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Renormalization-group-improved radiatively corrected Higgs masses in the minimal supersymmetric standard model at LEP2

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Abstract . Recently, using a very simple approximation scheme which includes the most important terms in the radiative corrections for the Higgs masses in the Minimal supersymmetric standard model (MSSM), Haber *et al* have estimated the Higgs masses over a very large fraction of MSSM parameter space. The purpose of this paper is to apply the method of solving the renormalization group equations for top quark and bottom quark Yukawa couplings in the two-Higgs doublet model given by Panda and Usmani to the above studies of Haber *et al*. Here the effects of the running vacuum expectation values (VEVs) in the two Higgs doublet model below μ M_{susy} have also been taken into account in terms of solutions of RGEs for the VEVs $v_1(\mu)$ and $v_2(\mu)$. It may be mentioned that at mass scales below M_{susy}, the solutions of the non-SUSY (Supersymetry) two Higgs doublet model are used. Interestingly, new results are obtained from systematic studies on the mass of lightest Higgs boson in MSSM which have importance in view of the LEP2 data and future LHC experiments

Keywords MSSM, renormalization group equation (RGE) Higgs masses

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1. Introduction

The standard model of particle physics is consistent with all experimental data except that the predicted Higgs bosons, required for giving masses to quarks and leptons, have not been observed. The discovery of Higgs boson will provide the proof that in the minimal standard model, the Higgs mechanism is the correct description of electroweak symmetry breaking [1]. The minimal standard model Higgs sector is not theoretically well-motivated due to the naturalness (or hierarchy) problem. The simplest and economical model realizing low energy supersymmetry is the minimal supersymmetric standard model (MSSM) [2] which can naturally accommodate elementary Higgs bosons. In MSSM, a supersymmetric partner is added to each quark, lepton and gauge boson. Moreover, it contains (i) minimal gauge group : $SU(3)_c \times SU(2)_L \times U(1)_Y$; (ii) minimal particle content : three generations of quarks and leptons and two Higgs doublet plus their superpartners; (iii) an exact discrete R-parity; (iv) supersymmetry breaking parametrised by explicit but soft breaking terms which includes gaugino, scalar masses and trilinear scalar couplings.

The MSSM Higgs sector is very much constrained by the concept of supersymmetry. The Higgs boson search will play a dominant role in the LEP2 programme and LHC. There are numerous works on the study of Higgs masses within the framework of MSSM [3].

The paper is organised as follows. The theory is given in Section 2. The RGEs for top quark, bottom quark couplings, $v_1(\mu)$ and $v_2(\mu)$ along with their solutions for two-Higgs doublet model are given in Section 3. The RG-modified Higgs masses without s-quark mixing effect in the framework of the model of Haber *et al* is given in Section 4 whereas the corresponding RG-modified Higgs masses with s-quark mixing is given in Section 5. The results and discussions are given in Section 6. Conclusions are given in Section 7.

2. Theory

The MSSM Higgs sector contains two Higgs doublet [2]. The scalar particles are two CP-even scalars h^0 and H^0 (with

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$m_{h^0} \le m_{H^0}$), a CP-odd scalar A^0 and a charged Higgs pair $H^{\pm}$. At the tree level, all Higgs masses depend on two parameters m_{A^0} and the ratio $\frac{v_2}{v_1}$ of Higgs vacuum expectation values (= tan β). The tree level bound is $(m_{h^0}^2) \le m_z\,|\cos 2\beta\,|$. The Higgs masses increase [4,5], when the radiative corrections are included. The method of calculation of radiative corrections are given in Haber and Hempling [6]. Following Ref. [6], the one-loop corrected Higgs mass is given (symbolically) by

$$M^2 = M_0^2 + \Delta M_{1LL}^2 + \Delta M_{mix}^2. \tag{1}$$

Here 0 refers to the tree level results. The subscript $1LL$ refers to the one-loop leading logarithmic approximation to the full one-loop calculation and the subscript 'mix' refers to the contributions due to $\tilde{q}_L - \tilde{q}_R$ mixing effects of the third generation s-quarks. The CP-even Higgs mass squared eigenvalues [6] are

$$m_{H^0,h^0}^2 = \frac{1}{2}\left[M_{11}^2 + M_{22}^2 \pm \sqrt{\left[M_{11}^2 - M_{12}^2\right]^2 + 4(M_{22}^2)^2}\,\right]. \tag{2}$$

The explicit formula for ΔM_{1LL}^2, and $(\Delta M^2)_{mix}$ given by Haber *et al* [6] are used for our studies.

