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A novel method for the measurement of dielectric constant at microwave frequency by using strip line embedded in rectangular waveguide

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Abstract : This paper deals with the introduction of a new method of measurement of dielectric constant at microwave frequency. The variation of cut-off frequency of strip line embedded in rectangular wave-guide is taken as a parameter for calibration of dielectric constant and a new method for measuring dielectric constant is suggested. By placing the material sample at various locations, theoretically the changes in cutoff frequency of fundamental and first overtone are worked out and the location of maximum changes is selected for further calculations. Using Finite Element Method (FEM) and sample of different dielectric constant located at the selected place, the variations in cut-off frequencies are calculated. Using these variations, a calibration curve is established and a new method of measurement of dielectric constant is suggested.

Keywords : Dielectric constant measurement, FEM, strip line.

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. Introduction

trip line has interesting potentialities for microwave ntegrated circuits on dielectric substrate. Riblet [1] escribed a strip line for the accurate determination of omplex reflection coefficient. Sinha [2] analyzed a nultiple strip discontinuity in rectangular waveguide. Barry 3] measured complex permittivity and permeability with -parameter, the measurement being made on a strip ansmission line device, loaded with material. Queffelec nd Gelin [4] proposed the use of discontinuity in iicrostrip for the measurement of permeability of material. arnard and Gautray [5] used a microstrip ring resonator r permittivity measurement.

In the present work, we used the strip line embedded rectangular crossectional waveguide, as a guiding ructure and proposed a method for the measurement of ielectric constant with calibration curve of cut-off equency, using finite element method.

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2. Theory

Consider a strip line structure with strip embedded in rectangular waveguide, with crossectional sides a and b as shown in Figure 1. These strip lines are placed in rectangular waveguide in such a way that strip is parallel to the longer side of cross section of waveguide. In such

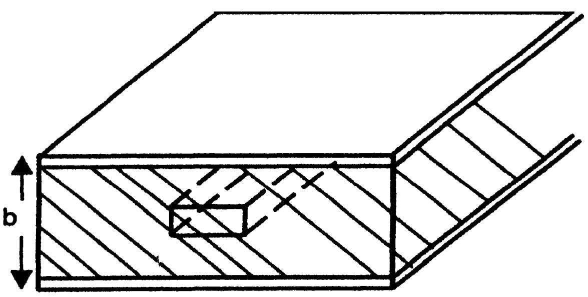

Figure 1. Strip line embedded in rectangular waveguide.

a structure, a sample of dielectric material is kept along Z-direction. Here, we have to determine the cut-off frequency of such guiding structure and check whether the cut-off frequency measurement with ε-variation would provide a method for dielectric constant measurement.

The electric and magnetic fields inside the wave guide, will satisfy the Maxwell's equations and assuming the time dependence of the fields as $e^{j\omega t}$, the Maxwell's equation are written as

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0\boldsymbol{H}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon_0\varepsilon\boldsymbol{E}. \tag{2}$$

The magnetic field vector formulation is used for the solution of the problem. Therefore, Maxwell's eq. (2) is expressed as

$$\varepsilon^{-1}\,(\nabla \times \boldsymbol{H}) = j\omega\varepsilon_0\boldsymbol{E}. \tag{3}$$

Taking the curl of eq. (3) and using eq. (1), eq. (3) can be written as

$$\nabla \times (\varepsilon^{-1}\nabla \times \boldsymbol{H}) = k_0^2\boldsymbol{H} \tag{4}$$

where $k_0^2 = \omega^2\mu_0\varepsilon_0$.

Here ω, ε_0 and μ_0 are the angular frequency; permittivity and permeability of free space, respectively and ε is the relative permittivity of the medium inside the wave-guide. For air-medium, the value of $\varepsilon = 1$.

