Indian J. Phys. **49**, 38–48 (1975)

Nonlinear interactions of electromagnetic waves in a hot magnetised plasma

T. P. KHAN

Dinabandhu Andrews College, Garia, Calcutta 743505

AND

S. R. ROY

Jadavpur University, Calcutta 700032

AND

R. K. ROYCHOUDHURY

Moulana Azad College, Calcutta

(*Received* 30 *May* 1974)

The nonlinear interactions of electromagnetic waves propagating in a hot plasma across a magnetic field is investigated. The excited waves due to nonlinear interactions may be found to be unstable above certain threshold value of the amplitude of the pump wave.

1. INTRODUCTION

Recently much effort has been made on understanding the nonlinear wave-wave interactions in plasma. Phelps (1971) observed a nonlinear coupling of electromagnetic wave and electrostatic wave resulting in generation of plasma wave with the difference frequency, where the frequency and wave number conservations were fulfilled between these interacting waves. Chang & Prokalab (1970) observed the decay of a finite amplitude cyclotron harmonic wave into two cyclotron harmonic waves. Etievant *et al* (1968) observed the nonlinear interaction process of interaction of three electrostatic waves in a cold plasma. As long as the plasma temperature is low this description is correct. For warm plasmas and working frequencies near the first harmonic of the electron cyclotron frequency the nonlinear processes considered by Etievant *et al* (1968) depend on the value of T_0 (Cano *et al* 1969).

In this paper we report new aspects besides the power factor which serve to detect the secondary radiation by conventional receivers. Among these are (1) the rapid variation of the power factor about different values of plasma frequency even when the frequency of the extraordinary wave is equal to the cyclotron frequency, (2) the threshold condition for decay process of three interacting waves. Agreement between the experimental results of Chang *et al* (1970) and the theoretical analysis is attained.

2. Basic Equation

We start with Maxwell equations coupled with the equation of motion for a hot plasma. The ions are treated as fixed uniform background. The unperturbed state is taken to be a spatially uniform plasma with a constant magnetic field along the z-axis

$$\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla \boldsymbol{V} = -v_t^2\frac{\nabla n}{n^0}-\frac{e}{m}\left[E+\frac{V\times B}{C}\right]-\nu \boldsymbol{V},$$

$$\nabla\times \boldsymbol{E} = -\frac{1}{C}\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla\times \boldsymbol{B} = \frac{1}{C}\frac{\partial \boldsymbol{E}}{\partial t}-4\pi en\boldsymbol{V}, \tag{1}$$

$$\nabla\cdot B = 0,$$

$$\nabla\cdot \boldsymbol{E} = -4\pi e(n-n^0),$$

where

v_t = thermal velocity of the electrons.

To solve eq. (1), we expand the equations by perturbation technique

$$\begin{bmatrix} v \\ E \\ B \\ n \end{bmatrix} = \begin{bmatrix} v^{(0)} \\ E^{(0)} \\ B^{(0)} \\ n^{(0)} \end{bmatrix} +\lambda \begin{bmatrix} v^{(1)} \\ E^{(1)} \\ B^{(1)} \\ n^{(1)} \end{bmatrix} +\lambda^2 \begin{bmatrix} v^{(2)} \\ E^{(2)} \\ B^{(2)} \\ n^{(2)} \end{bmatrix} + \dots \tag{2}$$

First order state

We look for stationary solutions of first order quantities of eq. (1) and by taking the space time dependence $\exp i(K.r-\omega t)$. We obtain

$$-i(\omega+i\nu)\boldsymbol{V}^{(1)}-\boldsymbol{\Omega}\times \boldsymbol{V}^{(1)} = -\frac{e}{m}E^{(1)}+\frac{iKv_t^2n^{(1)}}{n_0} \tag{3}$$

where Ω and ν are the cyclotron frequency and collision frequency respectively. In a similar way other first order equations are obtained from eqs. (1) and (2). These equations immediately give

$$\boldsymbol{V}^{(1)} = \boldsymbol{\mu}.\boldsymbol{E}^{(1)}$$

where μ is given by

$$\mu = \frac{e}{m}\frac{i\bar{\omega}}{(\Omega^2-\bar{\omega}^2)}\left[I-\frac{\Omega^2}{\omega^2}\hat{z}\hat{z}-\frac{i\Omega}{\omega}\left(1+\frac{K^2v_t^2}{\omega_p}\right)\hat{x}\hat{y}+\frac{i\Omega}{\omega}\hat{y}\hat{x}+\frac{K^2v_t^2}{\omega_p^2}\hat{y}\hat{y}\right], \tag{4}$$

and $\bar{\omega} = \omega+i\nu$ and I is the unit tensor.

