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Acoustic-wave amplification in semiconductors

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The theoretical analysis of the amplification of an acoustic-wave in a semiconductor which exhibits both piezoelectric and deformation potential coupling is made. The expression for the critical electron drift velocity for the acoustic amplification is derived. The effect of the ratio of the effective masses of the charge carriers on the transverse phonon-helicon wave interaction giving rise to the amplification is also studied.

1 INTRODUCTION

In their previous paper the authors (Ram Chandra & Verma 1974) studied theoretically the interaction of the drifted electrons with the acoustic waves in piezoelectric semiconductor which shows the negative differential mobility. It was pointed out that acoustic waves can be amplified even if their velocities are less than the drift velocity of the electrons in such a medium. However in the above analysis, the effect of the deformation potential coupling was neglected. Since all solids exhibit deformation potential it seems necessary to include the effect of this potential. Moreover, while studying the transverse acoustic-wave interaction generally the ratio of the effective masses of the charge carriers is not included. Since for semiconductors this ratio cannot be neglected and therefore its effect must be included. These are the few points of the present investigation. In sec. 2 an expression for the gain is derived which accounts for both deformation and piezoelectric couplings. The expression for the critical electron drift velocity for the acoustic-wave amplification is given in sec. 3. The effect of the ratio of the effective masses of the charge carriers on the amplification factor in transverse phonon-helicon wave interaction is given in sec. 4. It is shown that under some conditions the results agree with the earlier work.

2 THEORY

We assume that the acoustic-wave propagates along the x-direction of the semiconductor crystal assumed to be homogeneous, isotropic and infinite in extent. The d.c. electric field E_0 is also applied along the x-axis. The analysis is essentially one dimensional.

The expressions for the stress T and electric displacement D are given (Steele & Vural 1969) by

$$T = cS - eE - \left(\frac{C_{d}e}{q}\right)\frac{\partial E}{\partial x}, \quad \ldots (1)$$

$$D = eS + \epsilon E - \left(\frac{C_{d}\epsilon}{q}\right)\frac{\partial S}{\partial x}, \quad \ldots (2)$$

where c is the elastic constant at constant electric field, ϵ the dielectric permittivity at constant strain S, E the electric field, e the piezoelectric constant, C_d the deformation potential and q the electronic charge

With the help of eqs (1) and (2), the coupling equations can be written as

$$\epsilon\frac{\partial E_1}{\partial x} + e\frac{\partial^2 u}{\partial x^2} - \left(\frac{C_{d}\epsilon}{q}\right)\frac{\partial^3 u}{\partial x^3} = -qn_1 \quad \ldots (3)$$

and

$$\rho\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2} - e\frac{\partial E_1}{\partial x} - \left(\frac{C_{d}\epsilon}{q}\right)\frac{\partial^2 E_1}{\partial x^2}, \quad \ldots (4)$$

$$\frac{\partial n_1}{\partial t} + v_0\frac{\partial n_1}{\partial x} + n_0\mu'\frac{\partial E_1}{\partial x} + D_n\frac{\partial^2 n_1}{\partial x^2} = 0 \quad \ldots (5)$$

Eq (5) is the linearized continuity equation written in the collision-dominated limit under the approximation that diffusion D_n is independent of the electric field. In the above equations, E_1 is the rf electric field, n_1 the space charge density, n_0 the equilibrium density, ρ the mass density of the medium, u the lattice displacement which is in the x-direction, $\mu' = \partial(\mu E)/\partial E$ the differential mobility and $v_0 = -\mu_0 E_0$ the electron drift velocity

Assuming the variation of n_1, E_1 and u of the form $\exp[j(\omega t - kx)]$ and combining eqs (3) to (5), we obtain

$$\rho\omega^2 = c^* k^2, \quad \ldots (6)$$

where

$$c^* = c\left[1 + \frac{e^2}{\epsilon c}\,\frac{\left\{1 + \left(\frac{C_d k \epsilon}{eq}\right)^2\right\}}{1 + \frac{jqn_0\mu'}{\epsilon(\omega - kv_0 + jD_n k^2)}}\right] \quad \ldots (7)$$

Eq. (6) can be solved for the attenuation constant, α, after making an approximation

$$k = (\omega/v_s) + j\alpha \quad \ldots (8)$$

in eq (7). In terms of the complex elastic constant c, the attenuation constant is

$$\alpha = (\omega/v_s)c^{\frac{1}{2}}\,\mathrm{Im}(c^{*-\frac{1}{2}}) \quad \ldots \quad (9)$$

and is such that $|\alpha| << |\omega/v_s|$, where v_s is the sound wave velocity.

