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A study of refractive index structure constant from radar data

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Abstract : The work focuses on the measurement of refractive index structure constant from the backscattered signal strength. For this purpose, I have made use of the equatorial data measured by medium frequency (1.98 MHz) Partial Reflection Radar, located at Tirunelveli (8.7°N, 77.8°E, 0.3°N magnetic dip.). This radar has been operated continuously by the Indian Institute of Geomagnetism; the data obtained with this radar have been providing valuable information on middle atmosphere dynamics. This is a preliminary study conducted in the equatorial region and the Partial Reflection Radar is not an efficient tool for turbulent studies, hence the estimation of refractive index structure constant of turbulence *i.e.* C_n^2 is very difficult. The refractive index structure constants for turbulence is calculated numerically and plotted with the vertical wind velocity. C_n^2 values show an enhancement during post sunset hours. It may be due to the fact that the plasma within the geomagnetic flux tube becomes unstable because of pre-reversal enhancement ($E \times B$ electrodynamic drift) and takes part in the up-welling process thus making turbulence within the volume.

Keywords Refractive index structure constant, turbulence, radar data.

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1. Introduction

Radar is a means of collecting information about distant objects by sending electromagnetic waves on them. When transmitted energy falls on a distant object like clouds, winds, *etc.*, it reflects back some portion of this energy (echo). The reflected energy is scattered in all directions with intensities, a function of object's geometry and object's materials.

At the Equatorial Geophysical Research Laboratory (EGRL), Krishnapuram, a high-power medium frequency (MF) partial reflection radar was installed in 1992 under an Indo-Australian collaborative project, the main purpose being to obtain continuous measurements of neutral winds in the altitude region 60–100 km.

2. Mathematical analysis

If an antenna with radiating power P_t and with the gain G_t illuminates a target of cross-sectional area 'σ' at a distance 'r' away from the antenna, we can write the incident power P_i on the target as

$$P_i = P_t G_t s / 4\pi r^2. \tag{1}$$

For a monostatic radar system, the back scattered power received by the antenna is

$$P_r = \frac{P_t G_t \quad \sigma A_e L}{4\Pi r^2 \quad 4P r^2} \tag{2}$$

where A_e is the effective area of the antenna, L is the efficiency of the antenna and λ is the wavelength, and

$$G_t = 4\pi A_e \lambda^{-2}. \tag{3}$$

Then eq. (2) reduces to

$$p_r = \frac{P_t A_e^2 \sigma L}{4\pi \lambda^2 r^4} \tag{4}$$

this is the standard radar equation for a single scattering target situated in the antenna beam. For a randomly distributed scatterer,

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$$\sum \sigma = \frac{P_r 4\pi\lambda^2 r^4}{P_t A_e^2 L} \tag{5}$$

The radio refractive index of the atmosphere 'n' is defined as

$$n = (\mu_r \xi_r)^{1/2}, \tag{6}$$

where μ_r is the relative permeability of air ≈1 for the atmosphere, and ξ_r is the dielectric constant of the medium [1].

Again, a general formula for refractive index of radio waves propagating through middle atmosphere can be expressed as [2]

$$(n-1) = \frac{3.73\times10^{-1}e}{\ } + \frac{7.76\times10^{5}P}{\ } \ \frac{-N_e}{2N_c}, \tag{7}$$

where e is the partial water vapour pressure in mbars, P is the total atmospheric pressure in mbars, T is the absolute temperature in Kelvin, N_e is the number of density of electrons and N_c is the critical plasma density.

The first term in eq. (7) is the contribution due to water vapour, and is only relevant within the troposphere. The second term is due to dry air and is dominant above the tropopause. The third term is due to free electrons, and is dominant at ionospheric heights. From eq. (5), we have the backscatter cross section [3–5] :

$$\sigma = (4 \times 3.14) r^4 P_r \lambda^2 / P_t A_e^2 L. \tag{8}$$

The relationship between the strength of atmospheric radar backscatter and the intensity of refractive index structure constant for turbulence has been reformulated as

$$C_n^2 = \sigma / 0.00655 \times \pi^{4/3} \lambda^{-1/3}, \tag{9}$$

where C_n^2 is the potential refractive index structure constant and λ is the radar wavelength.

