Indian J. Phys. **80** (4), 409-411 (2006)

Note

Complex shape parameter for *s*-wave scattering from zero-energy wave functions

Anjana Acharya

Department of Physics, University College of Engineering,
Burla-768 018, Sambalpur, Orissa, India

E-mail : anjana@iopb.res.in

Received 19 December 2005, accepted 7 February 2006

Abstract : Using an extension of the method of Kermode and Van Dijk [*Phys. Rev.* **C42** 1891 (1990)] we show how a complex shape parameter for complex local potential can be determined in terms of zero-energy wave functions for *s*-wave scattering.

Keywords : Effective range theory, shape parameter, zero-energy wave functions

PACS Nos. : 21.10.-k, 21.90.+f

It has recently been shown that the shape parameter (P) and coefficients of higher powers of k^2 in the effective range expansion (ERE) functions may be obtained from the zero-energy wave functions for a real local potential which supports no more than one bound state[1]. The usefulness of this approach lies in the fact that calculation of P is not generally simple if the potential is not of a simple form and the shape parameter is small in magnitude.

In this note, we extend the approach of Kermode and Van Dijk[2] for complex local potentials and try to see whether the shape parameter in such a situation can also be obtained reasonably simply to permit its application to antiproton-proton scatterings. Hence, we have derived an expression for complex shape parameter in terms of integrals of the zero-energy wave functions.

In view of the present experimental works with low energy antiprotons scattering off protons both elastically and *via* annihilation channel, the determination of effective range parameters has been of considerable interest [3-6]. Kroll and Schweiger [7] have found a set of complex square well potential parameters for various partial waves which seem to fit the angular distributions and the ρ parameter reasonably satisfactorily. They use different ranges for real and imaginary parts of the potential in the presence of coulomb interaction.

While Kroll and Schweiger used different ranges for real and imaginary parts of the square well potential for realistic data fitting , we have simplified the problem by taking identical ranges for both, since at this stage, our intention is only to demonstrate the usefulness of this method and no claim is made about the actual determination of shape parameter for antiproton- proton scattering. We would need to include coulomb modification in this approach. At this stage of available information, it will be premature to lay any claim to that effect since even scattering lengths and effective ranges are still to be unambiguously identified. In view of this, for demonstration purposes, we have used potential parameters with identical ranges. We thus show that Kermode and Van Dijk's approach can be extended to complex potentials also with some modifications.

Complex shape parameter :

Here, we have taken a spherically symmetric local complex potential $(V(r) + iW(r))$ instead of a real potential. We define the real and imaginary parts of the s-state wave functions as $(u_r(r) + iu_i(r))$ for energy k^2 and $(u_{0r}(r)+iu_{0i}(r))$ for zero energy respectively and assume the same boundary conditions as given by Kermode and Van Dijk for both real and imaginary parts separately *viz.*

$$u_r(0) = u_{0r}(0) = u_i(0) = u_{0i}(0) = 0, \tag{1a}$$

$$u_r(R) = u_{0r}(R), u_i(R) = u_{0i}(R), \tag{1b}$$

$$u_r'(0) = u_{0r}'(0) = 1, u_i'(0) = u_{0i}'(0) = 0, \tag{1c}$$

where R is the distance beyond which both real and imaginary parts of the potential are negligible. As mentioned above, we are assuming for convenience, that R is identical in both cases. Defining a complex shape parameter $P = P_r + iP_i$ in the standard effective range expansion for the complex potential,

$$K \cot(\delta_r + i\delta_i) = -1/(a_r + ia_i) + 1/2(r_{0r} + ir_{0i})k^2$$

$$-(P_r + iP_i)(r_{0r} + ir_{0i})^3 k^4 + .. \quad (2)$$

with δ's as phase shifts, a's as scattering lengths and r_0's as effective ranges. It can be shown that P satisfies

$$(P_r + iP_i)(r_{0r} + ir_{0i})^3 k^2 - k^4(.....) = \int_0^R \Big[(u_{0r} + iu_{0i})^2$$

$$-(u_r + iu_i)(u_{0r} + iu_{0i}) - (\bar{u}_{0r} + i\bar{u}_{0i})^2 + (\bar{u}_r + i\bar{u}_i)(\bar{u}_{0r} + i\bar{u}_{0i})\Big] dr \quad (3)$$

and hence in the limit $k^2 \rightarrow 0$, we obtain expressions for P_r and P_i in terms of zero-energy wave functions as follows

$$(P_r + iP_i)(r_{0r} + ir_{0i})^3 = \lim_{k^2 \rightarrow 0} 1/k^2 \int_0^R \Big[(u_{0r} + iu_{0i})^2$$

$$-(u_r + iu_i)(u_{0r} + iu_{0i}) - (\bar{u}_{0r} + i\bar{u}_{0i})^2 + (\bar{u}_r + i\bar{u}_i)(\bar{u}_{0r} + i\bar{u}_{0i})\Big] dr. \quad (4)$$