Fourier tansforms of first order fields of Maxwell equations lead to

$$D.\boldsymbol{E}^{(1)} = 0,$$

where

$$D = \left(K^2 - \frac{\omega^2}{C^2}\right)I - K^2\,\hat{y}\hat{y} + \frac{i\,\omega\,\omega_p{}^2}{C^2}\,\boldsymbol{\mu}. \qquad \ldots \quad (5)$$

In the case of propagation perpendicular to $B^{(0)}$ we have the dispersion relation

$$\text{Det}\ D_0(\boldsymbol{K},\ \omega) = 0 \qquad \ldots \quad (5a)$$

where $D_0(\boldsymbol{K},\ \omega)$ is obtained by letting $K_x = K_z = 0$, $K_y = K$ in $D(\boldsymbol{K},\ \omega)$. Eq. (5*a*) has two independent solutions,

(i) $C^2K^2 = \omega^2 - \omega_p{}^2 + i\nu\,\dfrac{\omega_p{}^2}{\omega}$, ordinary curves,

(ii) $$C^2K^2 = \omega^2 - \omega_p{}^2 - \Omega^2\omega_p{}^2\left[(\omega^2-\omega_p{}^2)(1-\beta^2) - \Omega^2\left(1 - \frac{\beta^2\omega_p{}^2}{\omega^2-\omega_p{}^2-\Omega^2}\right)\right]^{-1}$$

$$+ \frac{i\nu\,\omega_p{}^2}{\omega}\ \frac{(\omega^2-\omega_p{}^2)^2+\omega^2\Omega^2}{(\omega^2-\omega_p{}^2-\Omega^2)^2}, \text{ extraordinary waves,} \qquad \ldots \quad (6)$$

where

$$\omega_p{}^2 = \frac{4\pi e^2 n^{(0)}}{m}, \quad \beta = \frac{v_t}{C}.$$

In the following we shall denote the extraordinary electric fields by $\boldsymbol{E}_e$ and the ordinary electric fields by $\boldsymbol{E}_0$. In the case of propagation perpendicular to $\hat{z}$ the first order solution for the extraordinary mode is given by (neglecting term with collision frequency)

$$\boldsymbol{E}_e = A[ia\hat{e} + \hat{K}]\exp i(k.r-\omega t) \equiv \epsilon_e \exp i(k.r-\omega t)$$

$$V_e = -\frac{cA\,\omega^2}{m\omega_p{}^2\Omega}\left[\left(1 - \frac{\omega_p{}^2+K^2v_t{}^2}{\omega^2}\right)\hat{e} + \frac{i\Omega}{\omega}K\right] \exp i(k.r-\omega t)$$

$$\boldsymbol{B}_e = -\frac{iaCK}{\omega}\,Az\exp i(k.r-\omega t), \qquad \ldots \quad (7)$$

$$n_e = -\frac{iKA}{4\pi e}\exp i(k.r-\omega t),$$

where A is the normalization constant

$$A = \frac{\epsilon_e}{|(1+a^2)^{\frac{1}{2}}|},\ \hat{K} = \frac{\boldsymbol{K}}{|\boldsymbol{K}|},\quad \hat{e} = \hat{K}\times\hat{Z}$$

$$a = \frac{\omega}{\omega_p{}^2\Omega}[\Omega^2 - \omega^2 + \omega_p{}^2 + K^2v_t{}^2]. \qquad \ldots \quad (8)$$

