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The tunnelling probability in a tunnel diode

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The tunnelling probability, Z of an electron from the conduction band on the n-side to the valence band on the p-side or vice versa of a tunnel diode has been derived by Franz (1956), Kane (1959, 1961), Price *et al* (1959), Fredkin *et al* (1962), Stratton (1964), Takeuti *et al* (1965) and Roy *et al* (1969, 1970). As great many features of tunnelling phenomena in soolids are essentially of one-dimensional nature, an expression for the tunnelling probability of electrons has been derived following Roy *et al* (1969), to deduce some important features of electron tunnelling in a p–n junction. The expression is found to predict the same $(n^*)^{-\frac{1}{2}}$ dependence of tunnelling probability but with a modified magnitude. The tunnelling probability is found to be almost independent of the kinetic energy of electron when its magnitude is large, but it is found to decrease to zero as the kinetic energy decreases and ultimately vanishes as shown by Shuey (1965). The wave-functions in the barrier region are also found to be of the characteristic decaying and growing type as in a rectangular barrier.

The variation of electron potential energy $V(x)$ with distance x is described by the following set of equations

$$
\begin{aligned}
V(x) &= 0 \text{ for } -\infty < x < 0 \text{ and } (a+b) < x < \infty \\
V(x) &= \frac{x}{a} E_g \text{ for } 0 < x < (a+b),
\end{aligned} \qquad \ldots \quad (1)
$$

where E_g = band gap energy of the semiconductor,

$$a = \text{depletion layer width} = \left(\frac{2K\epsilon_0 E_g}{q^2 n^*} \right)^{\frac{1}{2}},$$

(K = dielectric constant of the semiconductor, q = electron charge, $n^* = np/(n+p)$ = reduced carrier concentration, and $b = (E/E_g)a$, E = kinetic energy of electrons.).

The Schrödinger's time independent wave equations in the three regions of the barrier can then be written and solved as incidicated by Roy *et al* (1969) to obtain Z. The expression for tunnelling probability is

$$Z = \frac{4\gamma^2}{[U'_1(-\phi_1)U'_2(\phi_2)-U'_1(\phi_2)U'_2(-\phi_1)+\gamma^2\{U_1(-\phi_1)U_2(\phi_2)-U_1(\phi_2)U_2(-\phi_1)\}]^2+ \gamma^2[\{U'_1(-\phi_1)U_2(\phi_2)-U_1(\phi_2)U'_2(-\phi_1)\}-\{U_1(-\phi_1)U'_2(\phi_2)-U'_1(\phi_2)U_2(-\phi_1)\}]^2}, \quad \ldots (2)$$

where

$$\left.\begin{aligned} \phi_1 &= \gamma^2 = \left(\frac{2m^*E_ga^2}{\hbar^2}\right)^{1/3}\frac{E}{E_g} \\ \text{and} \quad \phi_2 &= \left(\frac{2m^*E_ga^2}{\hbar^2}\right)^{1/3}, \end{aligned}\right\} \quad \ldots (3)$$

U_1 and U_2 functions along with their derivatives are tabulated by Smirnov (1960).

Eq. (2) indicates that Z is expected to depend on E/E_g and n^*. Z is found to be almost independent of E (kinetic energy of electrons on either side of the barrier) except for very small values of $E/E_g < 0.1$. It then goes to zero monotonically with decrease in E. The variation of $\log_{10}Z$ with $(n^*)^{-\frac{1}{2}}$ is found to be linear as expected.

The wavefunctions in the barrier are $U_1(-\phi)$ and $U_2(-\phi)$. These wavefunctions are not the growing and decaying types as found in rectangular barrier. To find such a pair we formed linear combinations of the wave functions. If such functions are at all possible we must have,

$$\left.\begin{aligned} &\operatorname*{Lt}_{\phi\to\infty}[U_1(-\phi)+pU_2(-\phi)] = 0 \\ \text{and} \quad &\operatorname*{Lt}_{\phi\to\infty}[U'_1(-\phi)+pU'_2(-\phi)] = 0, \end{aligned}\right\} \quad \ldots (4)$$

where p is a numerical constant. These two equations would therefore require

$$p = -\frac{U'_1(-\infty)}{U'_2(-\infty)} = -\frac{U_1(-\infty)}{U_2(-\infty)}. \quad \ldots (5)$$

It is easy to see that such a constant p exists. From the property of U_1 and U_2 functions, it is known that

$$\frac{U_1(-\phi)}{U_2(-\phi)} = \frac{U'_1(-\phi)}{U'_2(-\phi)} + \frac{1}{U_2(-\phi)U'_2(-\phi)}.$$

For $\phi \to \infty$,

$$\frac{U_1(-\infty)}{U_2(-\infty)} \simeq \frac{U'_1(-\infty)}{U'_2(-\infty)} \quad \ldots (6)$$

as the contribution of the other term on the right hand side is negligible for large values of ϕ. The value of p works out to be 0.729. The required pair of wave-functions are therefore,

$$\psi_1 = [U_1(-\phi)+pU_2(-\phi)] \qquad \text{(decaying)}$$

and

$$\psi_2 = [U_1(-\phi)-pU_2(-\phi)] \qquad \text{(growing).}$$

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Comment on "The internal structure of a polytrope of constant gravitational mass density in general relativity."

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Sharma (1974) has obtained solutions of Einstein's field equations for the internal structure of a polytrope of constant gravitational mass density in general relativity. In that paper there are only three field equations while the number of variables are four viz., λ, ν, ρ and P. Hence to find out the solutions for the