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Transmission line representation of electromagnetic field in moving media*

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This paper presents a method using the transimission line theory for obtaining an expression for the propagation constant β for the case when the plasma is uniaxial in nature and is moving with a uniform velocity inside the cylindrical waveguide of inner radius r_0 and outer radius r_1 Both the E and H waves have been considered

1 INTRODUCTION

The propagation of electromagnetic waves along plasma columns has been investigated by Trivelpiece & Gould (1959), Sodomsky (1959), Paik (1962) and others and a great deal of literature is available on both fast and slow wave propagation However, the guiding systems employed previously have all possessed stop bands ranging from the plasma frequency to some higher frequency, the precise value of of the higher frequency depending upon the actual guiding system concerned The propagation of electromagnetic waves along a coaxial line partially filled with plasma was investigated (Pinder 1974) because it was thought that this guiding system would propagate at all frequencies higher than the plasma frequency, thereby enabling the propgation to be studied at frequencies close to the plasma frequency. The propagation of electromagnetic waves along such a coaxial line is best described in terms of a Brillouin diagram, a graph of operating angular frequency, ω against a propagation constant β Suzuki & Fukai (1969) have represented the electromagnetic fields in a moving medium by using the transmission line theory. In this paper we have presented a method using the transmission line theory for obtaining an expression for the propagation contstant β for the case when the plasma is uniaxial in nature and is also moving with a uniform velocity inside a cylindrical waveguide of inner radius r_0 and outer radius r_1.

2 THEORY

The geometry of the problem is as shown in figure 1 As mentioned above the medium is supposed to be a uniaxial, plasma (very similar to biaxial crystal

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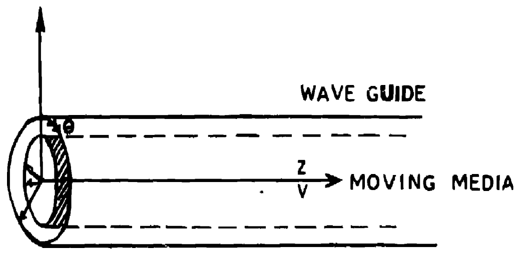

Fig. 1. Geometry of the problem

having a diagonal marix dielectric constant) moving uniformly in the z direction inside the cylindrical waveguide of inner radius r_0 and outer radius r_1 The dielectric constant and permeability for a uniaxial plasma can be written as

$$\overline{\overline{\epsilon}}' = \epsilon_0 \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}, \quad \overline{\overline{\mu}} = \mu \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots (1)$$

Now, for static coordinates the field equations can be written as (Suzuki & Fukai 1969)

$$\left.\begin{aligned} \nabla \times \boldsymbol{H} &= \omega \overline{\overline{\epsilon}} \boldsymbol{E} \\ \nabla \times \boldsymbol{E} &= -j\omega \overline{\overline{\mu}} \boldsymbol{H} \end{aligned}\right\} \quad \ldots (2)$$

$$\left.\begin{aligned} \overline{\overline{\epsilon}} &= \overline{\overline{y}} - \frac{\overline{\overline{\Omega}}_d \overline{\overline{Z}}^{-1} (\nabla + jw\overline{\overline{\Omega}}_d)}{jw} \\ \overline{\overline{\mu}} &= \overline{\overline{Z}} + \frac{\overline{\overline{\Omega}}_b \overline{\overline{y}}^{-1} (\nabla - j\omega\overline{\overline{\Omega}}_d)}{jw} \end{aligned}\right\} \quad \ldots (3)$$

and

$$\overline{\overline{y}} = \epsilon_0 \begin{bmatrix} a\epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & b\epsilon_3 \end{bmatrix}, \quad \overline{\overline{Z}} = \begin{bmatrix} b\mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & a\mu \end{bmatrix}$$

where

$$\overline{\overline{\Omega}}_b = \begin{bmatrix} 0 & 0 & -\Omega_3 \\ 0 & 0 & 0 \\ \Omega_1 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{\Omega}}_d = \begin{bmatrix} 0 & 0 & \Omega_1 \\ 0 & 0 & 0 \\ -\Omega_3 & 0 & 0 \end{bmatrix}$$

$$a = \frac{1-\beta_e^2}{1-\epsilon_1\beta_e^2}, \quad b = \frac{1-\beta_e^2}{1-\epsilon_3\beta_e^2}, \quad \Omega_1 = \frac{\beta_e}{c} \cdot \frac{\epsilon_1-1}{1-\epsilon_1\beta_e^2}, \quad \Omega_3 = \frac{\beta_e}{c} \cdot \frac{\epsilon_3-1}{1-\epsilon_3\beta_c^2}, \quad \beta = \frac{u}{c}$$

Now in the cylindrical coordinates the differential operator may be represented as

$$\bar{\bar{\nabla}} = \begin{bmatrix} 0 & -\frac{\partial}{\partial r} & \frac{\partial}{\partial z} \\ \frac{1}{r}\left(1+\frac{\partial}{\partial r}\right) & 0 & -\frac{1}{r}\frac{\partial}{\partial \theta} \\ -\frac{\partial}{\partial z} & \frac{1}{r}\frac{\partial}{\partial \theta} & 0 \end{bmatrix} \quad \ldots (4)$$

