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Asymmetric rotor model with angular-momentum dependent moments of inertia

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The rotational energies of the different positive parity states of even-even nuclei have been calculated on the basis of the Davydov and Filippov asymmetric rotor model using Sood's formula for the increase of moments of inertia with angular momentum. The results have been compared with experiment and other works.

1. Introduction

Davydov & Filippov (1958) postulated the existence of triaxial nuclei and calculated the energy spectra and $B(E2)$ transition probabilities on the basis of the asymmetric rotor model. Although recent theoretical investigations (Kumar & Baranager 1968, Gotz 1972) reveal that most of the deformed nuclei in the ground state are symmetric, the fact remains that calculations with the Davydov-Filippov model have led to quite impressive agreement with experimental data. Moreover, in some even-even nuclei the existence of 3^+ and 5^+ states does not fit into the picture of axially symmetric rotor. However, if a nucleus is oscillating about axial symmetry with an r.m.s. value γ, one would expect its rotational levels to be very like those of an axially asymmetric rotor with non-axiality parameter γ.

Although the Davydov-Filippov model gives quite good agreement with low-lying rotational spectra, the energy levels for higher I become somewhat greater than the experimental values. Abecasis & Hernandez (1972) have applied the variable moment of inertia (VMI) model of Mariscotti *et al* (1969) to axially asymmetric even-even nuclei. The calculated energy ratios $R_n = E(n^+)/E(2^+)$ in their AROVMI model agree well with experiment. This has tempted us to introduce the idea of variation of moments of inertia with angular momentum in the asymmetric rotor. In the AROVMI model, moments of inertia not only depend on angular momentum but also on energies of the different states having the same angular momentum. In our formulation moments of inertia are soleiy angular-momentum dependent. Thus the two 2^+ states have different moments of inertia in the AROVMI model while in our formulation they have the same moments of inertia.

2. Calculation and Results

For a triaxial nucleus the Hamiltonian is

$$H = \sum_{i=1}^{3} \frac{I_i^2}{2\mathcal{I}_i}, \qquad \ldots (1)$$

The principal moments of inertia are

$$\mathcal{I}_i = 4B\beta^2 \sin^2 \left(\gamma - \frac{i\,2\pi}{3}\right), \qquad \ldots (2)$$

where B is the mass parameter and β is the deformation parameter. Due to centrifugal stretching and the Coriolis antipairing effect $\mathcal{I}_i$ increases with I. We have used Sood's formula (Sood 1967).

$$\mathcal{I}_i = \mathcal{I}_i^0 \frac{1+NyI(I+1)}{1+(N-1)yI(I+1)}, \qquad \ldots (3)$$

where $N = 2.85 - 0.05\,I$ and y is some parameter, for the variation of moments of inertia with angular momentum. This formula gives good agreement with experiment for symmetric even-even nuclei. Supposing γ to be fixed, the energy levels for $I = 2$, 3 and 5 as given by Davydov & Filippov (1958) will be modified as

$$E_\tau(2^+) = \frac{1+10.5y}{1+16.5y} \frac{9(1 \mp (1-(8/9)\sin^2 3\gamma)^{\frac{1}{2}})}{\sin^2 3\gamma} A, \qquad \ldots (4)$$

$$E(3^+) = \frac{1+20.4y}{1+32.0y} \frac{18}{\sin^2 3\gamma} A, \qquad \ldots (5)$$

$$E_\tau(5^+) = \frac{1+48y}{1+78y} \frac{45 \mp 9(9-8\sin^2 3\gamma)^{\frac{1}{2}}}{\sin^2 3\gamma} A, \qquad \ldots (6)$$

where $A = \dfrac{\hbar^2}{4B\beta^2}$, $\tau = 1$ with the minus sign and $\tau = 2$ with the positive sign on the right hand side.

Solving eqs. (4) and (5) the parameters γ, y and A have been obtained as shown in table 1. The energy ratio R_n and the energy $E_1(5^+)$ have been calculated and compared with experiment and other works. These are given in table 2. The energy ratios calculated on the basis of the Davydov-Filippov model have been taken from Moore & White (1960). Sources of experimental data are Lederer (1967), Jett & Lind (1970), Sayer *et al* (1970), and Pathak *et al* (1970).

3. Discussions

With the same number of parameters as in the AROVMI model, the results fairly agree with experimental fiindings. Although experimental values of

Table 1. Parameters γ, y and A for even-even asymmetric nuclei

Nuclei	γ (degrees)	y	A (keV)
Mg^{24}	11.87	2.607×10^{-2}	118.30
Fe^{56}	20.41	3.454×10^{-3}	164.18
Ru^{102}	25.55	7.839×10^{-3}	87.00
Pd^{106}	26.47	1.762×10^{-2}	96.60
Dy^{160}	11.93	6.006×10^{-4}	19.98
Er^{164}	12.89	8.505×10^{-4}	20.76
Er^{166}	12.67	1.357×10^{-3}	18.42
Er^{168}	12.35	9.219×10^{-4}	18.29
Yb^{168}	11.85	1.191×10^{-3}	20.38
W^{184}	13.83	1.428×10^{-3}	24.96
W^{186}	16.03	$5.19\ \times10^{-4}$	26.32
Os^{192}	25.19	1.131×10^{-3}	36.45
Pt^{192}	30.00	1.528×10^{-3}	52.07
Pt^{196}	30.00	$9.96\ \times10^{-3}$	61.20
Th^{228}	9.73	5.823×10^{-4}	13.62
U^{234}	8.70	$8.66\ \times10^{-5}$	10.39
Pu^{238}	8.30	4.926×10^{-4}	10.61
Fm^{254}	10.05	1.188×10^{-3}	10.42

