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Effect of electron beam and temperature anisotropy on Alfvén waves

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Abstract : Dispersion relation and associated field-aligned currents are evaluated for Alfvén waves in the auroral region using particle aspect approach Effect of temperature anisotropy has been examined on the wave and the applicability of the finding is discussed for the magnetosphere-ionosphere coupling.

Keywords Alfvén wave, magnetosphere-ionosphere coupling, electron-beam effect

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1. Introduction

Among the first type of waves found to exist in plasma, Alfvén waves play an important role in energy transport, in particle acceleration and heating in the earth's magnetosphere, in solar flares and the solar wind. In particular, a series of spacecraft have directly detected strong Alfvén wave turbulence associated with particle energization in interplanetary space such as the auroral region [1–6].

Field-aligned currents play a dominant role in the study of magnetized plasmas of magnetosphere-ionosphere coupling. In the magnetohydrodynamic description (valid where time and spatial scales of motion are large compared to the gyroperiod and gyroradius, respectively), if perturbations of flow develop on one part of a flux tube, field-aligned currents must flow in order to communicate the charges to the entire flux tube. They are perhaps of most importance in magnetospheric physics in the study of coupling between regions where different dynamical conditions prevail but which are threaded by the same field [7]. Under this condition, Alfvén wave may be generated in the plasma sheet and propagate towards the ionosphere leading the auroral process.

In most of the theoretical work, the velocity distribution functions have been assumed to be ideal Maxwellian although most turbulent heating experiments like mirror devices, allow non-Maxwellian, particularly loss-cone distribution functions [8]. Thus, the object of the present work is to investigate what kind of effects the non-Maxwellian distribution may have on the current driven Alfvén waves.

In the recent past, particle aspect analysis has been developed for the analysis of Alfvén wave [9] which is based upon Dawson's theory of Landau damping which was further extended by Terashima [10], Misra and Tiwari [11], Tiwari *et al* [12], Varma and Tiwari [13] to the analysis of electrostatic and electromagnetic instabilities The present work is attributed to investigate the effect of electron beam in an anisotropic plasma on the Alfvén wave for the magnetosphere-ionosphere coupling.

2. Basic trajectories

We consider a plasma under static magnetic field B_0 (z-direction) in which collisions between particles are neglected. We shall discuss the behaviors of an Alfvén wave of plane polarization in the form :

$$k \parallel B_0,\ k.B = 0,\ k = (0,0,k_\parallel) \text{ and } E = (E_x,0,0) \quad (1)$$

with $\quad E_x = E_1 \cos(k_\parallel z - \omega t)$. (2)

$$B_y = \frac{E_1 k_\parallel c}{\omega} \cos(k_\parallel z - \omega t), \quad (3)$$

where E_x and B_y are the electric and magnetic fields of the wave. Here, the frequency ω is assumed to be real and the

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amplitude E_1 is treated as the slowly varying function of t. c is the velocity of light

$$\frac{1}{E_1}\frac{dE_1}{dt} \ll \omega .$$

B_0 is directed along the z-axis and wave propagates in the direction of ambient magnetic field. The wave is assumed to start at $t = 0$ when resonant particles are not disturbed. We begin with the equation of motion for the particles as

$$m\frac{dv}{dt} = q\left[E_1 + \frac{1}{c}v\times(B_0 + B_y\hat{y})\right], \tag{4}$$

where q is the charge and m is the mass of the particle. $\hat{y}$ is the unit vector along the y direction.

The Gaussian system of units is adopted in this paper and interactions between particles are neglected. The electric field E_1 on the right hand side is considered to be a small perturbation and v can be expressed as a sum of the unperturbed velocity V and the perturbed velocity u, *i.e.*, $v = V + u$, u is determined by the following set of equations :

$$\frac{du_+}{dt} + i\Omega u_+ = \frac{q}{m}\left[1 - \frac{V_\parallel k_\parallel}{\omega}\right]E_1\cos(k_\parallel z - \omega t),$$

$$\frac{du_\parallel}{dt} = \frac{q}{m}V_\perp\frac{E_1 k_\parallel}{\omega}\cos(\theta - \Omega t)\cos(k_\parallel z - \omega t), \tag{5}$$

where $u_+ = u_x + iu_y$ and $\Omega = qB_0/mc$.

