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Optical vortices produced by forked holographic grating and sign of their topological charge

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Abstract : We produce optical vortices using a computer generated hologram (CGH) having a forked grating. We propose to use free space trajectory of the vortex produced for assigning sign to topological charge associated with the vortex. We find experimental trajectory of the vortex that is matched with theoretical trajectory for negative as well as positive charge obtained with Collins integral formalism for vortex propagation. This match is used to determine the sign of the vortex charge unambiguously.

Keywords : Optical vortex, sign of topological charge, vortex trajectory, computer generated hologram

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The field associated with the vortices, manifestation of phase singularities and characterized by topological charges, possesses a helical wavefront. The direction of circulation of this wavefront around the phase singularity defines the sign of topological charge associated with the vortex. In optical field these are called optical vortices [1–3], which find variety of applications in optical tweezers [4,5], optical spanners [6], optical trapping of atoms [7], and also in quantum information and computation [8]. The sign of the topological charge associated with the vortices can play an important role in many of these applications. Therefore, unambiguous determination of the sign of the charge of an optical vortex is very important before we go for any of its applications.

An optical vortex nested in a Gaussian beam and centered at origin was produced in the laboratory by passing usual Gaussian beam of the He-Ne laser through branch point of the CGH [9]. This vortex, which acts as an input to the optical system can be written as

$$E_1(x_1,y_1) = (x + iy_1) \exp \left[\frac{ik}{2} r_1^T Q_1^{-1} r_1 \right], \tag{1}$$

where Q_1^{-1} is a complex matrix defining the complex beam parameters of the input beam [10], $k = (2\pi/\lambda)$ is the magnitude of the wave vector and r_1^T indicates transpose of r_1. By using Collins integral formalism for two-dimensional optical system [11,12], for given input $E_1(x_1, y_1)$ and the optical system described by *A, B, C, D* matrices, the output field can be written as [13,14]

$$E_2(x_2,y_2) = (F_1 + iF_2) \exp \left[\frac{ik}{2} r_2^T Q_1^{-1} r_2 \right], \tag{2}$$

where F_i indicates i-th component of the column vector

$$F = (AB^{-1} + Q_1^{-1})^{-1} B^{-1} r_1 \tag{3}$$

and

$$Q_2^{-1} = (C + DQ_1^{-1})(A + BQ_1^{-1})^{-1} \tag{4}$$

gives the output beam parameters in terms of *A, B, C, D* matrices of the optical system and the input beam parameters defined by Q_1^{-1}.

A, B, C, D matrices constituting the ray transfer matrix for free space propagation by a distance L can be written as

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$$A = I,\ B = LI,\ C = 0,\ D = I, \tag{5}$$

where I and 0 are identity and null matrices. Substituting eq. (5) into eqs. (2–4) gives the output field $E_2(x_2, y_2)$ of the system. An earlier theoretical work [15] that derives ABCD law for Laguerre-Gaussian beams, having similarity to the optical vortex, using Hankel transform of a Laguerre-Gaussian function justifies our theoretical treatment of the vortex propagation.

The experiment was performed with a He-Ne laser (Spindler-Hoyer ZL150) of power less than 1 mW and beam waist of 0.75 mm. The laser beam was normally incident on the CGH and passed through the branch point of the forked grating embedded in the CGH placed at 50 cm away from the laser aperture. The schematic has been shown in Figure 1. We also show optical micrograph of the forked grating (Figure 2) used in our experiment. The grating element was found to be ~180 μm. Figure shows

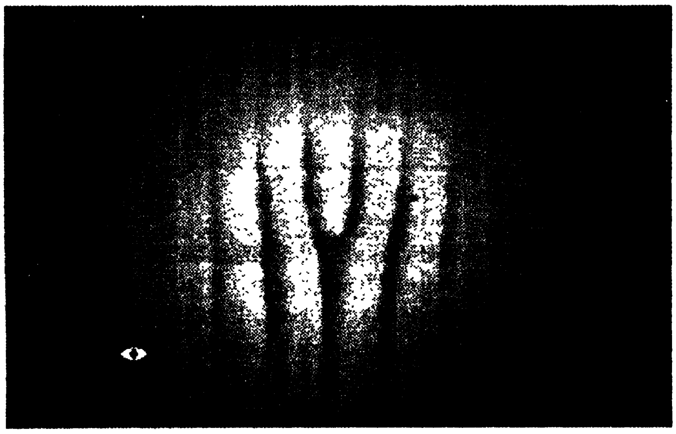

Figure 1. Schematic diagram of the experimental set up to generate an optical vortex using computer generated hologram. Figure is not made to scale.

only central portion of the grating having the fork. In the diffraction pattern generated from the grating, the first order diffraction, a vortex of charge 1 was selected by an aperture placed at 75 cm away from the CGH. The size of aperture was big enough to let the beam pass through the aperture without diffraction. By adjusting the position of the branch point with respect to the beam using x-y-z translation stage we were able to get an axial vortex. This vortex was imaged with a CCD (charg coupled device) (Starlight-Xpress, MX916) as it propagates in free space at different points along its path.

