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A dynamical problem on a piezoelectric plate transducer subjected to rigid backing and a constant displacement

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Abstract : The problem of a mechanical disturbance in a piezoelectric plate transducer owing to a periodic function of voltage under rigid backing as well as subjected to a constant impulsive displacement has been investigated in this paper. The method of operational calculus has been used to solve the problem. The variation of the mechanical disturbance with time ranging from 0 to 1 s exhibits a linear relationship and it is found to be of the order of 10^{-2} m

Keywords : Mechanical disturbance, piezoelectricity, plate transducer.

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The studies of piezoelectric transducers from the stand point of mechanics of continuous media have been initiated [1–3]. The relevant problems are important in view of their various practical applications in the field of science and technology. Earlier researchers simplified such problems by considering the transducer to be rigidly backed at one of its ends. It is, therefore, interesting to deal with a problem of piezoelectric transducer one end of which is supported by a backing as well as subjected to a prescribed mechanical displacement and this is what has been attempted in this paper.

The object of the paper is to determine mechanical disturbance in a piezoelectric plate transducer owing to a periodic function of voltage when one end of it is provided with rigid backing as well as subjected to a constant impulsive displacement. The problem has been solved with the aid of Laplace transform technique. Finally, taking the standard value of the material constants for quartz, the numerical values of the mechanical disturbance have been obtained and graph showing variation of mechanical response with small values of time has been plotted in Figure 1.

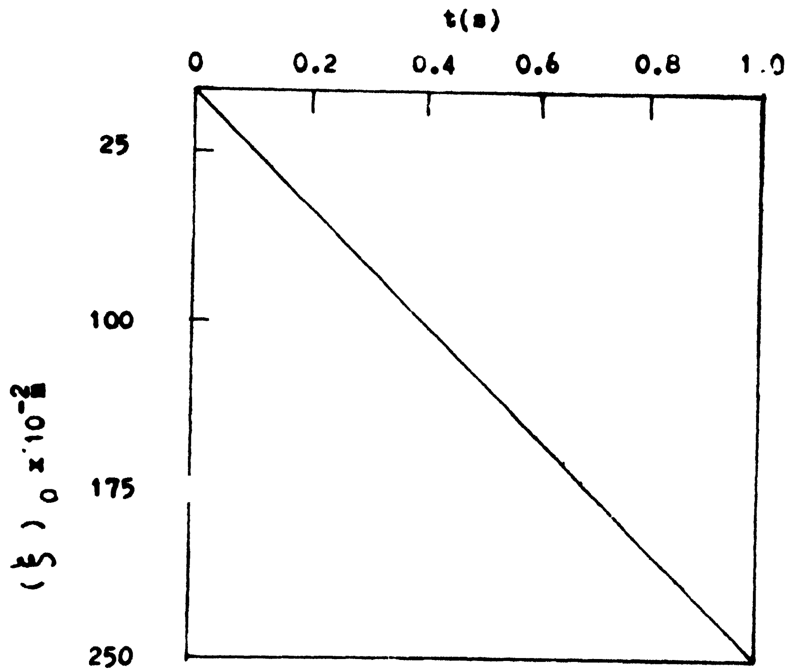

Figure 1. Variation of mechanical disturbance of piezoelectric quartz plate transducer with time.

Let us consider a piezoelectric plate transducer having its thickness direction coincident with the x-axis and let $x = 0$ and $x = X$ be the two extremities of the transducer. Our problem consists in determining the nature of the mechanical response at the end $x = 0$ when the other end of the transducer *i.e.* $x = X$ is always subjected to an impulsive displacement.

Following Redwood [1] the displacement ξ in the x-direction is given by

$$\frac{\delta^2 \xi}{\delta t^2} = v^2 \left(\frac{\delta^2 \xi}{\delta x^2} \right) \tag{1}$$

where $v = \sqrt{c/\rho}$ represents the velocity of the elastic wave in the transducer, c is the elastic constant and ρ is the density of the material of the transducer.

The Laplace transform to be used is

$$L[f(t)] = \bar{f}(p) \quad \int_0^\infty \exp(-pt) f(t) dt, \quad (\mathrm{Re}\, p > 0)$$

and it will be assumed throughout the present work that all variables are zero initially.

If $\bar{\xi}$ denotes the Laplace transform (of parameter p) of ξ, then from eq. (1) we get

$$\bar{\xi} = A\exp(-px/v) + B\exp(px/v) \tag{2}$$

where A and B are functions of p.

The mechanical force $\bar{F}$ exerted on any area normal to x, vide, Redwood [1], is given by

$$\bar{F} = pZ_e\left[-A\exp(-px/v) + B\exp(px/v)\right] - h\bar{Q} \tag{3}$$

and the electrical voltage V across the transducer is given by

$$\bar{V} = -h\left[(\bar{\xi})_x - (\bar{\xi})_0\right] + \frac{\bar{Q}}{C_0} \tag{4}$$

where Z_e is the characteristic impedance of the material; C_0, the static capacitance of the transducer; h, the piezoelectric constant of the material and Q, total charge.