The first order solution of the ordnary mode is given by

$$\boldsymbol{E}_0^{(1)} = \epsilon_{11}\hat{Z} \exp i(k.r-\omega t),$$

$$\boldsymbol{V}_0^{(1)} = -\frac{ie}{m\omega} \epsilon_{11}\hat{Z} \exp i(k.r-\omega t),$$

$$\boldsymbol{B}_0^{(1)} \quad \frac{CK}{\quad} \,\iota\hat{e}\, \epsilon_{11} \exp i(k.r-\omega t), \qquad \ldots (9)$$

$$n_0^{(1)} = 0.$$

Second order equations

In a similar way we can develop the second second order equations to obtain

$$\nabla \times (\nabla \times E^{(2)}) - \frac{\omega^2}{C^2} E^{(2)} + \frac{4\pi n^0 i\omega}{C^2} \mu . E^{(2)} = \frac{4\pi i\omega}{C^2} \boldsymbol{J}_s, \tag{10}$$

where

$$\boldsymbol{J}_s = -e \left[n^0 \mu . \left(\frac{v^{(1)} \times B^{(1)}}{C} + \frac{m}{e} v^{(1)} \Delta v^{(1)} \right) + n^{(1)} v^{(1)} \right]. \tag{11}$$

By method of the Fourier transforms of eq. (10) we obtain,

$$\boldsymbol{E}^{(2)}(x, t) = -\frac{4\pi\omega\, T.\boldsymbol{J}.}{C^2} \exp i(k.r-\omega t) \int_{-\infty}^{y} dy_0 g(x, y_0, z), \tag{12}$$

where

$$T \equiv (K'_y - K_y) D^{-1}$$

and

$$\boldsymbol{K}' = \boldsymbol{K} + \delta\boldsymbol{K}.$$

$$D = D_0(\boldsymbol{k}, \omega) + D_1(\omega, \boldsymbol{K}, \delta\boldsymbol{K}) \qquad \text{with} \quad D_0(\omega, \boldsymbol{K}),$$

defined by eq. (5) and

$$D_1 \doteq$$

$$2K\delta K_y - \frac{i\omega\Omega v_+^2 K\delta K_x}{C^2(\Omega^2-\omega^2)}, \; -K\delta K_x + \frac{\omega^2 v_t^2 K\delta K_x}{C^2(\Omega^2-\omega^2)} - \frac{i2\omega\Omega v_t^2 K\delta K_y}{C^2(\Omega^2-\omega^2)}, \; \frac{-i\omega\Omega v_t^2 K\delta K_z}{C^2(\Omega^2-\omega^2)}$$

$$-K\delta K_x + \frac{\omega^2 v_t^2 K\delta K_x}{C^2(\Omega^2-\omega^2)}, \; \frac{i\omega\Omega v_t^2 K\delta K_x}{C^2(\Omega^2-\omega^2)} + \frac{K^2 v_t^2 2K\delta K_y}{C^2(\Omega^2 \omega^2}, \; -K\delta K_y + \frac{\omega^2 v_t^2 K\delta K_e}{C^2(\Omega^2-\omega^2)},$$

$$0 \quad , \qquad -\left[1 + \frac{v_t^2}{C^2} K\delta K_z \right], \; 2K\delta K_y$$

We now give D^{-1} for ordinary wave and extraordinary wave respectively.

(a) Ordinary wave

$$D^{-1} = \frac{\hat{Z}\hat{Z}}{2K\delta K_y}, \qquad \ldots (13)$$

(b) Extraordinary wave

$$D^{-1} = \frac{M}{2K\delta K_y\left(M_{yy} + \frac{v_t^2\omega^2 M_{xx}}{C^2(\Omega^2-\omega^2)} + \frac{i\omega\Omega v_t^2 M_{yx}}{C^2(\Omega^2-\omega^2)}\right)}, \qquad \ldots (14)$$

where

$$M = \begin{bmatrix} -\frac{\omega^2(\Omega^2-\omega^2+\omega_p^2+K^2v_t^2)}{C^2(\Omega^2-\omega^2)}, & \frac{i\omega\Omega(\omega_p^2+K^2v_t^2)}{C^2(\Omega^2-\omega^2)}, & 0 \\ \frac{i\omega\omega_p^2}{C^2(\Omega^2-\omega^2)}, & \frac{(C^2K^2-\omega^2)(\Omega^2-\omega^2)+\omega_p^2\omega^2}{C^2(\Omega^2-\omega^2)}, & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In both cases above, the dependence of D^{-1} on $\delta K_y = K_y'-K_y$ comes in as a simple pole. For a point outside the interaction region the integral in eq. (12) can simply be deonoted as

$$R_0 = \int_{-\infty}^{\infty} dy_0 g(xy_0z).$$

Interaction of an Extraordinary Wave with an Ordinary Wave

If one of the incident wave is ordinary and other extraordinary the second order wave turns out to be an ordinary wave. In this case the second order electric field outside the source region is given by eqs. (11), (12) and (13) as

$$E^{(2)}(x,t) = \pm\frac{eR_0AK_1\omega_\pm}{4mC^2\omega_2K_\pm}\cos(k_\pm r-\omega t). \qquad \ldots (15)$$