We solve eq. (5) in the approximation by replacing the coordinates of the particles on the right hand side by those of free gyration, u_x and u_y are the perturbed velocities in the x and y directions respectively. E_1 and B_y are slowly varying quantities and treated as constants. Our method follows that of Terashima [10]. We start taking the trajectories of free gyration as,

$$z = z_0 + V_\parallel t,$$

$$V_x(t) = V_\perp\cos(\theta - \Omega t),$$

$$V_y(t) = V_\perp\sin(\theta - \Omega t),$$

$$V_z(t) = V_\parallel = \text{constant}, \tag{6}$$

where z_0 is the initial position of the particles in the z direction, θ is initial phase of gyration. Eq. (5) is solved for perturbed velocities $u(r, t)$ of charged particles in the presence of Alfvén wave as described by Baronia and Tiwari [9] which is given as

$$u_x(r,t) = \frac{q}{m}\left[1 - \frac{V_\parallel k_\parallel}{\omega}\right]E_1\left[\frac{\Lambda_0}{a_0^2}\sin(k_\parallel z - \omega t)\right.$$

$$\frac{\delta}{2(\Lambda_0 - \Omega)}\sin(k_\parallel z - \omega t - \Lambda_0 t + \Omega t)$$

$$\left. - \frac{\delta}{2(\Lambda_0 + \Omega)}\sin(k_\parallel z - \omega t - \Lambda_0 t - \Omega t)\right],$$

$$u_y(r,t) = \frac{q}{m}\left[1 - \frac{V_\parallel k_\parallel}{\omega}\right]E_1\left[\frac{\Lambda_0}{a_0^2}\cos(k_\parallel z - \omega t)\right.$$

$$2(\Lambda_0 - \Omega)\cos(k_\parallel z - \omega t - \Lambda_0 t + \Omega t)$$

$$\left. + \frac{\delta}{2(\Lambda_0 + \Omega)}\cos(k_\parallel z - \omega t - \Lambda_0 t - \Omega t)\right],$$

$$u_z(r,t) = \frac{q}{m}\frac{V_\perp E_1 k_\parallel}{\omega}\frac{1}{2}\left[\frac{1}{(\Lambda_0 - \Omega)}\sin(k_\parallel z - \omega t + \theta - \Omega t)\right.$$

$$- \frac{1}{(\Lambda_0 + \Omega)}\sin(k_\parallel z - \omega t - \theta + \Omega t)$$

$$- \frac{\delta}{(\Lambda_0 - \Omega)}\sin(k_\parallel z - \omega t - \Lambda_0 t + \theta)$$

$$\left. - \frac{\delta}{(\Lambda_0 + \Omega)}\sin(k_\parallel z - \omega t - \Lambda_0 t - \theta)\right] \tag{7}$$

where $\delta = 0$ for the non-resonant particles and $\delta = 1$ for resonant one and

$$\Lambda_0 = V_\parallel k_\parallel - \omega, \quad a_0^2 = \Lambda_0^2 - \Omega^2 . \tag{8}$$

The resonant particle condition is given [9] by $V_\parallel k_\parallel - \omega = 0$ These equations represent the perturbed velocities of the charged particles in the presence of Alfvén waves and have vast application in plasma heating processes, confinement devices and the space plasmas.