3. Renormalization group equations (RGEs) for top and bottom quark couplings and their solutions for two-Higgs doublet model

At the beginning, we like to mention that we use the solutions of the non-SUSY two Higgs doublet model below the SUSY scale. In this model, the quark mass is the product of the Higgs-quark Yukawa coupling (h_q) and the appropriate vacuum expectation value. Haber and Hempling [6] have solved the RGEs for top and bottom quark couplings using some approximate values for $m_t(\mu)$ and $m_b(\mu)$ for two scales *viz*. $M_{A^0} = O(M_z)$ and $M_{A^0} = O(M_{susy})$ where the values of $m_t(\mu)$ and $m_b(\mu)$ are given in the two-Higgs doublet non-supersymmetric model by

$$m_b(\mu) = \frac{1}{\sqrt{2}} h_b(\mu)\, v_1(\mu), \tag{3}$$

$$m_t(\mu) = \frac{1}{\sqrt{2}} h_t(\mu)\, v_2(\mu), \tag{4}$$

along with the well-known normalization condition *viz*.

$$v_1^2 + v_2^2 = 4m_{\omega}^2 / g^2 = (246\, GeV)^2. \tag{5}$$

The renormalization group equations for the top quark, bottom quark and τ lepton Yukawa couplings in the two-Higgs doublet model are solved by Parida and Usmani [7] for $m_t \le \mu \le M_c$ where μ is the mass scale and M_c is the quark lepton unification scale. We consider the effect of mass scale (μ) on the running VEVs $v_1(\mu)$ and $v_2(\mu)$ [8,9]. Now, the top (h_t) and bottom (h_b) quark Yukawa couplings satisfy the following coupled renormalization group equations (RGEs) [7].

$$16\pi^2 \frac{dh_t}{dt} = h_t \quad \frac{9}{2}h_t^2 + \frac{1}{2}h_b^2 - \sum C_i^{(b)} g_i^2 \tag{6}$$

$$16\pi^2 \frac{dh_b}{dt} = h_b \quad \frac{9}{2}h_b^2 + \frac{1}{2}h_t^2 - \sum C_i^{(b)} g_i^2 \tag{7}$$

On the other hand, the vacuum expectation values $v_1(\mu)$ and $v_2(\mu)$ satisfy the following renormalization group equations [8, 9]

$$16\pi^2 \frac{dv_2}{dt} = v_2 \quad -3h_t^2 + \sum C_i^{(v)} g_i^2 \tag{8}$$

$$16\pi^2 \frac{dv_1}{dt} = v_1 \quad -3h_b^2 + \sum C_i^{(v)} g_i^2 \tag{9}$$

In eqs. (6) – (9) we have $t = \ln\mu$, $C_i^{(t)} = \left(\frac{17}{20}, \frac{9}{4}, 8\right)$, $C_i^{(b)} = \left(\frac{1}{4}, \frac{9}{4}, 8\right)$ and $C_i^{(v)} = \left(\frac{9}{20}, \frac{9}{4}, 0\right)$; $i = Y, 2L, 3C$. Following Parida and Usmani [7], for $\mu > m_t$, the solutions of eqs. (6) and (7) are

$$h_t(\mu) = \frac{h_t(m_t)}{A_t} e^{\frac{9}{2}I_t + \frac{1}{2}I_b}, \tag{10}$$

$$h_b(\mu) = \frac{h_b(m_t)}{A_b} e^{\frac{1}{2}I_t + \frac{9}{2}I_b}, \tag{11}$$

where

$$I_i = \int_{\ln m_t}^{\ln\mu} \frac{h_i^2(t)}{16\pi^2}\, dt, \quad i = t, b \tag{12}$$

and

$$A_f = \Pi\left(\frac{\alpha_i(\mu)}{\alpha_i(m_t)}\right)^{\frac{c_i}{2a_i}}. \tag{13}$$