We would like to express our results in terms of the incoming and outgoing Poynting vectors. We obtain the dimensionless power factor

$$F_1(\omega_1,\omega_2,\omega_3) = \left(\frac{m}{e}\right)^2 \frac{C^5\bar{S}_3}{(\pi/2)R_0^2\bar{S}_1\bar{S}_2} = \frac{n_1}{n_2n_3}\frac{1}{a^2}\left(\frac{\omega_1}{\omega_2}\right)^2, \qquad \ldots (16)$$

where

$$n_1 = \left[1-\frac{\omega_p^2}{\omega_1^2}-\frac{\omega_p^2}{\omega_1^2-\omega_p^2(1-\beta^2)-\left(1-\frac{\beta^2\,\omega_p^2}{\omega_1^2-1-\omega_p^2}\right)}\right]^{\frac{1}{2}},$$

$$n_2 = \left(1-\frac{\omega_p^2}{\omega_2^2}\right)^{\frac{1}{2}}, \qquad \ldots \quad -(17)$$

$$n_3 = \left(1-\frac{\omega_p^2}{\omega_3^2}\right)^{\frac{1}{2}},$$

where bar denotes time averaging and $\bar{S}_t = \hat{K}.\bar{S_t}$, 1 is the incoming extraordinary wave, 2 is the incoming ordinary wave, and 3 is the outgoing ordinary wave and all frequencies have been normalised in terms of cyclotron frequency.

Interaction of two ordinary waves

Interaction of two ordinary waves produces an extraordinary wave. Using eqs. (11), (12), (14), we obtain

$$E^{(2)} = \frac{eR_0\epsilon_{\parallel 1}\epsilon_{\parallel 2}(\omega_p^2+K^2v_t^2)\omega_{\pm}\Omega}{4mC^2[\Omega^2-\omega^2+\omega_p^2+K^2v_t^2-\beta^2(C^2K^2-\omega^2+\omega_p^2)\omega_1\omega_2]}$$

$$\times\left[K_{\pm}\times\hat{Z}\sin(K_{\pm}x-\omega_{\pm}t)+\hat{K}_{\pm}\cos(\hat{K}_{\pm}x-\omega_{\pm}t)\frac{\omega_{\pm}^2-C^2K_{\pm}^2-\omega_p^2}{\omega_{\pm}\Omega}\right] \qquad \ldots \quad (18)$$

The power factor in this case is

$$F_2(\omega_1\,\omega_2\,\omega_p) = \frac{\bar{S}_3}{\bar{S}_1\bar{S}_2}\,\frac{C}{\pi/2}\left(\frac{mC^2}{eR_0}\right)^2$$

$$= \frac{n_3}{n_2n_1}\,\frac{[\Omega\omega_3(\omega_p^2+K^2v_t^2)]^2}{[\omega_1\omega_2(\Omega^2+\omega_p^2+K^2v_t^2-\omega^2+\beta^2(C^2K^2-\omega^2+\omega_p^2)]^2} \qquad \ldots \quad (19)$$

where 1 and 2 are the incoming ordinary waves and 3 is the outgoing extraordinary wave and n_1, n_2, n_3 are obtained from dispersion relation (17).