More precisely, these are the clear air echoes produced by fluctuations of the atmospheric index of refraction. Partial Reflection Radar means transmitting signal from the earth is partially reflected in the E-region of ionosphere and remaining signal transverses into the higher region of the ionosphere. Then the receiver collects the reflected signal from the ionosphere. The received power is very small as compared to transmitted power. The radar receiver is of super-heterodyne type. Input of the receiver is a low-noise device which has the capability of receiving weak signals. Details are given in Section (3). The radar operating at VHF-UHF bands is mainly sensitive to inhomogeneities in the radio refractive index associated with small-scale atmospheric turbulence, the backscattered signal can be effectively used for the quantification of turbulence [5,6].

3. System details

The transmitting array is arranged in a square, and consists of four center-fed half wave dipoles, 75 m in length. The antennas are constructed from 3 mm pre-stretched copper wire. One set of dipole is arranged parallel to the Earth's magnetic field and the other is arranged orthogonal to it. This is because Tirunelveli is close to the magnetic equator and the ordinary and the extra ordinary rays are linearly polarized parallel to, and at right angles to the magnetic field, respectively. One set of dipoles is used for *O*-mode during the day time and the other is used for transmitting E-mode during the night time. The *O*-mode dipoles are oriented in the north-south direction. The *e*-mode dipoles are oriented in the east-west direction. The ideal height of the transmitting antenna is one quarter of the transmitting wavelength (30 m) above the ground plane.

The receiving antennas are of the inverted V-type, and are situated at the vertices of an equatorial triangle in which the basic spacing is 180 m. The centers of both the transmitting and receiving arrays coincide, making this truly monostatic system. The impedance of each receiving antenna was 50 ohm to match the co-axial cable taking the signal back to the receiving system.

The transmitter operates at a frequency of 1.98 MHz, although this is adjustable by ~0.2 MHz, and is fully phase coherent. The transmitter power is 25 kW peak to peak, with a variable pulse length of 10–50 micro seconds, usually operated at 30 micro seconds which corresponds to a 4 km height resolution. The pulse repetition frequency is 50 Hz with 32 point integration. Super-heterodyne receivers are used in the system. In each receiver, signal of 1.98 MHz is mixed with a 2.475 MHz local oscillator signal to produce a 477 KHz Intermediate Frequency which is then fed to the signal processing unit. This signal is fed to phase detectors and then digitized to 16-bit resolution [2]. Receiver details are provided in Table 1.

The Partial Reflection Radar is mainly used for studying wave processes; it can be inferred from the wind motion which is directly recorded. Wave effects on the atmosphere are the changes in density, temperature and air flow. Here, the density and temperature changes result in the variation of pressure which subsequently results in the flow of air.

Table 1. Receiver details of Partial Reflection Radar.

No.	Specification	Details
1	Number of receivers	3
2	Center frequency	1.98 MHz
3.	Band width (3 dB)	~39 KHz
4	Sensitivity	~10 µv
5	Maximum gain	60 dB
6.	Gain control	Programmable in steps of 1 dB
7	Gain control range	60 dB
8	Input impedance	50 Ω
9	Signal processors	The digitizers have 16-bit resolution
10	Sampling starting height	60–98 km (day); 70–98 km (night)

4. Middle atmosphere and turbulence

Wind arises due to temperature, density and pressure gradients. Depending upon the nature and time scales of external restoring forces, different types of wave motions are produced in atmosphere. When these atmospheric waves travel from one region to another region of the globe both vertically and horizontally, a dynamical coupling is created among the different regions of atmosphere [7]. A study of perturbation in any of the aforesaid parameters would be helpful in understanding the dynamics of Earth's atmosphere too.

The tropical atmosphere is influenced by solar radiation which drives both large scale circulation and cumulus convection. The coriolis force is smaller, hence this allows certain wave modes which get trapped in equatorial region. Since the gas density exponentially decreases with height, the atmospheric waves grow in amplitude upwards, and become significant contributors to the momentum and energy budgets of the middle upper atmosphere (80–100 km).