The functions $A_f\ (f = t, b)$ are obtained by integrating out the gauge coupling contributions in eqs. (6, 7) and keeping terms upto one-loop out of the two-loop approximation

$$\frac{1}{\alpha_i(\mu)} \quad \frac{1}{\alpha_i(m_t)} - \frac{a_i}{2\pi} \ln\frac{\mu}{m_t} \tag{14}$$

where

$$\begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix} \tag{15}$$

Next, following the method of Parida and collaborators [8, 9], the solutions of eqs. (8) and (9) for the VEVs $v_1(\mu)$ and $v_2(\mu)$ are given by

$$v_2(\mu) = v_2(m_t)\prod \left(\frac{\alpha_i(\mu)}{\alpha_i(m_t)}\right)^{\frac{a_i^{(1)}}{2a_i}} \times e^{-3I_t} \tag{16}$$

$$v_1(\mu) = v_1(m_t)\prod \left(\frac{\alpha_i(\mu)}{\alpha_i(m_t)}\right) \times e \tag{17}$$

Now eqs. (16) and (17) give

$$\tan\beta(\mu) = \tan\beta(m_t) \times e^{(-3I_t + 3I_b)}. \tag{18}$$

The eqs. (10), (11), (16) and (17) can be used in eqs. (3) and (4) for calculating $m_b(\mu)$ and $m_t(\mu)$, which occur in the formulas for the Higgs masses. For a given $\tan\beta (= \frac{v_2}{v_1})$, the h_t and h_b Yukawa couplings are computed at $\mu = m_t$ using the following relations

$$h_t(m_t) = \frac{m_t(m_t)}{174\sin\beta} \tag{19}$$

$$h_b(m_t) = \frac{m_b(m_t)}{174\sin\beta} \tag{20}$$

The eqs. (10-20) are used in the model of Haber and Hempling [6] for the calculation of RG-modified Higgs masses

4. RG-modified Higgs Masses without s-quark mixing

In our work, we use the analytic solutions of RGEs given by Parida *et al* [7-9] which are summarized in Section 3. We apply these techniques to the one loop leading logarithmic CP-even Higgs squared mass matrix given by [6]

$$M_{1LL}^2 = M_0^2 + \Delta M_{1LL}^2, \tag{21}$$

where the dominant effects of RG-improvement is given by

$$M_{1RG}^2 = M_{1LL}^2\left(m_t(\mu_t), m_b(\mu_t)\right), \tag{22}$$

$$\mu_t \equiv \sqrt{m_t M_{SUSY}}\ , \quad \mu_b \equiv \sqrt{m_b M_{SUSY}}\ . \tag{23}$$

In our study, we vary the parameters over a wide range relevant to LEP2 and LHC.

5. RG-modified Higgs masses with s-quark mixing

Here, we consider the effects arising from mass splitting and $\tilde{q}_L - \tilde{q}_R$ mixing in the third generation s-quark sector. We follow the formalism of Haber *et al* [6] except that we consider the solutions of renormalization group equations given by Parida *et al* [7-9]. We also consider the parameters at the LHC range. At one-loop, the s-quark mixing introduces the shifts ΔM_{mix}^2 and $(\Delta M_{H^\pm}^2)_{mix}$. For $M_Q = M_U = M_D = M_{susy}$, the relevant formulas are given in Ref. [6] For non-zero squark mixing, M_{1RG}^2 is generalized to

$$M_{1RG}^2 \cong M_{1LL}^2 + \Delta M_{mix}^2 \equiv M_{1LL}^2\left(m_t(\mu_t), m_b(\mu_t)\right)$$

$$+\Delta M_{mix}^2\left(m_t(\mu_t), m_b(\mu_t)\right). \tag{24}$$

The details of calculations of Higgs mass squared are given in Ref. [6].