3. Numerical Computation and Results

Having exhibited, in the previous section, the basic formulas to be used we now wish to compute $F(\omega_1, \omega_2, \omega_p)$ for different values of ω_1, ω_2, ω_p. The computational problem is one of search and optimization. We wish to find largest F for a given ω_p with some combination of ω_1 and ω_2 which will give meaningful θ_{12} and θ_{23}, $-1 \leqslant \cos\theta_{12} \leqslant 1$ and $-1 \leqslant \cos\theta_{23} \leqslant 1$, where θ_{12}, θ_{23} a e the angles between the wave vectors $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ and $\boldsymbol{K}_2$, $\boldsymbol{K}_3$. It has been observed that the power factor $F_1(\omega_1, \omega_2, \omega_p)$ and $\cos\theta_{12}$, $\cos\theta_{23}$ depend appreciably on the plasma temperature in the case of $\omega_1 = \Omega$ for different values of ω_2. In figures 1 and 2, plots of $F_1(\omega_1, \omega_2, \omega_p)$ and $\cos\theta_{12}$, $\cos\theta_{23}$ respectively are shown against $(\omega_p/\Omega)^2$ for $\omega_1 = \Omega$ and $\omega_2 = 0.5\Omega$. It is evident from the figures that this time quantities are very much sensitive to $(\omega_p/\Omega)^2$ when the plasma temperature is sufficiently high. It is interesting to note that $F_1(\omega_1, \omega_2, \omega_p)$, $\cos\theta_{12}$ and $\cos\theta_{23}$ become practically independent of temperature when ω_1 is less than Ω, and the graphs are very much similar to those obtained earlier by Etievant *et al* (1968).

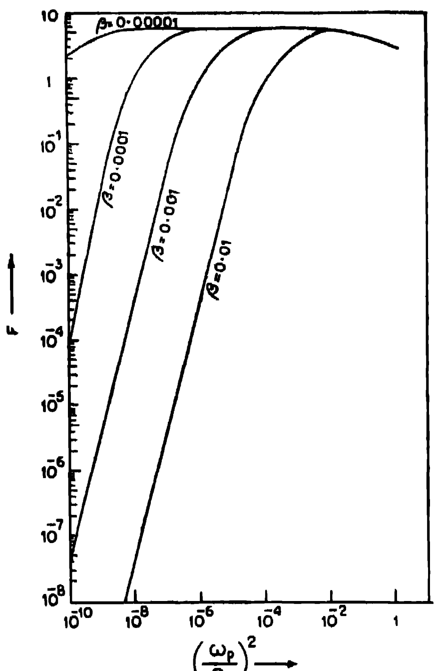

Fig. 1. Power factor F_1 versus $(\omega_p/\Omega)^2$ with $\omega_2 = 0.5\Omega$ and $\omega_1 = \Omega$, for different values of β as indicated in the plot.

Criteria for instability

We look for plane wave solutions with varying amplitudes in the direction of propagation *i..e*,

$$E\omega(r) = \hat{a}_w A_w \left(\frac{k.r}{K} \right) e^{ik \cdot r}, \qquad \ldots \quad (20)$$

where $\hat{a}_w$ is a unit vector and in accordance with our assumption

$$\frac{1}{KA} \left| \frac{dA}{dr} \right| << 1.$$

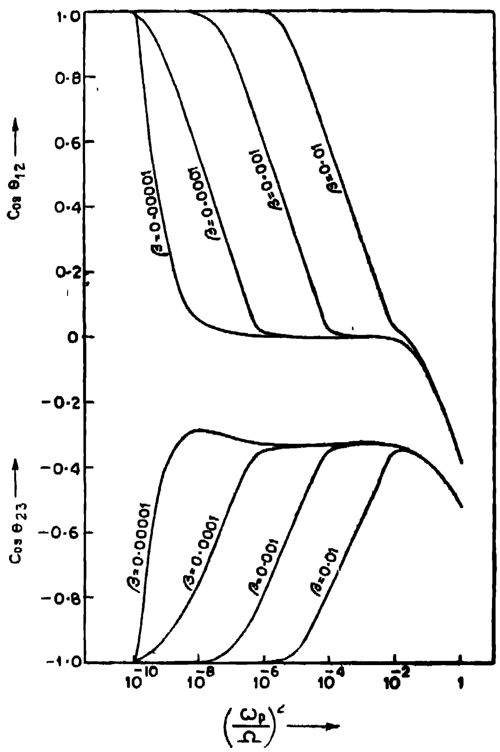

Fig. 2. Plot of $\cos \theta_{12}$ and $\cos \theta_{23}$ versus $(\omega_p/\Omega)^2$ with $\omega_1 = \Omega$ and $\omega_2 = 0.5\Omega$ for different values of β as indicated on the curves.