Table 2. Values of $E_1(5^+)$ and R_n for even-even asymmetric nuclei. For $E_1(5^+)$ the first row gives the calculated values and the second row, the experimental values in keV for each nucleus. For R_n, the four rows represent the ARM, the calculated, the experimental and the AROVMI model values respectively for each nucleus. $R'_4 = E_2(4^+)/E_1(2^+)$.

	$E_1(5^+)$	R_4	R'_4	R_6	R_8	R_{10}	R_{12}	R_{14}
		3.31	13.58					
Mg^{24}	3479	2.84	11.65					
	—	3.01	—					
		—	—					
		3.10	6.24					
Fe^{56}	5848	2.96	5.96					
	—	2.46	—					
		—	—					
		2.81	5.50					
Ru^{102}	2688	2.58	5.05					
	—	2.33	—					
		—	—					
		2.76	5.50					
Pd^{106}	2709	2.41	4.81					
	—	2.40	—					
		—	—					
		3.31	13.45	6.88	11.63	17.49	24.37	32.25
Dy^{160}	1295	3.28	13.32	6.74	11.22	16.54	22.57	29.15
	1290	3.27	13.32	6.70	11.14	16.46	22.49	28.98
		3.29	13.41	6.77	11.32	16.80	—	—

Table 2—(contd)

	$E_1(5^+)$	R_4	R'_4	R_6	R_8	R_{10}	R_{12}	R_{14}
		3.30	11.81	6.83	11.50	17.20	23.88	31.51
Er^{164}	1203	3.26	11.66	6.64	10.94	15.96	21.54	27.58
	1197.4	3.28	11.59	6.72	11.21	16.61	22.79	29.54
		3.28	11.55	6.71	11.14	16.44	22.46	29.16
		3.30	12.18	6.85				
Er^{166}	1076	3.24	11.94	6.56				
	1074	3.29	11.87	6.77				
		3.25	12.08	6.61				
		3.31	12.71	6.86				
Er^{168}	1104	3.26	12.54	6.66				
	—	3.31	12.48	6.88				
		3.30	12.63	6.80				
		3.31	13.62	6.88	11.64	17.50		
Yb^{168}	1351	3.25	13.53	6.62	10.89	16.04		
	1304	3.27	13.36	6.67	11.04	16.37		
		3.29	13.33	6.67	11.04	16.23		
		3.29	10.54	6.78				
W^{184}	1308	3.23	10.32	6.48				
	1287	3.27	10.21	6.73				
		3.28	10.41	6.77				
		3.25	8.40	6.59				
W^{186}	1206	3.23	8.33	6.47				
	—	3.27	—	6.73				
		3.23	—	6.60				
		2.83	5.51					
Os^{192}	1280	2.78	5.42					
	—	2.82	—					
		—	—					
		2.67	5.67	5.00	8.00			
Pt^{192}	1798	2.61	5.54	4.80	7.47			
	—	2.48	—	4.39	6.51			
		—	—	—	—			
		2.67	5.67					
Pt^{196}	1833	2.44	5.13					
	—	2.47	—					
		—	—					
		3.32	19.32	6.95				
Th^{228}	840.4	3.30	19.15	6.81				
		3.25	18.99	6.57				
		—	—	—				
		3.33	23.74	6.97	11.90			
U^{234}	1094	3.32	23.71	6.94	11.84			
	1087	3.30	23.45	6.82	11.47			
		—	—	—	—			
		3.33	25.89	6.97	11.92			
Pu^{238}	1193	3.30	25.69	6.86	11.56			
	—	3.31	—	6.88	11.65			
		—	—	—	—			
		3.32	18.82					
Fm^{254}	847	3.26	17.82					
	—	3.36	—					
		—	—					

$E_1(5^+)$ are available only for a few nuclei, the agreement is quite good. Since the nucleus is asymmetric, centrifugal stretching occurs in all directions and so the shape of the nucleus remains approximately unchanged and the value of γ remains constant. With the increase of angular momentum the product $B\beta^2$ increases. The increase of β with I is due to centrigfugal stretching while the increase of the mass parameter B is due to the Coriolis antipairing (CAP) effect. Pairing gives extra binding to the nucleus which effectively decreases the mass and so the moments of inertia. Antipairing on the other hand would lead to increase of moments of inertia. In the Davydov-Filippov model the values of γ as well as $B\beta^2$ do not change with I but here although γ remains unchanged, the value of $B\beta^2$ increases with I. Simultaneous parametrization of rotational bands built on the ground state and the γ-vibrational state has also been achieved in our formulation.

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