From our experimental set up mentioned above, we obtained center of vortices at different distances with CCD images displayed on the monitor. Four representative CCD

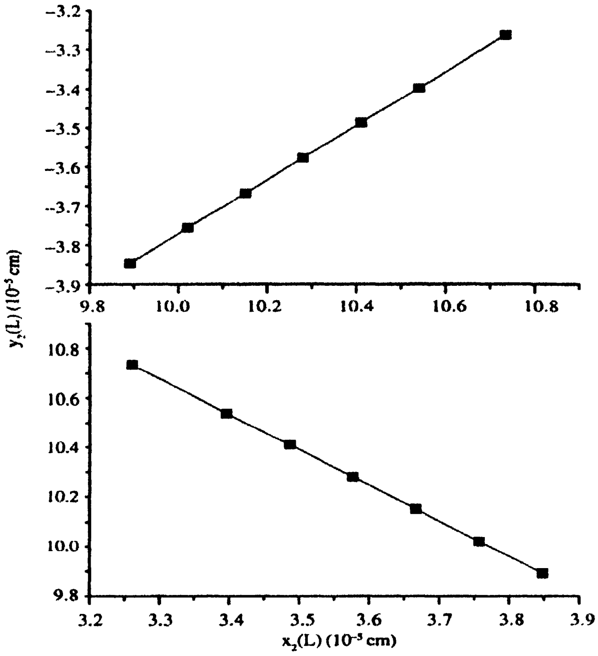

Figure 2. Optical micrograph of the grating used in our experiment Grating element $\simeq$ 180 μm.

images have been shown in Figure 3 with figure caption mentioning the center in each case. These are the points of minimum intensity in the images. Figure 4 shows the trajectory of these points and thus depicts the experimental trajectory followed by the vortex. One can see that these points do not lie in a horizontal plane. It should be noted that the diffracted beams from a grating propagate in a

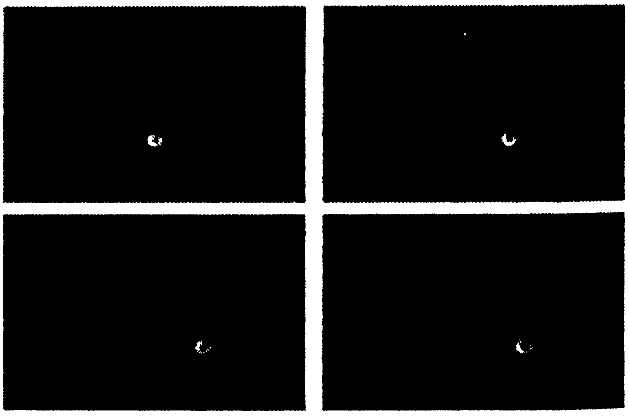

Figure 3. Four representative CCD images of the vortex as it moves in free space. These images correspond to experimental points on the trajectory shown in Figure 4. Top : (left) (3990.4 × 10^{-4} cm, 2542.4 × 10^{-4} cm), (right) (4118 × 10^{-4} cm, 2508.8 × 10^{-4} cm); Bottom : (left) (4234 × 10^{-4} cm, 2464 × 10^{-4} cm), (right) (4442.8 × 10^{-4} cm, 2408 × 10^{-4} cm).

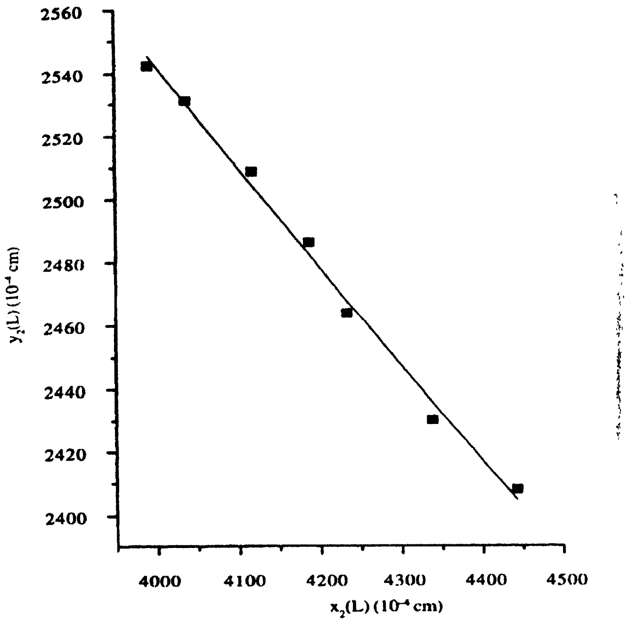

Figure 4. Trajectory of the vortex - Experimental. The points in the graph correspond to measurements at 12.5, 17.5, 22.5, 27.5, 32.5, 37.5, 45 cm from the aperture that selects the vortex (refer to text).

dispersion plane that is parallel to the grating vector. In our case, the grating lines are having fork orientation (Figure 2). Therefore, the grating vector which is normal to the grating lines will not lead to a horizontal dispersion plane, in contrast to usual grating with parallel vertical lines and a horizontal dispersion plane.