Using eq. (17) in eq. (10) and since $D_0(k, \omega).E_\omega = 0$ when $\boldsymbol{K}$ and ω satisfy the linear dispersion relation we obtain dropping the second derivative of A in comparison with K^2A

$$2i\left(1-\left|\frac{K.\hat{a}_w{}^2}{K}\right|^2\right)K \cdot \frac{\partial A\omega_1}{\partial r} = -\hat{a}\,\omega_1{}^* \frac{4\pi i\omega_1}{C^2} J_s e_\omega{}^{ik\cdot r}, \qquad \ldots \quad (21)$$

where $\boldsymbol{J}_{s\omega}$ can be written as,

$$\hat{a}\,\omega_1{}^*.\mathrm{J}_{s\omega} = iC^2V(\omega_1|\,\omega_2|\,\omega_3)$$

$$V(\omega_1|\,\omega_2|\,\omega_3) = -\frac{4\pi e n_0\omega_1}{2C^2}\left\{-\frac{1}{\omega_3}[\hat{a}_{\omega_3}.\boldsymbol{\mu}_{\omega_2}.\hat{a}\,\omega_2\hat{a}\,\omega_1{}^*.\ \boldsymbol{\mu}\,\omega_1K_3\right.$$

$$-\boldsymbol{K}_3.\boldsymbol{\mu}\,\omega_2.\hat{a}\,\omega_2.\hat{a}\omega_1{}^*.\,\boldsymbol{\mu}.\hat{a}\omega_1]-\frac{im}{e}\boldsymbol{K}_3.\boldsymbol{\mu}\,\omega_2.\hat{a}\omega_2.\hat{a}^*\omega_1.\boldsymbol{\mu}\,\omega_1(\mu\omega_3.\hat{a}\omega_3)$$

$$\left.-\frac{1}{\omega_2}\boldsymbol{K}_2.\boldsymbol{\mu}\,\omega_2.\hat{a}\omega_2.\hat{a}^*\omega_1.\boldsymbol{\mu}\,\omega_3.\hat{a}\omega_3+2 \rightleftharpoons 3\right\},$$

for each of the three interacting waves.

We can give the expression for V obtained from the interaction of two ordinary waves with an extraordinary wave. Let

$$a_{\omega_1} = a_{\omega_3} = \hat{z} \qquad \text{ordinary waves}$$

$$a_{\omega_2} = \frac{1}{(1+a_2^2)^{\frac{1}{2}}} (ia_2\hat{e}+\hat{K}_2) \qquad \text{extraordinary waves}$$

and now define

$$\alpha = \frac{K_1 \cdot n}{K_1^2} \left| V(\omega_1 | \omega_2 | \omega_3) \right|$$

$$\beta = -\frac{K_2 \cdot n}{K_2^2} \left(\frac{\omega_2}{\omega_1}\right)^2 \frac{\left(1+\frac{K_2^2 v_t^2}{\Omega^2-\omega^2}\right)\left(1+\frac{K_2^2 v_t^2}{\omega_p^2}\right)}{\left[1-\left|\frac{K_2 . a\omega_2}{K_2}\right|^2\right]} \left| V(\omega_1 | \omega_2 | \omega_3) \right| \quad \dots \quad (22)$$

$$\gamma = -\frac{K_3 \cdot n}{K_3^2} \left(\frac{\omega_3}{\omega_1}\right)^2 \left| V(\omega_1 | \omega_2 | \omega_3) \right|$$

$$\left| V(\omega_1 | \omega_2 | \omega_3) \right| = \frac{e}{m} \frac{\omega_p^2 \Omega \omega_1 K_2}{2C^2 \omega_2 \omega_3} \left[\frac{\Omega^2 \omega_p^4}{\omega_2^2} + (\omega_p^2+\Omega^2-\omega_2^2+K^2 v_t^2)^2\right]^{-\frac{1}{2}},$$

and write equations for three waves following Sagdeev & Galeev (1969)

$$\frac{\partial A_{\omega_1}}{\partial S} = \alpha A_{\omega_1} A_{\omega_2},$$

$$\frac{\partial A_{w_2}^*}{\partial S} = \beta A_{\omega}{}^*{}_1 A_{\omega_3} \quad \dots \quad (23)$$

$$\frac{\partial A_{\omega_3}^*}{\partial S} = \gamma A_{\omega_1}^* A_{\omega_2}$$

where $\boldsymbol{r} = \boldsymbol{n} . S$.