6. Results and discussion

The results of our systematic studies on the lightest Higgs boson masses (m_h) in MSSM are given in Figures (1-8). The variation of m_h with different parameters like M_{susy}, μ, μ_H, $\tan\beta$, m_A for mixing as well as non-mixing cases are studied. First, Figure 1 shows that for $\tan\beta = 1.5$ in the non-mixing case, m_h can vary from about 40 to 94 GeV, when M_{susy} is varied from 200 to 2000 GeV. In the context of LEP2 bounds on the mass of the lightest Higgs in MSSM, $\tan\beta = 1.5$ is ruled out. For large $\tan\beta = 20$ and 50, m_h varies from 93 to 122 GeVs as M_{susy} varies from about 200 to 2000 GeVs for $\mu = 1$ TeV, $m_A = 1$ TeV. In this case, M_A is very large (1 TeV) and $\mu = 1$ TeV

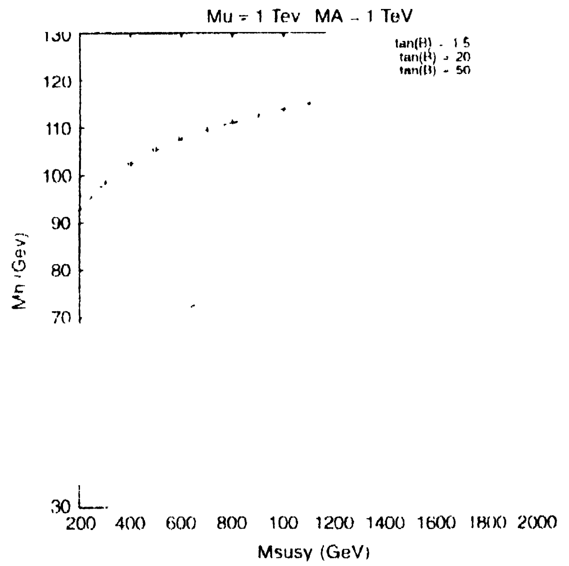

Figure 1. The radiatively corrected light CP-even Higgs mass (m_h) is plotted against M_{susy} for $\mu = 1$ TeV, $M_A = 1$ TeV in the cases of $\tan\beta = 1.5, 20, 50$

Next Figure 2 shows that for $M_A = 100$ GeV, m_h varies from 92 to 100 GeV in the non-mixing case for M_{susy} varying from about 200 to 2000 GeV for $\tan\beta = 50$ and $\mu = 200$ GeV. For $M_A = 500$ and 1000 GeVs, m_h can vary from 93 to 130 GeVs for M_{susy} varying from 200 to 2000 GeVs for $\mu = 200$ GeV. This scenario holds for $\tan\beta = 50$. It may be mentioned that in Figure 1 also, $\tan\beta = 50$ gives similar results on m_h in conformity with LEP2 bounds.

Again Figure 3 gives similar studies as in Figure 2 except that tan β (= 1.5) is very small in this case. Interestingly, the m_h values are much lower than the LEP2 bound on m_h. Here m_h

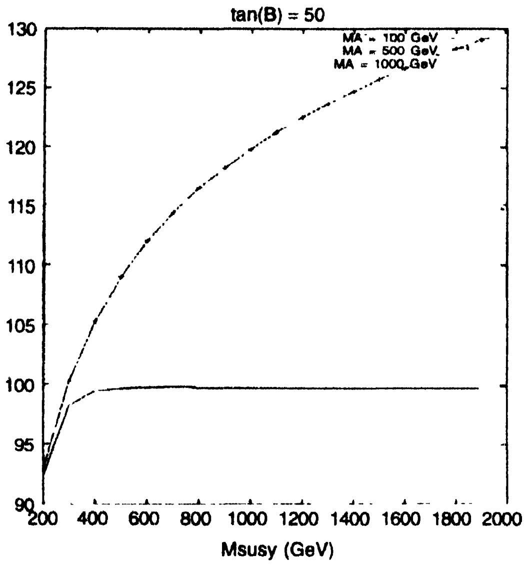

Figure 2. m_h is plotted against M_{susy} for tan β = 50, μ = 200 in the cases of M_A = 100, 500, 1000 GeV