However, the above expressions still contain u_r, u_i and $\bar{u}_r$, $\bar{u}_i$ which are not zero-energy objects. In order to get rid of them, we do a Taylor expansion in k^2 and write the wave functions in terms of complex objects $\beta_r + i\beta_i$ and $\bar{\beta}_r + \bar{\beta}_i$

$$u_r(r) + iu_i(r) = \left[1 + (\beta_r(r) + i\beta_i(r))k^2(u_{0r}(r) + iu_{0i}(r))\right] \quad (5a)$$

$$(\bar{u}_r(r) + i\bar{u}_i(r)) = \left[1 + (\bar{\beta}_r + \bar{\beta}_i)k^2\right](\bar{u}_{0r}(r) + i\bar{u}_{0i}(r))\big]. \quad (5b)$$

This leads to

$$(P_r + iP_i)(r_{0r} + ir_{0i})^3$$

$$= \int_0^R dr\Big[(\beta_r + i\beta_i)(u_{0r} + iu_{0i})^2 - (\bar{\beta}_r + \bar{\beta}_i)(\bar{u}_{0r} + i\bar{u}_{0i})^2\Big] \quad (6)$$

which after separation, gives the real and imaginary values of the shape parameter.

However, $(\beta_r + i\beta_i)$ and $(\beta_r + i\beta_i)$ are still to be determined.

Determination of functions $\beta(r)$ *and* $\bar{\beta}(r)$:

Since we have,

$$(\bar{u}_r(r) + i\bar{u}_i(r)) = \sin(kr)\cot(\delta_r + i\delta_i) + \cos(kr)$$

with

$$\bar{u}_r(r) = 1 - \left\{r\, a_r / (a_r^2 + a_i^2)\right\} + r r_{0r} k^2/2$$

$$+ \left\{k^2 r^3 a_r / (6(a_r^2 + a_i^2))\right\} - k^2 r^2/2,$$

$$\bar{u}_r(r) = \left\{r\, a_i / (a_r^2 + a_i^2)\right\} + r r_{0i} k^2/2 + \left\{k^2 r^3 a_i / 6(a_r^2 + a_i^2)\right\} \quad (7b)$$

to order of k^2, and u_0's being the following

$$\bar{u}_{0r}(r) = 1 - \left\{r\, a_r / (a_r^2 + a_i^2)\right\}, \quad (8a)$$

$$\bar{u}_{0i}(r) = \left\{r\, a_i / (a_i^2 + a_i^2)\right\}. \quad (8b)$$

Using equations (5),(7) and (8), we obtain

$$\bar{\beta}_r(r) = r\Big[\Big\{3(a_i^2 - a_i^2 - r\, a_r)(r_{0r} - r) + r^2(a_r - r) + 3r\, a_i r_{0i}\Big\} /$$

$$6\big((a_r - r)^2 + a_i^2\big)\Big], \quad (9a)$$

$$\bar{\beta}_i(r) = r\Big[\Big\{3r_{0i}(a_r^2 + a_i^2 - r\, a_r) + 2r^2 a_i - 3r\, a_i r_{0i}\Big\} /$$

$$6\big((a_i - r)^2 + a_i^2\big)\Big]. \quad (9b)$$

These appear to be correct expressions because $\bar{\beta}_i(r)$ reduces to Kermode-Van Dijk's $\bar{\beta}_r(r)$ when imaginary part is switched off while $\bar{\beta}_i(r)$ becomes zero under that condition. $\beta_r(r)$ and $\beta_i(r)$ can now be determined by substituting $(u_r + i_{ui})$ from eq (5) into the appropriate Schrödinger equation giving

$$(\beta_r + i\beta_i)''(u_{0r} + iu_{0i}) + 2(\beta_r + i\beta_i)''(u_{0r} + iu_{0i})''$$

$$+ \big(1 + (\beta_r + i\beta_i)k^2\big)(u_{0r} + iu_{0i}) = 0. \quad (10)$$

Letting $k^2 \rightarrow 0$, we get a differential equation for complex β'' as

$$(u_{0r} + iu_{0i})^2(\beta_r + i\beta_i)' = -(u_{0r} + iu_{0i})^2$$

$$= -\int^r (u_{0r}(s) + iu_{0i}(s))^2 ds.$$

The constant of integration is taken to be zero for convenience.