The measured wind field over Tirunelveli (8.7°N, 77.8°E) exhibits variability on several time scales. Several wave events lasting for a few days to few tens of days have been observed, revealing the influence of lower atmospheric wave excitation processes on the region up to 100 km [8–10].

Tides are known for many years to play an important role in the dynamics of the middle atmosphere. Atmospheric tides are global scale oscillations in temperature, density, pressure and wind at periods which are sub-harmonics of a solar/lunar day. Solar tides are primarily generated by thermal force due to absorption of solar radiation by water vapour or ozone [11,12]. They are further classified into two : (i) migrating tides and (ii) non-migrating tides.

Tides have in general, amplitudes larger than other wave motions and they dominate in the atmospheric dynamics. They transport momentum and wave energy upward from their source regions to the regions in which they are displaced by various instabilities such as (i) Inertial instability (ii) Shear instability (iii) Barotropic and Baro clinic instability and hence, affect the mean circulation and structure of the atmosphere [13–15].

Many have concentrated on large scale motions which are prevalent in the atmosphere. In fact, the most common state of any fluid is turbulent flow and only some of the flow can be termed laminar. It is only because the atmosphere is turbulent that we can use partial reflection radar to study it in a limited manner.

Turbulence can exist only if energy available for its generation is greater than the buoyancy effects. The determination of the structure of the turbulence is often one of the primary objectives of atmosphere dynamics studies. In that case, the velocity and density fluctuations can often be thought of as being composed of a number of sinusoidal fluctuations of varying scales, and varying spatial orientations. We can therefore, by analyzing the wind or density fluctuations, discover something about the spectrum of turbulence. The energy spectrum is divided into three distinct regions based on the distinct characteristics of turbulence within each region, known as the buoyancy range, inertial range and viscous range.

Middle atmosphere turbulence (80–120 km) is thought to be due to gravity waves and tides, which generate turbulence through many different processes including non-linear breaking, shear instabilities, convective over-turning, and critical level interactions. Turbulence is important because it is a large contributor to the diffusion of momentum, heat and matter. Mesospheric turbulence can be both spatially and temporally intermittent and may occur in patches or in horizontal layers. The layers may be upto several kilometers thick and may be in a 'sandwich'-type arrangement such that the turbulent layers are bound by non-turbulent regions. Around an altitude of 110 km, the molecular viscosity becomes large enough to damp out any turbulence, and this region is referred to as the turbopause. While turbulence in the neutral atmosphere has been observed to heights of ~130 km, above the turbopause it only plays a minor physical role.

Turbulence is one of the primary generators of both spatial and temporal variations in the refractive index of the atmosphere; it creates a medium which scatters electromagnetic radiation thereby enabling one to probe the atmosphere and study both the long-term and short-term dynamics as well as turbulence itself.

5. Refractive index structure constant

In the theory of electromagnetic wave propagation in turbulence, the refractive index structure constant is taken as a measure of the strength of the turbulence. A lot of efforts have been made to measure this parameter at different regions and seasons. However, this parameter does not indicate the fluctuation strength directly. It is found that the strength of turbulence is the variance of the refractive index. In the inertial sub-range, the structure function of refractive index generally depends on both the variance and the turbulence outer scale [3].

6. Methodology

This is a preliminary study conducted in the equatorial region and since the Partial Reflection Radar is not an efficient tool for turbulent studies, the estimation of C_n^2 is very difficult. However, this paper tried to develop a method to derive C_n^2 profiles from available high resolution partial reflection radar data. To simplify the analysis, we make the assumption that there are no errors in the radial velocities and the radial velocity terms have been perfectly converted into horizontal components. Starting with the horizontal components, there is an in-built post-processing software to compute the horizontal divergence at each level which is then integrated to get the vertical velocity. Software program has been developed based on the eqs. (8) and (9) to compute C_n^2 values from the received signal power (P_r). Radar can measure the refractive index structure function, because C_n^2 is proportional to the radar reflectivity [12].

$$C_n^2 = (\eta/0.38)\lambda^{1/3},$$

where η is the radar reflectivity and λ is the radar wavelength.