varies from 32 to 70 GeVs as M_{susy} is varied from 200 to 2000 GeVs. This study suggests that the combination of low tan β (= 1.5) and low μ (= 200) are ruled out in MSSM. On the other hand, in Figure 4, the parameters are same as in Figure 3 except that tan β (= 20) is larger in this case. For M_A = 100 GeV, m_h varies from 90 to 100 GeVs which lie below the LEP2 bounds on m_h. Hence, $M_A \simeq M_Z$ is ruled out in MSSM. But for M_A = 500 and 1000 GeVs, m_h can take values from 92 to 129 GeVs for M_{susy} varying from 200 to 2000 GeVs. Thus, large tan β and low $\mu (\simeq m_t)$ seem to be favored by MSSM.

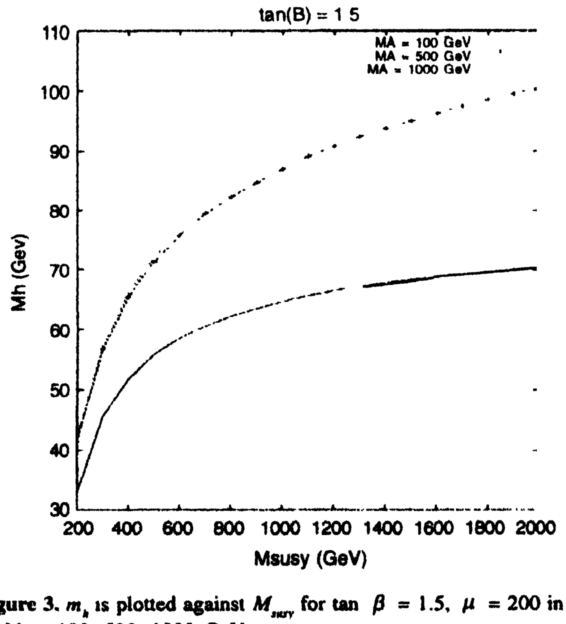

Figure 3. m_h is plotted against M_{susy} for tan β = 1.5, μ = 200 in the cases of M_A = 100, 500, 1000 GeV.

Again, Figure 5 gives variation of m_h with M_{susy} for M_A = 1000 GeVs, tan β = 20 in the non-mixing and mixing cases. In the non-mixing case, m_h varies from 92 to 130 GeVs for M_{susy}

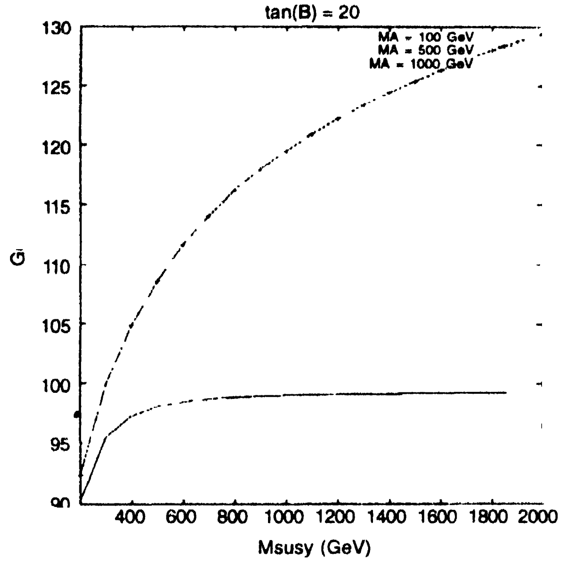

Figure 4. m_h is plotted against M_{susy} for tan β = 20, μ = 200 in the cases of M_A = 100, 500, 1000 GeV

varying from 200 to 2000 GeVs. In the mixing case for the same values of M_A (= 1000 GeV) and tan β (= 20), there is no change in the variation of m_h when $\mu_H = M_{susy}$. These results are allowed by LEP2 bounds on the lightest Higgs boson mass.