3. Density perturbation

To evaluate the density perturbation affected by velocity perturbation due to Alfvén waves, we consider a group of particles with the same initial condition and let its number density be

$$n(r,t,V) = N(V) + n_1(r,t,V), \tag{9}$$

where N is the zeroth order distribution and n_1 is perturbed density which can be derived from the equation [9,10]

$$\frac{dn_1}{dt} = -N(V)\nabla_z u_z . \tag{10}$$

The expression on the right hand side is converted as function of t and initial parameters [9,10] and after integration the non-resonant particle density is given as

$$n_1(r,t,V) = \frac{qV_\perp E_1 k_\parallel^2 N(V)}{2m\omega}\left[\frac{1}{(\Lambda_0 - \Omega)^2}\sin(k_\parallel z - \omega t + \theta - \Omega t)\right.$$

$$\left. - \frac{1}{(\Lambda_0 + \Omega)^2}\sin(k_\parallel z - \omega t - \theta + \Omega t)\right] \tag{11}$$

Similarly, for the resonant particles

$$n_1(r,t,V) = -\frac{qV_\perp E_1 k_\parallel^2 N(V)}{2m\omega}$$

$$\times\left[\frac{1}{(\Lambda_0-\Omega)^2}\sin(k_\parallel z-\omega t+\theta-\Omega t)\right.$$

$$+\frac{1}{(\Lambda_0+\Omega)^2}\sin(k_\parallel z-\omega t-\theta+\Omega t)$$

$$-\frac{1}{(\Lambda_0-\Omega)^2}\sin(k_\parallel z-\omega t-\Lambda_0 t+\theta)$$

$$-\frac{1}{(\Lambda_0+\Omega)^2}\sin(k_\parallel z-\omega t-\Lambda_0 t-\theta)$$

$$-\frac{t}{(\Lambda_0-\Omega)}\cos(k_\parallel z-\omega t-\Lambda_0 t+\theta)$$

$$\left.-\frac{t}{(\Lambda_0+\Omega)}\cos(k_\parallel z-\omega t-\Lambda_0 t-\theta)\right]. \quad (12)$$

To evaluate the dispersion relation and current, we can hereafter take the zeroth order distribution [13,14] $N(V)$ of the form

$$N(V) = N_0 f_\perp(V_\perp) f_\parallel(V_\parallel), \quad (13)$$

$$f_\perp(V_\perp) = \left[\frac{m}{2\pi T_\perp}\right] \exp\left\{-\frac{mV_\perp^2}{2T_\perp}\right\}$$

$$f_\parallel(V_\parallel) = \left[\frac{m}{2\pi T_\parallel}\right]^{1/2} \exp\left\{-\frac{m(V_\parallel - V_0)^2}{2T_\parallel}\right\}$$

where $T_\parallel$ and $T_\perp$ are the parallel and perpendicular temperatures with respect to the ambient magnetic field, V_0 is beam velocity and N_0 is the background plasma density.

4. Current

To evaluate the perturbed current per unit wavelength in the presence of Alfvén wave, we use the following set of equations

$$J_{i,e} = \int_0^\lambda ds \int_0^\infty 2\pi V_\perp \, dV_\perp \int_{-\infty}^{+\infty} dV_\parallel$$

$$\times e\left[(N+n_1)(V+u) - NV\right]_{i,e}$$

$$\text{and} \quad J_{i,e} = J_i - J_e, \quad (14)$$

where ds represents the length of magnetic field line elements and $ks = k.r$.

Eq. (14) considers current per unit wavelength and not the current density per square centimeter [9,10,17]. The right hand side terms of eq. (14) involves density perturbation n_1 and the velocity perturbation u which contain the oscillatory terms. If the integral $\int ds$ is not performed, the evaluated current will be oscillatory. To find out the average values, the current is evaluated per unit wavelength. These currents represent the distributed currents over the entire wavelength and eq. (14) represents the average value of this distributed current over a wavelength of the Alfvén wave.

