a bending stiffness $D = Et^3/12(1-\nu^2)$ and a mass per unit length $\overline{m} = \rho t$ equivalent to the beam described in References [9,10]. The problem discretisation was set to $1/h = 4\,\mathrm{m}^{-1}$ using a proper element order to ensure a nodal density per fluid wavelenght $d_\lambda = 6$.

Firstly, the phase velocity of the FSI problem was computed to investigate the propagating modes of the system. Besides, the phase velocities were compared with those obtained from the structure and the fluid cavity neglecting the interaction between them.

Figure 3 shows the phase velocity $C = \omega/k_z$ for a frequency range until 10240 Hz. The slab phase velocities (Figure 3.(a)) exhibits several cut-off frequencies which are in accordance with the natural frequencies of the equivalent simply-supported beam, $f_n = 0.5\pi n^2(EI/\overline{m}W^4)^{0.5}$. Later, the waves become propagatives and tend to the phase velocity C_L of the propagating plane waves in the slab [8]. The phase velocity curves were found to be dispersive over all the frequency range.

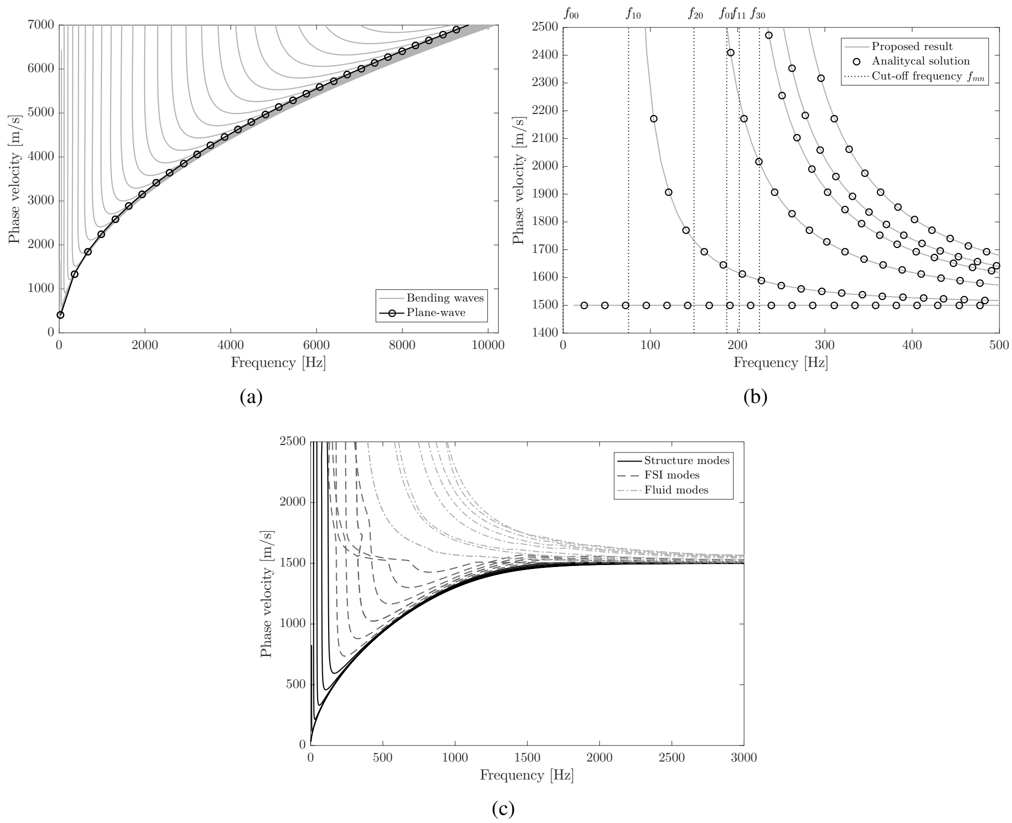


Fig. 3: Phase velocity for (a) the structure, (b) the fluid and (c) the coupled fluid-structure system.

On the other hand, the phase velocities for a rectangular acoustic waveguide with dimension $W \times H$ are given by $C_{mn} = c_f^2 \left((m\pi/k_zW)^2 + (n\pi/k_zH)^2 + 1\right)^{0.5}$, where C_{mn} is the phase velocity related to the propagating acoustic mode at the cut-off frequency $f_{mn} = c_f/2\pi((m\pi/W)^2 + (n\pi/H)^2)^{0.5}$. For the sake of simplicity, Figure 3.(b) only presents the phase velocity curves for the first six acoustic modes given by the frequencies $f_{00} = 0$ Hz, $f_{10} = 75$ Hz, $f_{20} = 150$ Hz, $f_{01} = 187.5$ Hz, $f_{11} = 201.9$ Hz and $f_{31} = 225$ Hz. The first acoustic mode has a non-dispersive phase velocity equals to the sound propagation velocity, c_f, while higher modes are dispersive with a similar behaviour going to c_f above the cut-off frequency f_{mn}.

The phase velocity of the coupled problem provides the structural and the acoustic propagating modes. Figure 3.(c) presents three kinds of phase velocity curves: (i) structure propagating modes with low fluid interaction, (ii) acoustic propagating modes with low interaction with the structure and (iii) coupled fluid-structure interaction modes. The first type of propagating modes exhibits a similar trend to those of the uncoupled slab, with the lowest cut-off frequencies and become non-dispersive for high frequencies. These modes did not reach the plane wave phase velocity C_L, but they went to the sound propagation velocity c_f in a similar way. The second kind of fluid propagating modes also shows similitude with the obtained for the isolated cavity. Finally, the third group has a behaviour between the plate and the fluid responses. The interaction effect between the fluid and the slab modifies the slab response until a frequency where the fluid becomes dominant behaving as a non-dispersive waveguide.