We now investigate the process of decay instability in the nonlinear interactions of three waves. The perturbations $A\omega_2$ and $A\omega_3$ are assumed to be small compared to the amplitude of incident wave $A\omega_1$. Assuming $A\omega_2$ and $A\omega_3$ to be slowly varying functions of S such that $A\omega_2 \sim A\omega_3 \sim e^{\lambda S}$ and $\frac{\partial A\omega_1}{\partial S} \sim A\omega_2 . A\omega_3 \sim 0$ i.e., $A\omega_1$ being constant, the instability may be possible with the maximum growth rate given by

$$\lambda = [\beta\gamma]^{\frac{1}{2}} A\omega_1$$

$$= \frac{e\omega_p^2 \Omega A\omega_1}{2mC^2(\omega_p^2+\Omega^2-\omega_2^2+K^2 v_t^2)} \left[\frac{K_2}{K_3}\left(1+\frac{K_2^2 v_t^2}{\Omega^2 \omega_2^2}\right)\left(1+\frac{K_2^2 v_t^2}{\omega_p^2}\right)\right]^{\frac{1}{2}}, \quad \dots \quad (24)$$

provided the inequality

$$\lambda > Im\, K_3 > \frac{\nu\omega_p^2}{2C^2}\left(\frac{1}{K_2\omega_2} - \frac{1}{K_1\omega_1}\right), \tag{25}$$

is satisfied which gives the threshold value of $A\omega_1$. It may be mentioned that Prokalab & Chang (1970) had calculated the threshold value of electron Bernstein wave decay instability using Vlasov equation. Though it is generally believed that such instability cannot be obtained from fluid equations because it involves electrostatic Bernstein waves it is interesting to note that our calculation are in good agreement with experimental results. Substituting the following experimental values of Chang & Prokalab (1970).

Temperature of electron plasma $= 3.7$ eV

Number density of electrons $= 2.5 \times 10^{10}$ cm^{-3}

Electron-neutral collision frequency $\nu = 5 \times 10^{6}$ sec^{-1}.

The frequencies and wave number of three waves

$\omega_1 = 758$ MHz $\qquad K_1 = 19.4$ cm^{-1},

$\omega_2 = 335$ MHz $\qquad K_2 = 39.6$ cm^{-1},

$\omega_3 = 420$ MHz $\qquad K_3 = 20.7$ cm^{-1}.

We find the threshold value of the amplitude of the incident wave to be

$$A\omega_1 = 2.7 \text{ volt/cm}$$

which is in good agreement with experimental value $A\omega_1 \sim 2$–3 V/cm of Chang & Prokolab (1970)

4. Conclusion

Our numerical computation shows that the power of the generated wave is a rapidly varying function of ω_p^2. It may be mentioned that, contrary to the case of cold plasma the power is not independent of ω_p^2 when the frequency of the extraordinary wave is equal to the cyclotron frequency. It is not much sensitive to plasma temperature when $\omega_1 < \Omega$. The excited wave is found to be unstable above certain threshold value of the pump wave. Our analysis may help to interpret the results of instabilities of waves in a nonlinear decay process.

Acknowledgment

We are grateful to Mr. Amitananda Das, Department of Radio Physics, Calcutta University for helping us in the numerical computations, one of us (T.P.K.) likes to thank Professor T. Roy of Jadavpur University, Calcutta-32, for constant encouragement and helpful suggestions.

References

Chang R. P. H. & Prokolab M. 1970 *Phys. Fluids* **13**, 2766.

Cano R., Etievant C., Fidone I. & Granta G. 1969 *Nucl. Fusion* **9**, 223.

Etievant C., Ossakow S., Ozizmir E. & Su C. H. 1968 *Phys. Fluids* **11**, 1778.

Phelps D., Rynn N. & Van Hoven G. 1971 *Phys. Rev. Letters* **26**, 688.

Prokolab M. & Chang R. P. H. 1970 *Phys. Fluids* **13**, 2054.

Sagdeev R. Z. & Galeev A. A. 1969 *Nonlinear Plasma Theory*, Edited by T. M. O'Neil and D. L. Book, Benjamin, New York.