After combining eqs. (7) to (9), we obtain

$$\alpha \cong \frac{\frac{1}{2}\{K^2+L^2(\omega/v_s)^2\}(\omega_c/\omega)Y(\mu'/\mu_0)}{Y^2+(\omega_c/\omega)^2\{(\mu'/\mu_0)+\omega^2/\omega_c\omega_D\}^2}, \quad \ldots \quad (10)$$

where $K^2 = e^2/\epsilon c << 1$, $\omega_c = qn_0\mu_0/\epsilon$ is the dielectric relaxation frequency, $\omega_D = v_s^2/D_n$ is the diffusion frequency, $Y = 1-(v_0/v_s)$ and $L^2 = (C_d/q)^2(\epsilon/e)$. The amplification mechanism is the same as discussed in the earlier work (Ram Chandra & Verma 1974). From the expression (10), it is clearly seen that the inclusion of the deformation potential in the coupling equations help in the increment of the gain and it increases monotonically with the frequency. For $\alpha = 0$, i.e., in the limit when the deformation potential is neglected, the expression for α reduces to that obtained by Ram Chandra & Verma (1974).

From eq. (10), we can compare the gain per unit length obtained by purely piezoelectric coupling and by deformation potential coupling. We have

$$\frac{(\mathrm{Gain})_{deform.}}{(\mathrm{Gain})_{piezo.}} = \frac{L^2\omega^2}{K^2v_s^2} = \frac{\omega^2}{\omega_{c1}{}^2} << 1 \quad . \quad (11)$$

3. Critical Electron Drift Velocity

As it is well known that acoustic amplification occurs in piezoelectric semiconductors when the electron drift velocity exceeds a critical value. If the medium exhibits the negative differential mobility, the amplification condition requires that the acoustic-wave velocity must be greater than this critical electron drift velocity. Moore & dePian (1967) pointed out that an exact solution for the critical velocity can be found as a function of the frequency of the acoustic signal. They obtained a condition for which c^*, the effective elastic constant, becomes a real quantity so as to obtain purely real velues of k. The condition so obtained determines the critical electron drift velocity. As a result of their analysis they obtained the following expression for the critical drift velocity v,

$$v = v_s\left(1+\frac{e^2}{\epsilon c}\,\frac{(\omega^2/\omega_c\omega_1)}{\{1+\omega^2/\omega_c\omega_1\}}\right)^{\frac{1}{2}}, \quad \ldots \quad (12)$$

where

$$\omega_1 = \frac{v_s^2(1+e^2/\epsilon c)}{fD_n} \quad \ldots \quad (13)$$

Here f denotes the fraction of the mobile electrons. If all the space charge is due to the conduction electrons (in absence of traps) then $f = 1$

Following Moore & dePian (1967), we obtain the following expression for the critical drift velocity, v_1,

$$v_1 = v_s \left[1 + \frac{(e^2/\epsilon c) A_1 \left\{ A_2 + \dfrac{2L^2\omega^2}{v'^2_1} \right\} + \dfrac{A_1 L^2 \omega^2}{v'^2_1}}{A_1 \left\{ A + \dfrac{L^2\omega^2}{v'^2_1} \right\} + A_2} \right]^{\frac{1}{2}} \quad \ldots \quad (14)$$

where $A_1 = (\mu_0/\mu')(\omega^2/\omega_c \omega_1)$, $A_2 = 1 + e^2/\epsilon c$, $v'^2_1 = v_s^2 A_2$ and ω_1 is given by eq (13) for $f = 1$

The critical electric field strength E'_0 is given by

$$E'_0 = v_1/\mu_0.$$

It can be seen that for $\alpha = 0$, i e, when the coupling is purely piezoelectric, expression (14) for the critical drift velocity v_1 reduces to that obtained by Moore & dePian (1967) for $\mu_0 = \mu'$. For the purely deformation potential coupling, eq (14) reduces to

$$v_1 = v_s \left[1 + \frac{(\mu_0/\mu')(\omega^2/\omega_c \omega_D) \dfrac{L^2\omega^2}{v_s^2}}{(\mu_0/\mu')(\omega^2/\omega_c \omega_D) \left\{ 1 + \dfrac{L^2\omega^2}{v_s^2} \right\} + 1} \right]^{\frac{1}{2}} \quad \ldots \quad (15)$$

which strongly depends upon the frequency of the acoustic-signal. It can be noted that v_1 approaches v so long as ω_{ct} is of the order of ω.

4 Transverse Phonon-Helicon Wave Interaction

A lot of research has been reported on the subject of helicon wave propagation and phonon-helicon interaction in semiconductors. Since the phase velocity of the helicon wave lies in the range of sound wave velocity, they can interact and can lead to the amplification of the sound wave (Steele & Vural 1969). In metals and gaseous plasmas the electron-ion mass ratio is often ignored while studying the helicon wave propagation and its interaction with the sound waves. However, in semiconductors the ratio of the effective masses of charge carriers plays an important role and cannot be ignored (Singh & Pandey 1975). Browne (1964), Solymar & Lashmore-Davies (1967) and Singh (1973) have studied the transverse-acoustic interaction in piezoelectric semiconductors but they have not included the effect of effective masses of the charge carriers on the gain

In this section we have studied the transverse phonon-helicon interaction in piezoelectric semiconductor which accounts for the effect of the electron-hole effective mass ratio. Here, we are neglecting the deformation potential just for simplicity. The amplification occurs when the drift velocity of the electrons exceeds the sound wave velocity. We have shown that under certain conditions the result agree with the earlier work.