With the help of eqs. (7), (11) and (14), we obtain the ionic current per unit wavelength as

$$J_{zi} = -\frac{3e\omega_{pi}^2 E_1^2 k_\parallel^2 V_{T\perp i}^2}{8m_i \omega \Omega_i^4}. \quad (15)$$

Similarly, the electron current per unit wavelength can be written as

$$J_{ze} = \frac{3e\omega_{pe}^2 E_1^2 k_\parallel^2 V_{T\perp e}^2}{8m_e \omega \Omega_e^4}. \quad (16)$$

Thus, the total current per unit wavelength flowing along the magnetic field in the presence of Alfvén wave is given as

$$J_z = -\frac{3ek_\parallel^2 E_1^2}{8m_e\omega}\left[\frac{\omega_{pe}^2 V_{T\perp e}^2}{\Omega_e^4} + \frac{\omega_{pi}^2 V_{T\perp i}^2}{\Omega_i^4(m_i/m_e)}\right] \quad (17)$$

The average values of perpendicular currents become zero in the first order. Here $q = +e$ for ions and $-e$ for electrons is used and $\omega_{pi,e} = (4\pi N_0 e^2/m_{i,e})^{1/2}$ is the plasma frequency. $V_{T\perp i,e} = (2T_{\perp i,e}/m_{i,e})^{1/2}$ is the perpendicular component of thermal velocity where the temperature T is expressed in the unit of energy, $\Omega_{i,e}$ is the cyclotron frequency.

5. Dispersion relation

The dispersion relation is evaluated with help of wave equation in the form

$$\nabla(\nabla.E_1) - \nabla^2 E_1 = -\frac{4\pi}{c^2}\frac{\partial J_1}{\partial t} - \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2} \quad (18)$$

where J_1 is the first order current density. In the case of plane polarized Alfvén wave the wave equation becomes

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{4\pi}{c^2}\frac{\partial J_x}{\partial t} + \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \quad (19)$$

Substituting the value of J_x in terms of perturbed velocity u_x and perturbed density n_1, the dispersion relation for Alfvén wave is evaluated as

$$\omega = k_\parallel V_A \left[1 - \frac{(V_{T\parallel e}^2 + V_0^2)}{V_A^2}\right]^{1/2} \quad (20)$$

where $V_{T\parallel e} = (T_{\parallel e}/m_e)^{1/2}$ is the electron thermal velocity parallel to the magnetic field. In the anisotropic plasma the dispersion relation can be written as

$$\omega = k_\parallel V_A \left[1 - \frac{\{(T_{\parallel e}/T_{\perp e}).V_{T\perp e}^2/2 + V_0^2\}}{V_A^2}\right]^{1/2} \quad (21)$$

Here, we notice that the dispersion relation of Alfvén wave is modified by thermal and beam velocities of electrons. In case both the terms are zero, the dispersion relation reduces to well known form. The current driven by Alfvén wave is modified through the dispersion relations. $V_A = B_0/(4\pi N_0 m_i)^{1/2}$ is the Alfvén wave speed. In this model, the wave frequency ω is considered as real and the principal part of plasma dispersion function is used and coupling of compressional mode is not considered [9].

6. Results and discussion

The following plasma parameters are used to estimate the dispersion relation and associated current per unit wavelength which may be suitable to auroral acceleration region [15–17]. B_0 = 4300 nT; Ω_i = 412 s^{-1}; N_0 = 5.0 × $10^3/m^3$; E_1 = 50 mV/m; $V_{T\perp i}$ = 3.5 × 10^4 m/s. Expressions for field-aligned current per unit wavelength and the dispersion relations are evaluated and results are presented in figures.

The electron beams are injected from the tail side of the magnetosphere at the substorm times constituting field-aligned currents and auroral acceleration [15–17]. In the same event, the Alfvén waves are also observed by various rockets and satellites, therefore, the electron beam may be the cause of Alfvén wave generation which modifies the wave frequency and the anisotropy of plasma sheet and auroral acceleration region may affect the field aligned-current and the wave spectrum.