7. Results and discussion

Data from first January 2005 to thirty first January 2005 had been taken for the present analysis. 24 hours data is used for the evaluation of C_n^2. Partial Reflection Radar normally provides information from 80–110 km, for the sake of computation windowing is done between 80–88 km ranges. The receiver-1 value is given as C_{n1}^2 and receiver-2 values are given as C_{n2}^2. A comparative study of different C_n^2 (average) has been made for this equatorial station.

The C_n^2 values were studied to see what activity proceeded in the thermosphere during the sun set and sun rise hours in this zone. To show the effect, we selected three type *viz.* severe, moderate and weak having different C_n^2 values. It was found that most of the days showed the post-sunset peak at 20.00–21.00 IST as depicted in Figure1. The figure depicts the refractive index structure constant C_{n1}^2, C_{n2}^2 and vertical velocity (data of 1st January 2005); both C_{n1}^2 and C_{n2}^2 showed an enhancement. Moderate C_n^2 values were also observed on 15th, 17th and 20th January as shown in Figure 2. The minimum values were noted in rare cases and depicted as such in Figure 3.

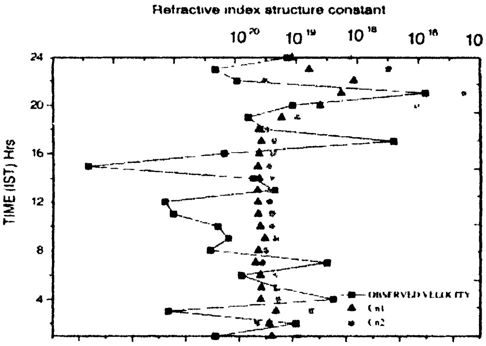

Figure 1. The refractive index structure constant C_{n1}^2 (▲), C_{n2}^2(•) and vertical velocity (—) [data from partial reflected echoes].

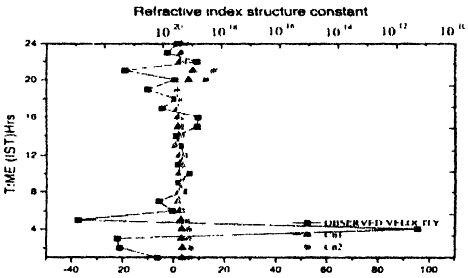

Figure 2. The refractive index structure constant C_{n1}^2 (▲), C_{n2}^2(•) and vertical velocity (—) [data from partial reflected echoes].

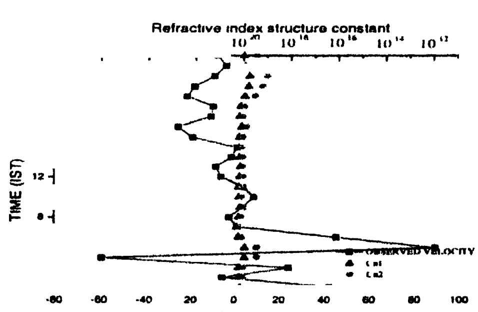

Figure 3. The refractive index structure constant C_{n1}^2 (▲), C_{n2}^2(•) and vertical velocity (—) [data from partial reflected echoes].

The study reveals that the association between the refractive index structure function C_n^2 is proportional to the radar reflectivity. The quantification of turbulence in this regime may provide the Physics of E-layer dynamics after the post-sunset hours. However, the detail examination and classification of the cause will require further investigations.

8. Conclusions

A study of radar based measurements of C_n^2 is carried out in this paper. The determination of the refractive index structure constant of the turbulence is often one of the primary objectives in any study of turbulence. In the theory of electromagnetic wave propagation in turbulence, the refractive index structure constant is taken as a measure of the strength of the turbulence. The refractive index structure constants for turbulence is calculated numerically and plotted with the vertical wind velocity. C_n^2 values show an enhancement during post-sunset hours. It may be due to the fact that plasma within the geomagnetic flux tube becomes unstable because of pre-reversal enhancement ($E \times B$ electrodynamic drift) and take part in the up-welling process thus creating turbulence within the volume.

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