Finally, this section examines the time domain response of the comprehensive problem produced by an impulsive bending moment. The excitation source was modelled as a Ricker pulse with a characteristic frequency of f_m = 3000 Hz, which was defined by $B_f(\omega) = 2\omega^2/\omega_m^3 \exp(-\omega^2/\omega_m^2)$ [12], being $\omega_m = 2\pi f_m$. The 3D solution was assessed as the superposition of problems with different wavenumbers in the interval $-20.1 \leq k_z \leq 20.1$ rad/m. The wavenumber sampling $\Delta k_z = 0.157$ rad/m allowed to compute the solution for a maximum distance $z = \pi/\Delta k_z = 20$ m from the source into a regularly spaced points with $\Delta z = 2\pi/\max(k_z) = 0.312$ m. The time domain solution was evaluated from an inverse Fourier transform of several harmonic problems at the frequency range from 10 Hz to 10240 Hz, with a frequency sampling of $\Delta f = 10$ Hz.

Figures 4 and 5 present the slab deformation and the pressure field at three time steps. Two kinds of waves can be distinguished at $t = 0.00489$ s (Figure 4): a low amplitude wavefront travelling at c_f followed by a slower circumferential wavefront with higher amplitude. This last mentioned wave reached the cavity bottom and was reflected toward the surface generating a high amplitude pulse at the time $t = 0.01095$ s (Figure 5). Then, a new wavefront caused by this pulse spreading along the slab with a typical cone Mach distribution.

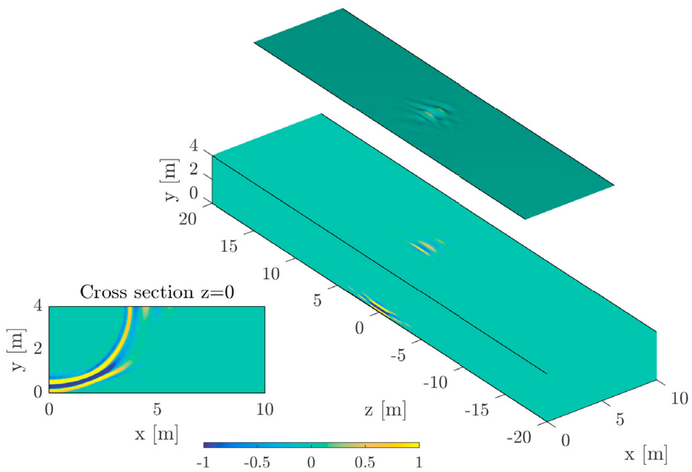


Fig. 4: Time responses of the fluid pressure (4.0×10^{-5} Pa) and the vertical plate displacement (2.3×10^{6} m) wave propagation at $t = 0.00489$ s.

4. Conclusions

This paper has presented a spectral based element method to study guided waves on thin-walled structures with fluid-acoustic interaction. The method was formulated in 2.5D and it is suitable for 3D problems whose material and geometric properties were homogeneous in one direction. Two spectral elements were proposed to represent thin plates and acoustic cavities using Lagrange interpolation polynomials as shape functions at the Legendre-Gauss-Lobatto points.

The wave propagation in a fluid cavity with a flexible gate was analysed. The response of the FSI system showed several structural and acoustic propagating modes above the cut-off frequencies with different phase velocities. These propagating modes became non-dispersive from a frequency where they propagate at the fluid sound propagation velocity. The wave propagation exhibited a complex pattern due to the FSI effects.

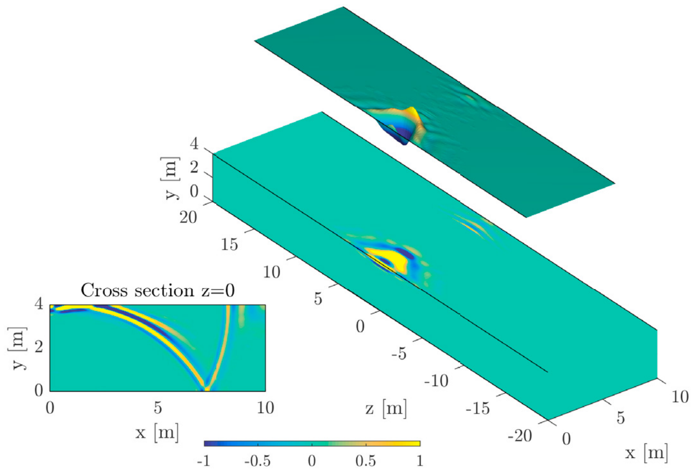


Fig. 5: Time responses of the fluid pressure (4.0×10^{-5} Pa) and the vertical plate displacement (2.3×10^{6} m) wave propagation at t = 0.01095 s.

Acknowledgements

The research work presented herein was supported by the Spanish Ministry of the Economy and Competitiveness (Ministerio de Economía y Competitividad) through research project BIA2013-43085-P and BIA2016-75042-C2-1-R. This research was also supported by the project POCI-01-0247-FEDER-003344, funded by Portugal 2020 through the Operational Programme for Competitiveness Factors (COMPETE 2020). Financial support is gratefully acknowledged. The authors also wish to acknowledge the support provided by the Andalusian Scientific Computing Centre (CICA).

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