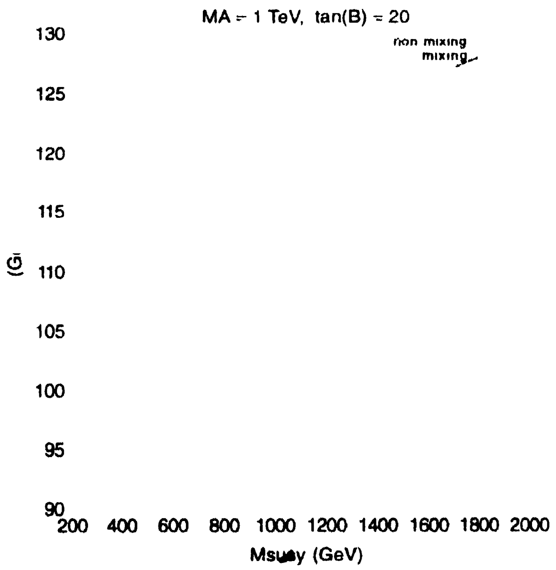

Figure 5. m_h is plotted against M_{susy} for M_A = 1 TeV, tan β = 20, μ = 200 and $\mu_H = M_{susy}$ in the mixing and non-mixing cases.

Figure 6 also gives similar studies to that given in Figure 5 except that here tan β is very small (= 1.5). The values of m_h in the non-mixing case, vary from 42 to 102 GeVs for M_{susy} varying from 200 to 2000 GeVs in case of M_A = 1 TeV, μ = 200 GeVs and these results are not possible according to LEP2 data. In the mixing case for the same values of M_A (= 1 TeV) and tan β (=

1 5). m_h varies from about 52 to 106 GeVs for $\mu_H = M_{susy}$ in the case of M_{susy} varying from 200 to 2000 GeVs. Hence, probably the combination of large M_A and small tan β are not possible in MSSM. Thus, Figure 6 shows that for large M_A (= 1 TeV) and very small tan β (= 1.5), the results in both the non-mixing and mixing cases are unallowed in MSSM by LEP2 data. Hence, we should restrict tan β to large values.

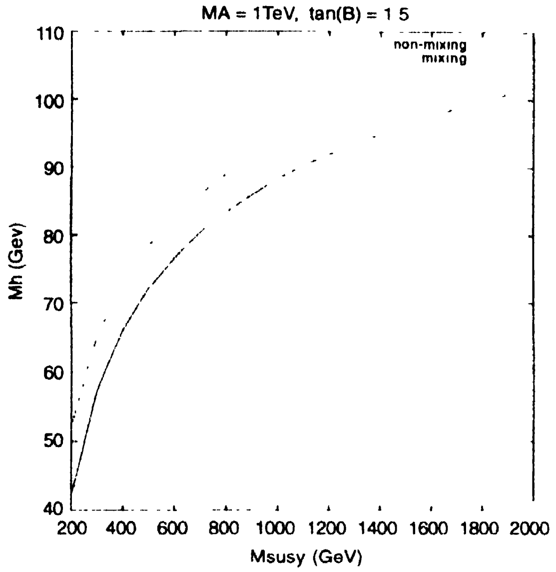

Figure 6. m_h is plotted against M_{susy} for M_A = 1 TeV, tan β = 1 5, μ = 200 and $\mu_H = M_{susy}$ in the mixing and non-mixing cases

Figure 7 gives m_h in the non-mixing and mixing cases for M_A = 100 GeV, tan β = 20 for M_{susy} varying from 200 to 2000 GeV. In both the cases, the maximum values of m_h lies between 90 and 99 GeVs. Hence, these results are ruled out by the LEP2 data, suggesting that the combination of low M_A and large tan β are not possible. Lastly, Figure 8 gives the variation of m_h with tan