The Figure 1 shows the variation of real frequency ω with $k_{\|}$ for different values of electron beam velocity V_0. It is seen

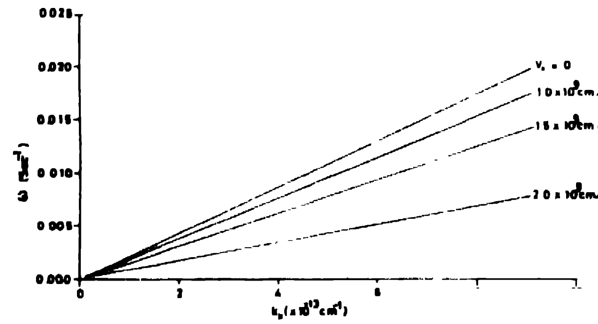

Figure 1. Wave frequency ω *versus* wave number $k_{\|}$ for different V_0

that the effect of the electron beam is to reduce the frequency when the electron beam is along the magnetic field and in the direction of propagating vector $k_{\|}$. It is the wave velocity which gets modified in the presence of beam and anisotropy in the plasma which is a dispersionless medium for the Alfvén wave.

Figure 2 predicts the variation of real frequency ω with $k_{\|}$ for different values of temperature anisotropy ($T_{\|e}/T_{\perp e}$). Here, we notice that the wave frequency increases with $k_{\|}$ but decreases with the increase of temperature anisotropy. These curves show that the phase velocity is smaller in hot plasmas. Thus, the temperature anisotropy observed in the distant magnetotail and the auroral acceleration region also modifies the wave spectrum.

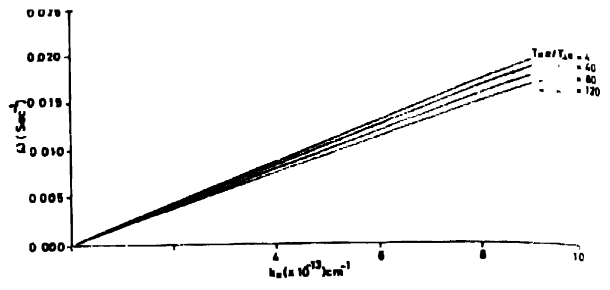

Figure 2. Wave frequency ω *versus* wave number $k_{\|}$ for different temperature anisotropy

Figure 3 shows the variation of field-aligned current per unit wavelength with $k_{\|}$ for different values of electron beam velocity. It is observed that current decreases with increase of $k_{\|}$ as well as V_0. The effect of electron beam parallel to

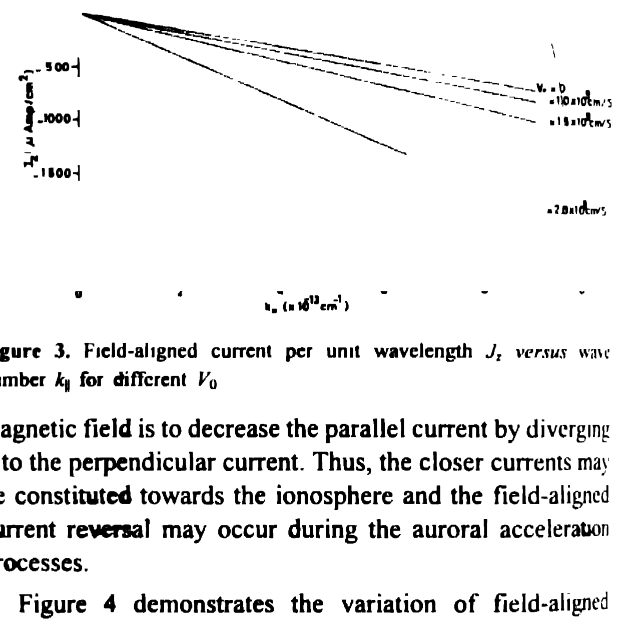

Figure 3. Field-aligned current per unit wavelength J_z *versus* wave number $k_{\|}$ for different V_0

magnetic field is to decrease the parallel current by diverging it to the perpendicular current. Thus, the closer currents may be constituted towards the ionosphere and the field-aligned current reversal may occur during the auroral acceleration processes.