Theoretical trajectory is obtained as discussed in [13,14] and is shown in Figure 5. It shows the theoretical

Figure 5. Theoretical trajectory of the vortex - Charge -1 (top), Charge +1 (bottom). L values being same as in the experiment.

trajectories for the vortex of charge +1 as well as of −1. A comparison with Figure 4 immediately gives the charge +1 for our experimental vortex. We would like to mention that a vortex of charge +1 *i.e.* $(x + iy)$ will correspond to an anticlockwise circulation of the helical wavefront while going around the phase singularity, in the process changing phase from 0 to 2π. For a vortex of charge −1 *i.e.* $(x - iy)$ it is reverse. This fact has been illustrated by the Figure 6. The experiment was verified with charge −1 also and found in agreement with theory. The data points in the curves correspond to L = 87.5, 92.5, 97.5, 102.5, 107.5, 112.5, 120 cm. Trajectories were plotted at other L values also with same increment in L. The relative movement of the vortex was found to be same. Experimental values for $x_2(L)$, $y_2(L)$ were obtained by multiplying the horizontal and vertical pixel numbers by size of the pixel. The values are higher than theoretical by orders of magnitude. This can be attributed to limitations of the experimental observation, each pixel size of CCD being 11.6 × 11.2 μm. However, trend of the trajectory is unmistakably the same.

In conclusion, we have determined the sign of the charge of a vortex produced experimentally. The assignment of the charge is unambiguous in contrast to the interferometric methods [16] where order is clear but sign remains ambiguous [17]. The result has direct application in finding out the handedness of rotation for an optical spanner before one applies it to rotate a molecule and also for the processes which preserve topological charge.

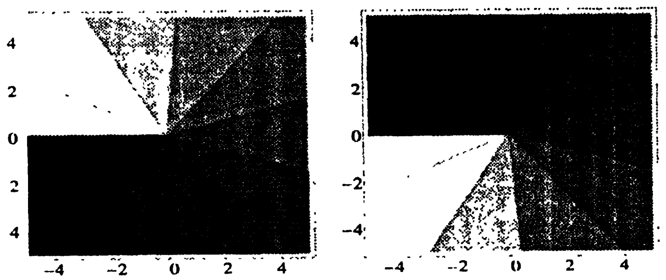

Figure 6. Contourplot of the phase of a vortex of Charge +1 (left), Charge-1 (right). Phase changes from 0 to 2θ while going from darker to brighter region.

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References

[1] L Allen, M J Padgett and M Babiker *The Orbital Angular Momentum of Light* in *Progress in Optics* E Wolf (ed) **Vol. 39** p291 (Amsterdam : North-Holland) (1999)

[2] Y S Kivshar and A Ostrovskaya *Opt. Phot. News* **12** (April) 24 (2001)

[3] M S Soskin and M V Vasnetsov *Singular Optics* in *Progress in Optics* E Wolf (ed) **Vol. 42** (Amsterdam : North-Holland) (2001)

[4] H He, M Friese, N R Heckenberg and H Rubinsztein-Dunlop *Phys. Rev. Lett.* **75** 826 (1995)

[5] K T Gahagan and G A Swartzlander *Opt. Lett.* **21** 827 (1996)

[6] N B Simpson, K Dholakia, L Allen and M J Padgett *Opt. Lett.* **22** 52 (1997)

[7] T Kuga, Y Torii, N Shiokawa, T Hirano, Y Shimizu and H Sasada *Phys. Rev. Lett.* **78** 4713 (1997)

[8] A Mair, A Vaziri, G Weighs and A Zeilinger *Nature* **412** 313 (2001)

[9] J Arlt, K Dholakia, L Allen and M J Padgett *J. Mod. Opt.* **45** 1231 (1998)

[10] J A Arnaud *Hamiltonian Theory of Beam Mode Propagation* in *Progress in Optics* E Wolf (ed) **Vol.11** (Amsterdam : North-Holland) (1973)

[11] S A Collins (Jr.) *J. Opt. Soc. Am.* **60** 1168 (1970)

[12] N Hodgson and H Weber *Optical Resonators Fundamentals, Advanced Concepts and Applications* (London : Springer) (1997)

[13] R P Singh and S R Chowdhury *Opt. Commun.* **215** 231 (2003)

[14] R P Singh and S R Chowdhury *J. Opt. Soc. Am.* **A20** 573 (2003)

[15] J P Taché *Appl. Opt.* **26** 2698 (1987)

[16] M Padgett, J Arlt and N Simpson *Am. J. Phys.* **64** 77 (1996)

[17] I V Basistiy, M S Soskin and M V Vasnetsov *Opt. Commun.* **119** 604 (1995)