To ascertain the constants A and B, we must enumerate the boundary conditions of the problem. The most general type of this problem can be thought of by having a transducer of impedance Z_e situated between two non-piezoelectric materials of impedance Z_1 and Z_2. Then the conditions of continuity of force and displacement at $x = 0$ and at $x = X$ provide the following equations.

We write at $x = 0$, $(\bar{F}_1)_0 = (\bar{F})_0$; $(\bar{\xi}_1)_0 = (\bar{\xi})_0$

$$\text{at } x = X,\quad (\bar{F}_2)_x = (\bar{F})_x;\ (\bar{\xi}_2)_x = (\bar{\xi})_x \tag{5.1}$$

where

$$\bar{\xi}_1 = A_1\exp(-px/v_1) + B_1\exp(px/v_1) \tag{5.2}$$

$$\bar{F}_1 = pZ_1\left[-A_1\exp(-px/v_1) + B_1\exp(px/v_1)\right] \tag{5.3}$$

$$\bar{\xi}_2 = A_2\exp(-px/v_2) + B_2\exp(px/v_2) \tag{5.4}$$

$$\bar{F}_2 = pZ_2\left[-A_2\exp(-px/v_2) + B_2\exp(px/v_2)\right] \tag{5.5}$$

The suffixes 1 and 2 stand for the entities of materials attached to the ends $x = 0$ and $x = X$ respectively.

Now in order to obtain mechanical response, we assume the voltage input in the form $V = V_0 \sin \omega t$, so that

$$\bar{V} = \frac{V_0\omega}{(p^2 + \omega^2)} \tag{6}$$

When the transducer is connected to a high input impedance of resistance R, we assurance after Redwood [1],

$$A_1 = A_2 = B_2 = 0 \tag{7}$$

and in an accordance with our problem, we take

$$(\xi)_x = \xi_0 \delta(t) \tag{8}$$

Where $\delta(t)$ is the Dirac delta function and ξ_0 is a constant so that

$$(\bar{\xi})_x = \xi_0$$

From eqns. (3–6) with the aid of boundary conditions (7) and (8) we get the following relations :

$$\bar{V} = -p\bar{Q}R \tag{9.1}$$

$$B_1 = A + B \tag{9.2}$$

$$pZ_1 B_1 = pZ_e(\ A + B) - h\bar{Q} \tag{9.3}$$

$$\xi_0 = A\exp\left(-px/v\right) + B\exp\left(px/v\right) \tag{9.4}$$

$$\frac{V_0\,\omega}{p^2+\omega^2} = h(A+B) + \frac{\bar{Q}}{C_0} \tag{9.5}$$

Solving these equations we get A and B and substitution of these values in eq. (2) we obtain $(\bar{\xi})_0$.

Using the standard results, *vide* [3,4] and simplifying, the inverse transform is given by

$$\begin{aligned}(\xi)_0 = {} & 2\xi_0 Z_c/(Z_c+Z_1)\left[\alpha\exp\left\{\alpha(t - x/v)\right\}H(t - x/v)\right] \\ & + \delta\left[(t - x/v)\right] - hV_0C_0\omega/(Z_c+Z_1)(\alpha^2+\omega^2)\left[\exp(\alpha t)\right. \\ & \left. - \cos\omega t - (\alpha/\omega)\sin\omega t\right]\end{aligned} \tag{10}$$

where $\alpha = \dfrac{h^2C_0}{(Z_e+Z_1)}$

and $H(t)$ is the Heaviside Unit function.

The final expression for mechanical displacement is obtained from eq. (10) for small values of time only.

For numerical calculations, the values of the material constants have been taken from [3–8] while values like V_0, C_0, Z_c, Z_1 and X have been chosen suitably to facilitate numerical calculations as follows.

$$V_0 = 5\times10^4\,\text{V},\ C_0 = 16.6\times10^{-8}\,\text{f},\ Z_c = 1,\ Z_1 = 2,\ X = 0.10\,\text{m}$$

The numerical values of the mechanical disturbance and time are shown in Table 1.

Table 1. Numerical values of the mechanical disturbance of a piezo-quartz plate transducer with time.

t (s)	$(\xi)_0 \times 10^{-2}$ m
0.1	−23 17
0 2	−46 34
0 3	−69.51
0.4	−92.68
0.5	−115.85
0 6	−139.02
0.7	−162.19
0.8	−185.36
0.9	−208.53
1.0	−231 70

The mechanical disturbance is found to be of the order of 10^{-2} m (Figure 1) and reduces to zero at $t = 0$. In eq. (10) the contribution of other terms is insignificant compared to the first term in the disturbance. As a result, the disturbance gives out a linear relationship with time. This treatment is valid only within the investigated range of time.

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