Assuming the phase dependence as $\exp(\omega t-\beta z)$ and representing the differential operator as $(\partial/\partial r) = \nabla_r$, the equivalent dielectric constant and permeability can be represented as

$$\bar{\bar{\epsilon}} = \begin{bmatrix} a\epsilon_0\epsilon_1 - \frac{\Omega_1(\omega\Omega_1+\beta)}{\omega\mu a} & 0 & 0 \\ 0 & \epsilon_0\epsilon_2 & 0 \\ 0 & -\frac{\Omega_3\nabla r}{j\omega\mu b} & \epsilon_3 b\epsilon_0 - \frac{\Omega_3(\omega\Omega_3+\beta)}{\omega\mu b} \end{bmatrix} \quad \ldots (5)$$

$$\bar{\bar{\mu}} = \begin{bmatrix} b\mu - \frac{\Omega_3(\omega\Omega_3+\beta)}{\omega\epsilon_0\epsilon_3 b} & 0 & 0 \\ 0 & \mu & 0 \\ & -\frac{\Omega_1\nabla r}{j\omega\epsilon_0 \epsilon_1 a} & a\mu - \frac{\Omega_1(\omega\Omega_1+\beta)}{\omega a\epsilon_0\epsilon_1} \end{bmatrix} \quad \ldots (6)$$

Now in order to represent the electromagnetic field in terms of the line parameters we assume that when $H_z = 0$, the propagation wave is E wave In the same way when $E_z = 0$, it is the H wave which is propagating Further for E waves we assume that mode voltage V_1 is proportional to the electric field E_z and the mode current I_1 is proportional to the electric field $E\phi$ Also for H waves, current I_2 is proportional to the field H_z and the mode voltage V_2 is proportional to E_0. Thus we can obtain the propagation equations for E and H wave which will be the function of r For E waves we have

$$-\frac{\partial V_1}{\partial r} = j\frac{a_{11}}{r} I_1 + b_{11} V_1, \quad \ldots (7)$$

$$-\frac{\partial I_1}{\partial r} = jC_1 r V_1 \quad \ldots (8)$$

$$a_{11} = \frac{\omega^2\epsilon_{33}\mu_{11} - \beta^2}{\omega\epsilon_{33}}, \quad b_{11} = -j\beta\frac{\epsilon_{32}}{\epsilon_{33}}, \quad C_{11} = \omega\epsilon_{12}$$

From eqs (7) and (8) we can get

$$\left\{ \frac{1}{r} \nabla_r (r \nabla_r) + k_c^2 \right\} v_1 = 0 \,, \quad \ldots \quad (9)$$

where

$$k_c^2 = \frac{1}{b} \frac{\epsilon_2}{\epsilon_3} \{(k_0 b)^2 \epsilon_3 - (\omega\Omega_3 + \beta)^2\}, \quad k_0^2 = \omega^2 \mu_0 \epsilon_0$$

Now rewriting eq (8) as

$$I_1 = \frac{j\omega\epsilon_0\epsilon_2\gamma}{k_0^2} \nabla_r v_1, \quad \ldots \quad (10)$$

We can calculate the voltage V_1 and the current I_1 and these can be represented as

$$\left.\begin{aligned} V_1 &= A J_0(k_0 V) \\ I_1 &= -j \frac{A\,\omega\epsilon_0\epsilon_2 r}{k_c} J_1(k_c r) \end{aligned}\right\}, \quad \ldots \quad (11)$$

where J_0 and J_1 are the Bessel functions of zeroth and first order respectively

Thus the input impedance at $r = r_0$ may be represented as

$$Z_{in}(r_0) = -\frac{V_1(r_0)}{I_1(r_0)} = \frac{-jk_c}{j\omega\epsilon_0\epsilon_2} \frac{J_0(k_0 r_0)}{J_1(k_0 r_0)} \,, \quad \ldots \quad (12)$$

and similarly the output impedance can be represented as

$$Z_0(r_0) = j \frac{\Omega_c}{\omega\epsilon_0 r_0} \epsilon_n (\Omega_0 r_0, \Omega_0 r_1), \quad \ldots \quad (13)$$

$$\Omega_0^2 = k_0^2 - \beta^2$$

$$\epsilon_n(x, y) = \frac{J_0(x) N_0(y) - J_0(y) N_0(x)}{J_1(x) N_0(y) - J_0(y) N_1(x)}$$

Then in order that the longitudinal resonance may take place inside the wave-guide the total impedance should be zero So, $Z_{in} + Z_0 = 0$ and from eqs (12) and (13) we get

$$\frac{k_c}{\epsilon_2} \cdot \frac{J_0(k_c r_0)}{J_1(k_c r_0)} = \Omega_0 \epsilon_n (\Omega_0 r_0, \Omega_0 r_1), \quad \ldots \quad (14)$$

from which the propagation constant β may be calculated Similarly for H waves we have

$$-\frac{\partial V_2}{\partial r} = j a_{22} I_2 \quad \ldots \quad (15)$$

$$-\frac{\partial I_2}{\partial r} = j \frac{C_{22}}{r} V_2 + d_{22} I_2 \quad \ldots \quad (16)$$

where

$$a_{22} = \omega\mu_{22}, \quad C_{22} = \frac{\omega^2\epsilon_{11}\mu_{33} - \beta^2}{\omega\mu_{33}}, \quad d_{22} = -j\beta\,\frac{\mu_{32}}{\mu_{33}}$$

Proceeding as before, from eqs (15) and (16) we may get an expression for the propagation constant β as

$$\left\{\frac{1}{r}\nabla r(V\nabla_r) + k'^2_c\right\} I_2 = 0$$

$$k'^2_c = \frac{1}{a}\{\epsilon_1 k_0^2 a^2 - (\omega\Omega_1 + \beta)^2\}. \qquad \ldots \quad (17)$$

3. Summary

In this paper we have presented a method using the transmission line theory for obtaining an expression for the propagation constant β for the case when the plasma is uniaxial in nature and is moving with a uniform velocity inside a cylindrical waveguide of inner radius r_0 and outer radius r_1. Both the E and H waves have been considered.

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