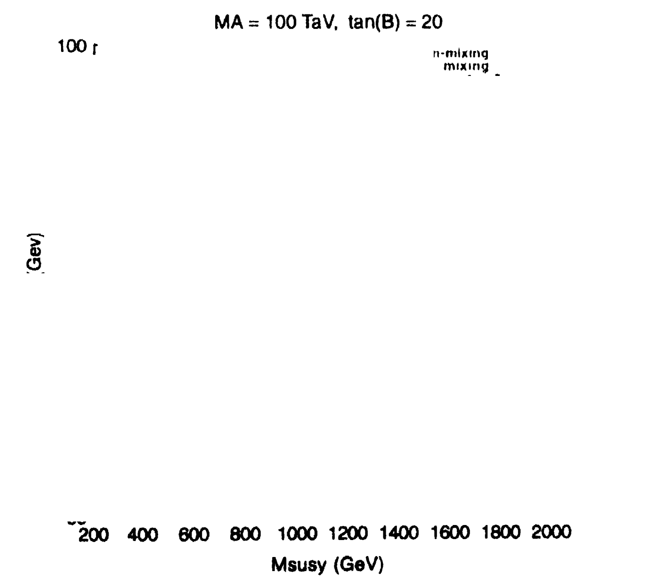

Figure 7. m_h is plotted against M_{susy} for M_A = 100 GeV, tan β = 20, μ = 200 and $\mu_H = M_{susy}$ in the mixing and non-mixing cases

β (1-50) for M_A = 200 GeV, μ = 200 GeVs and $\mu_H = M_{susy}$ = 1 TeV in the non-mixing and mixing cases. In the non-mixing case, m_h varies from 75 to 120 GeVs as tan β is varied from 1 to 50. Here, m_h is very sensitive to tan β for tan β varying from 1 – 10. The results on m_h are not so sensitive to tan β for tan β varying from 10 to 50. For example, in the mixing case, m_h varies from 118 to 119 GeV as tan β varies from 11 to 50. For 1 < tan β < 3, m_h varies from 75 to 102 GeVs which are ruled out in MSSM. These results on the variation of m_h with M_{susy} for tan β = 20, M_A = 1 TeV are comparable with the results of other authors [6, 10].

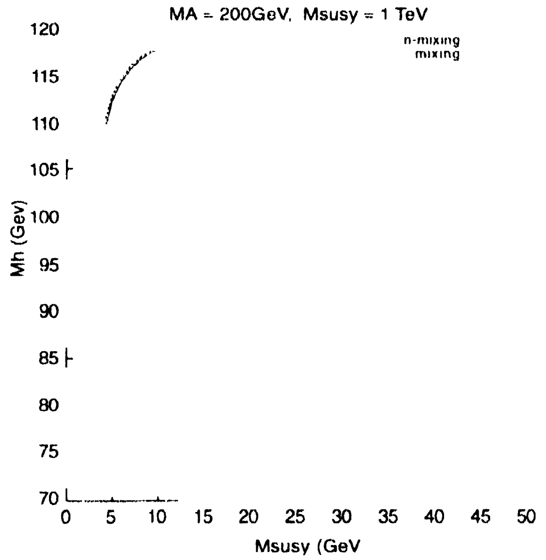

Figure 8. m_h is plotted against tan β for M_A = 200 GeV, $\mu_H = M_{susy}$ = 1 TeV, μ = 200 in the mixing and non-mixing cases

7. Conclusions

Thus, on the basis of our studies with the RG-improvement in the Higgs sector of the MSSM, we observe the following :

- (i) the lightest Higgs mass in the MSSM depends sharply on the parameters like M_A, tan β and M_{susy}.
- (ii) for M_{susy} varying from 200 to 2000 GeV, very small values of tan β (= 1.5) and M_A (≃ 100 GeV) are ruled out.
- (iii) for M_{susy} varying upto 2 TeV, large values of tan β (20–50) and M_A (500–1000 GeV) are allowed by LEP2 data.
- (iv) the effects of running VEVs on the Higgs masses are small (~ few Gevs).

We find that the upper limit of the value of m_h is equal to 130 GeV (Figure 2). The Higgs masses under certain sets of parameters lie well within the reach of the LEP2 Higgs search. We hope that some of the results given in our studies will be verified by the future experiments.

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