The magnetic field $\boldsymbol{H}$ must satisfy suitable boundary condition at the conducting boundaries *i.e.* the normal component of the magnetic field is zero on conductor boundaries :

$$\boldsymbol{H}.\boldsymbol{n}|_{\text{boundary}} = 0,\ H_x = 0,\ H_y = 0\ .$$

Variational formulation :

To develope a functional Π for the variational formulation of the given problem, the eq. (4) is multiplied by an arbitrary test function v and then integrated over the domain Ω of the problem. This leads to an expression for functional as

$$\prod = \int_\Omega \boldsymbol{v} \bullet \nabla \times \left(\varepsilon^{-1}\nabla \times \boldsymbol{H}\right) d\Omega - \int_\Omega \boldsymbol{v} \bullet k_0^2 \boldsymbol{H}\, d\Omega. \tag{5}$$

Using vector identity for the first term in eq. (5) and converting volume integral into surface integral, we get

$$\prod = \int_\Omega \boldsymbol{n} \bullet \left(\varepsilon^{-1}\nabla \times \boldsymbol{H} \times \boldsymbol{v}\right) d\Gamma$$

$$+\int_\Omega \boldsymbol{n} \bullet \left(\varepsilon^{-1}\nabla \times \boldsymbol{H}\right) \bullet \left(\nabla \times \boldsymbol{v}\right) d\Gamma - \int_\Omega \boldsymbol{v} \bullet k_0^2 \boldsymbol{H}\, d\Omega\ , \tag{6}$$

where $\boldsymbol{n}$ is outward unit normal vector.

This can be transferred to

$$\prod = \int_\Gamma \boldsymbol{v} . \bullet \left[\boldsymbol{n} \times \left(\varepsilon^{-1}\nabla \times \boldsymbol{H} \times \boldsymbol{v}\right)\right] d\Gamma$$

$$+\int \left(\varepsilon^{-1}\nabla \times \boldsymbol{H}\right) \bullet \left(\nabla \times \boldsymbol{v}\right) d\Gamma - \int \boldsymbol{v} \bullet k_0^2 \boldsymbol{H}\, d\Omega\ . \tag{7}$$

The coefficients of boundary integral constitute the natural boundary condition. So it is included in the formulation. Since tangential component of $\boldsymbol{E}$ is continuous at the boundary, the first term in eq. (7) will be zero. Using Galerkin's criterion as $\boldsymbol{v} = \boldsymbol{H}^*$ (complex conjugate of H), the functional Π will become bilinear and symmetric. The bilinear terms are multiplied by 1/2, we get functional as

$$\prod = \frac{1}{2}\int \left[\left(\nabla \times \boldsymbol{V}\right) \bullet \left(\varepsilon^{-1}\nabla \times \boldsymbol{H}\right) - k_0^2 \boldsymbol{H}^* \bullet \boldsymbol{H}\right] d\Omega \tag{8}$$

Now, function H can be determined which makes Π stationary. For stationary functional in eq. (8), its first variation must vanish *i.e.* $\delta\Pi = 0$.

The domain of the cross section of a waveguide is divided into rectangular elements with four nodes. The functional over each element, is taken in the derivative form of mapping function and hence, functional is given in the form of S_e, T_e.

Therefore, $\prod^e = \sum \left(S_e H_e - k_0^2 B_e H_e\right) = 0$,

where H_e is a vector with three unknown components H_x^e, H_y^e, H_z^e, B_e is an integral of simple mapping function and S_e, T_e are the integral mapping functions of derivative. Π^e is a functional over all elements.

The condition $(\partial\Pi / \partial\{H\}) = 0$ leads to the following matrix equation.

$$[S]\{H\} - K_0^2[T]\{H\} = 0. \tag{9}$$

Eq. (9) is the matrix equation to be solved for eigen values $K_0^2 = \lambda$.

3. Numerical calculations

For the measurement of dielectric constant, strip line embedded in rectangular waveguide, is used. The waveguide dimensions are a = 2.4 cm, b = 2.4 cm and has one strip line of length 0.8 cm placed in parallel with cross sectional sides of rectangular waveguide. The dielectric sample with height 0.8 cm and width 0.8 cm is used. The cross sectional waveguide is divided into 576 rectangular elements. It contains 625 nodes and 1875 unknown components. Out of these, 98 numbers of unknowns are specified by the boundary conditions. Hence, the matrix eigen equation will contain matrices of the size 1777 × 1777. For a strip line having 1875 unknown components, out of which 106 number of unknowns are specified by the boundary conditions, hence the matrix eigen equation will contain matrices of size 1769 × 1769. Time required for running the program is about 40 sec, on P-IV, 1.7 GHz system.