Figure 4 demonstrates the variation of field-aligned current per unit wavelength with $k_{\|}$ for different values of temperature anisotropy. The current decreases with the increase of $k_{\|}$ as well as temperature anisotropy. It is also clear that the current increases with the phase velocity of the

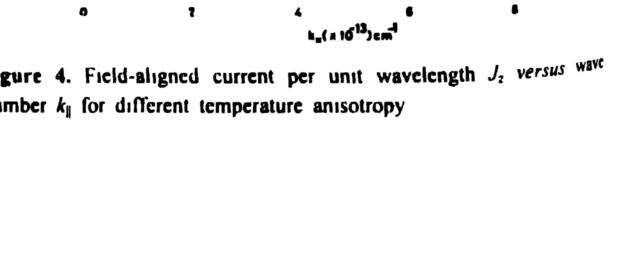

Figure 4. Field-aligned current per unit wavelength J_z *versus* wave number $k_{\|}$ for different temperature anisotropy

Alfvén wave. Thus, the field-aligned current is limited by the electron beam in the anisotropic magnetosphere.

The theory may be applicable to the auroral ionospheric regions wherever the field-aligned currents are reported alongwith the particle precipitation [15–17]. The presence of field-aligned current in auroral ionosphere can permit short wavelength instabilities to lower altitudes. Down flow of electron beam alongwith energetic particle precipitation may reduce the Alfvén wave frequencies. The study may also be useful to the experimental devices with current carrying plasmas in the presence of Alfvén waves. Although, many theoretical attempts have been made to investigate the Alfvén wave propagation, the single particle theory may be able to explain some of the plasma phenomena where other theories are not well suited [12].

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References

[1] A V Marchenko, R E Denton and M K Hudson *Phys Plasmas* 3 3861 (1996)

[2] E M Dubinin, A S Volokitin, P L Israelvich and N S Nokolaeva *Planet Space Sci* **36** 949 (1988)

[3] P Louarn, J E Wahlund, T Churt, H Feraudy and A Roux *Geophys. Res. Lett* **21** 1847 (1994)

[4] J E Wahlund, P Louarn, T Churt, H Feraudy and A Raux *Geophys. Res Lett.* **21** 1831 (1994)

[5] M Volwerk, P Louarn, T Churt, H Feraudy, A Raux and B Holback *J Geophys Res* **101** 1335 (1996)

[6] G L Huang, D Y Wang, D J Wu, H Feraudy, D Le-Queau, M Volwerk and B Holback *J Geophys Res* **102** 7217 (1997)

[7] D J Sothwood and M G Kivelson *J. Geophys Res* **96** 67 (1981)

[8] M S Tiwari and P Varma *Planet Space Sci* **41** 199 (1993)

[9] A Baronia and M S Tiwari *Indian J. Phys* **73B** 499 (1999)

[10] Y Terashima *Prog Theor Phys* **37** 775 (1967)

[11] K D Misra and M S Tiwari *Can J Phys* **57** 1124 (1979)

[12] M S Tiwari, R P Panday and K D Misra *J Plasma Phys* **34** 163 (1985)

[13] P Varma and M S Tiwari *Phys Scr.* **44** 296 (1991)

[14] X Y Wang, X Z Liu, Z Y Li and X B Zhang *Phys Plasmas* **5** 836 (1998)

[15] M S Tiwari and G Rostoker *Planet Space Sci.* **32** 1497 (1984)

[16] A Baronia and M S Tiwari *Planet Space Sci* **47** 1111 (1991)

[17] A Baronia and M S Tiwari *J Plasma Phys* **63** 311 (2000)