We assume that the sound wave propagates as a shear wave along the z-axis which is also the direction of the applied d.c magnetic field, of a semiconductor (n-InSb), assumed to be homogeneous, isotropic and infinite in extent. The lattice displacement is along the x and y axis (transverse sound wave) and the electrons are drifted along the z-axis. Following the standard method (Steele & Vural 1969) and using all the necessary coupling equations together with the equation of motion for the electrons accounting for the ratio of the effective masses of the charge carriers (Singh & Pandey 1975), we write the dispersion relation in the following form

$$[A]\cdot[\omega^2-k^2v_s^2] = -\frac{v_s^2}{c^2}\epsilon_l\omega(\omega+jD_nk^2)K^2k^2, \qquad \dots (16)$$

Drifted helicon — Sound — Coupling

where

$$A = k^2-\frac{\omega^2}{c^2}c_l\left(1+\frac{jD_nk^2}{\omega}\right)+\frac{\omega_p^2}{c^2}\frac{(\omega-kv_0)B}{(\omega-kv_0-j\nu\mp\omega_B+B_1)},$$

$$B = 1\pm\frac{\omega_B d}{(\omega-j\nu)}; \quad B_1 = \mp\omega_B d-\frac{\omega_B^2\alpha}{(\omega-j\nu)}.$$

In above equations v_s is the sound velocity, ϵ_l the semiconductor lattice dielectric constant, c the velocity of light, D_n the diffusion constant, $\omega_p[=(n_2^2/m_e^*\epsilon_0\epsilon_l)^{\frac{1}{2}}]$ the semiconductor plasma frequency, v_0 the drift velocity of the electrons, ν and ω_B the collision frequency and the cyclotron frequency of the electrons respectively, $d = m_e^*/m_h^*$ and K the dimensionless coupling coefficient. The $+$ and $-$ sign corresponds to the two polarizations. In absence of diffusion and in the limit as $d\to 0$ eq. (16) reduces to that obtained by Steele & Vural (1969) and they have discussed its solution in detail.

In terms of the effective elastic constant c^*, eq. (16) can be rewritten as

$$\rho\omega^2 = c^*k^2, \qquad \dots (17)$$

where

$$c^* = c_{44}\left[1-\frac{\omega}{c^2}\frac{(\omega+jD_nk^2)\epsilon_lK^2}{A}\right].$$

To obtain the solution of eq. (16), we neglect the fast electromagnetic branch ($k^2c^2 >> \omega^2c_l$) and consider only slow-wave branches of the dispersion equation. Steele & Vural (1969) have shown that the positive energy sound wave can interact with the negative energy drifted helicon wave (left handed polarized wave). Passing directly to the collision-dominated limit and following Steele & Vural (1969) or Singh (1973), we obtain the following expression for the gain α in absence of diffusion

$$\alpha \cong \frac{K^2}{2}(\omega_c/v_s)(v_s/c')^4\left[\frac{Y}{\left\{1+\frac{\omega_c}{\omega}\frac{v_s}{c'}Yd\frac{\omega_B}{\nu}\right\}^2+\left(\frac{\omega_c}{\omega}\right)^2 Y^2\left(\frac{v_s}{c'}\right)^4}\right] \quad \ldots (18)$$

where $c' = (c/c_l)^{\frac{1}{2}}$ is the velocity of light in the medium, $\omega_c\,(=\omega_p^2/\nu)$ is the dielectric relaxation frequency and $Y = 1-v_0/v_s$

This is the modified expression for the gain in absence of diffusion but accounting for the parameter d. In the limit as $d \to 0$, eq. (18) reduces to

$$\alpha = (K^2/2)(\omega_c/v_s)\left[\frac{Y}{(c'/v_s)^4+Y^2(\omega_c/\omega)^2}\right] \quad \ldots (19)$$

Singh (1973) in the similar conditions has obtained the following expression for the gain

$$\alpha = (K^2/2)(\omega_c/v_s)\left[\frac{\mu}{1+\mu^2(\omega_c/\omega)^2}\right], \quad \ldots (20)$$

where $\mu = Y$

If we approached to the last electromagnetic branch ($c' \cong v_s$), eq. (19) reduces to the same expression [eq. (20)] as obtained by Singh (1973). But since in the semiconductor d cannot be equal to zero and therefore its affect on the acoustic-wave amplification must be included

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