Initially, for the empty waveguide, cut-off frequencies, for fundamental and first overtone modes are obtained by using the FEM software, for H-formulation of electromagnetic propagation problems. The dielectric sample material of $\varepsilon = 5$ is placed at the position shown in Figure 2 with the labeled number; again the FEM programme is run to find the cut-off frequency for

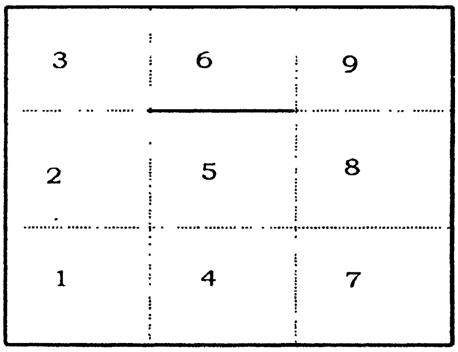

Figure 2. Number indicates the location number.

fundamental and overtone. The same procedure is repeated for different locations and the mode for which higher changes are indicated, are selected for the best location measurements. The results are indicated in Table 1.

Table 1. Variation of cut-off frequency due to different locations of dielectric sample.

Location no. of dielectric	Fundamental mode cut-off (GHz)		Change in frequency due to dielectric (GHz)	
	I mode	II mode	I	II
0	6.2544	9.9294	0	0
1	3.0717	4.7824	3.1827	5.1470
2	3.9011	3.0872	2.3533	6.8422
3	3.8787	2.8917	2.3757	7.0377
4	3.9011	3.0872	2.3533	6.8422
5	3.5662	3.2249	2.6882	6.7045
6	3.9011	3.0872	2.3533	6.8422

At the selected location, samples with different values of dielectric constant, are placed and the cut-off frequency for each sample is obtained by analyzing with FEM software. These results are indicated in Table 2. Calibration curve is obtained as shown in Figure 3, which is used for the measurement of ε. This ascertains the usefulness of the procedure for the measurement of dielectric constant.

Table 2. Variation of cut-off frequency due to dielectric.

Values of dielectric constant	Change in frequency due to dielectric (GHz)		Change in cut-off frequency (GHz)	
	I mode	II mode	I	II
1	6.2544	9.9294	0	0
1.5	5.2735	8.2965	0.9809	1.6329
2	4.8424	7.4594	1.4120	2.4700
2.5	4.4317	6.9541	1.8227	2.9753
3	4.1894	6.2519	2.065	3.6775
4	3.5219	5.4247	2.7325	4.5047
5	3.0717	4.7824	3.1827	5.1470
6	2.7248	4.2379	3.5296	5.6915
7	2.4324	3.8637	3.822	6.0657
8	2.2769	3.4454	3.9775	6.4840
9	2.0479	3.0547	4.2065	6.8747
10	1.8397	2.8030	4.4147	7.1264

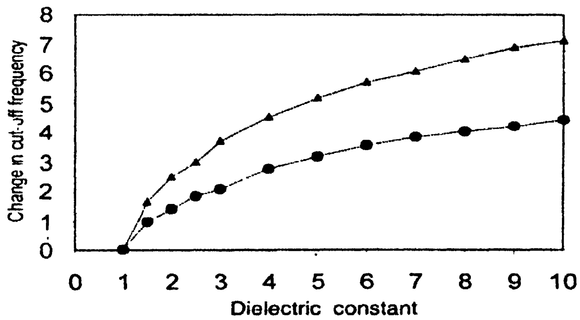

Figure 3. Calibration curve.

4. Suggestion of new method

It is observed that due to change in dielectric constant of the material, the change in cut-off frequency in fundamental mode, is small. Whereas such changes in first over tone mode are appreciable, changes in cut-off frequencies in first overtone mode, are used as parameter for evaluations of dielectric constant. The smooth and continuous nature of cut-off frequency and dielectric constant is visible from Figure 3. It can be used for introducing a new method for the measurement of dielectric constant. Using the above information, a new method suggested, may have following three steps.

(i) Experimental determination of calibration curve of cut-off frequency with dielectric constant. (ii) Finding the cut-off frequency for sample material. (iii) Obtaining dielectric constant of specimen by interpolation, using calibration curve.

5. Conclusion

By using FEM technique, a new method of measurement of dielectric constant, based on the property of cut-off frequency of slot line embedded in rectangular waveguide, from calibration curve is suggested. This method has the following special points over the other methods.

(i) New property used for measurement of dielectric constant. (ii) Less operation in measurement as compared to reflection method. (iii) No restriction of smallness of the sample size as in resonance method. (iv) Large changes in cut-off frequency, as compared to the changes in resonance frequency, can provide high accuracy.

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