Application to complex square-well potential of identical ranges :

The appropriate informations for a complex square-well potential of depth

$K^2 = V + iW$ and range R are,

$K = K_r + iK_1$,

$K_r = (V + (W^2 + V^2)^{1/2}/2)^{1/2}$

$K_i = W/(2K_r)$,

with

$$u_{0r}(r) = A_r \sin(k_r r)\cosh(k_i r) - A_i \cos(k_r r)\sinh(k_i r)$$

$$u_{0i}(r) = A_r \cos(k_r r)\sinh(k_i r) - A_i \sin(k_r r)\cosh(k_i r),$$

$$u_{0r}(r) = 1 - \left\{r\, a_r \Big/ \left(a_r^2 + a_i^2\right)\right\},$$

$$u_{0i}(r) = \left\{r\, a_i \Big/ \left(a_r^2 + a_i^2\right)\right\},$$

$$a_r = R - \left[K_r \tan(K_r R)\left(1 - \tanh^2(K_i R)\right) + K_i \tanh(K_i R)\right.$$
$$\left. \times \left(1 + \tan^2(K_r R)\right)\right] \Big/ \left(K_r^2 + K_i^2\right)\left(1 + \tanh^2(K_i R)\tan^2(K_r R)\right),$$

$$a_i = \left[K_i \tan(K_r R)\left(1 - \tanh^2(K_i R)\right) - K_r \tanh(K_i R)\right.$$
$$\left. \left(1 + \tan^2(K_r R)\right)\right] \Big/ \left(K_r^2 + K_i^2\right)\left(1 + \tanh^2(K_i R)\tan^2(K_r R)\right).$$

With the square-well potential for which R is taken to be the range of the potential, we used $R = 2.02$ fm and $K = 0.9343$ fm^{-1} and we found $P_r = -0.040$ and $P_i = -0.006$. The finiteness to P_i is interesting because the imaginary part of the potential is very large. We show in Figure 1 the β - functions and we note that while the behaviour of β_r and $\bar{\beta}_r$ is very similar to that obtained for purely real case, β_i and β_i show a distinctly different behaviour including some oscillations. Further, their magnitude is an order less than β_r 's. Again, we have shown that the coefficients of the effective range expansion can all be expressed in terms of the zero energy wave function. This feature is especially useful in the determination of the complex shape parameter, which is calculated in a straightforward manner by integrating simple functions made up of the square of the zero-energy wave functions. We have thus shown that Kermode and Van Dijk method can be conveniently extended to complex local potentials as well and a complex shape parameter can be defined.

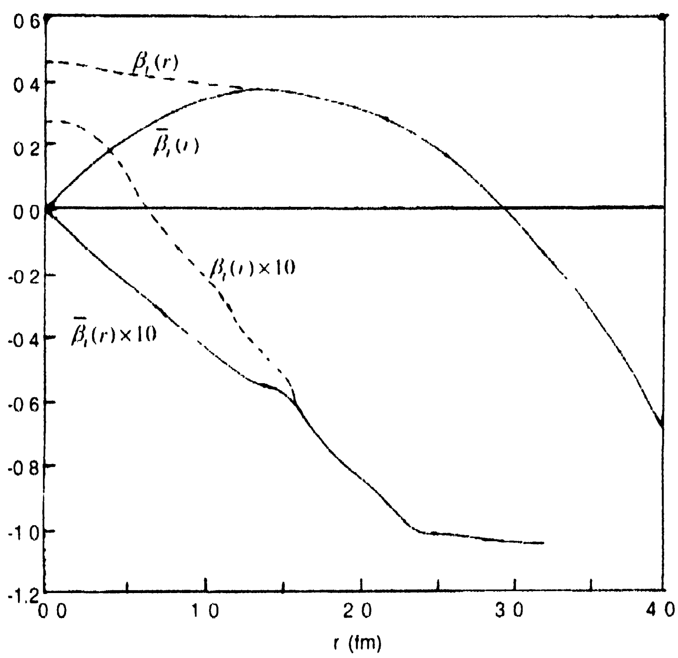

Figure 1. The real and imaginary parts of the β and $\bar{\beta}$ functions for the complex square well potential

References

[1] L S Rodberg and R M Thaler *Introduction to the Quantum Theory of Scattering* (New York : Academic) (1967)

[2] M W Kermode and W Van Dijk *Phys. Rev* **C42** 1891 (1990)

[3] J Mahalanabic, H J Pirner and T A Shibata *Nucl Phys.* **A485** 546 (1988)

[4] H J Pirner, B Kervikov and J Mahalanabic *Z Phys.* **A338** 111 (1991)

[5] A V Lagu in *Physics at Lear with Low Energy Antiprotons* (eds) C Amsler *et al* (New York . Harwood) p251 (1988)

[6] B O Karvikov *Nucl. Phys. (Proc. Suppl)* **B8** 273 (1989)

[7] P Kroll and W Schweiger *Nucl Phys.* **A503** 865 (1989)