

## On eigen values of simple harmonic motion

M S Ansari\* and G M Mir

Regional Engineering College, Srinagar-190 006, Kashmir, India

Received 29 August 2000, accepted 30 November 2000

**Abstract** In order to establish correspondence between Bohr's method of quantization and Schrödinger wave function approach, we have solved wave equation for simple harmonic oscillator, quartic potential and hydrogen atom with the assumption that the particle is totally confined within the classical limits. For certain values of the energy, the wave functions have maximum divergence at large distance from the classical turning points. We call such energy levels as Anti-eigen values. Anti-eigen values are also reported for harmonic and quartic potentials.

**Keywords** Harmonic and quartic potentials, anti-eigen and classical eigen values

**PACS Nos.** 02.70.e, 03.65.Ge

Power series potential has been a subject of intensive investigation by a large number of workers [1–10]. The power potential is rather easy to work with, and it represents a wide class of problems in atomic and solid state physics.

In order to bring about some sort of relation between the old quantum mechanics and the Schrödinger wave approach, we solved the wave equation for simple harmonic oscillator,

quartic potential and hydrogen atom by assuming that the particle is totally confined within the classical limits. The wave function outside this range has been neglected for the calculations, and it might be considered even undefined for our purpose. We imposed the condition, either (i)  $\psi = 0$  or (ii)  $\psi' = 0$  at the classical turning points. For the sake of reference, we call such energy levels as classical eigen values.

Table 1. Anti-eigen and classical eigen values.

no	Harmonic potential*			Quartic potential		
	A	B	C	A	B	C
0	1.954677	1.681600	2.263119	3.0000	1.876774	2.423841
1	3.964555	3.672290	4.287225	5.98978	4.967735	5.941667
2	6.161803	5.669857	6.297685	10.45800	8.810237	9.995725
3	8.177761	7.668809	8.303724	14.79601	13.148673	14.498511
4	10.323579	9.668243	10.3077281	20.0314	17.889980	19.374609
5	12.349202	11.667894	12.3106071	25.1610	22.974250	24.574625
6	14.49010	13.667661	14.3128011	30.9910	28.360154	30.062724
7	16.5411	15.667496	16.3145221	36.724	34.017010	35.811564

A: anti-eigen values

B: eigen values with  $\psi = 0$  at the classical turning points

C: eigen values with  $\psi' = 0$  at the classical turning points

\* For simple harmonic oscillators, energy values are in the unit of  $hw/2$ .

\*Corresponding Author

Out of curiosity, we also determined so called anti-eigen values of simple harmonic and quartic oscillators, energy values for which the wave function at a large distance from the classical turning points has the maximum divergence in comparison with that of any adjacent value of energy.

We have followed the same methodology as reported earlier [1,2]. In the present case, the wave function was monitored at the classical turning point. The energy was determined such that either (i)  $\psi = 0$  or (ii)  $\psi' = 0$ .

For determining the anti-eigen values, we monitored the wave function at a large distance from the classical turning points. The energy was slowly varied as an input parameter to the numerical programme, till we got the desired result.

For calculating classical eigen values of hydrogen atom, the radial component of Schrödinger wave function was solved with the condition  $\psi = 0$  at the classical turning point.

Table 1 gives the classical and anti-eigen values for simple harmonic and quartic potentials. Contrary to our expectations, classical eigen values do not reproduce the Bohr's quantized energy levels for simple harmonic motion. It is also strange that Bohr's energy levels are closer to what has been termed as anti-eigen values *i.e.* it corresponds to maximum divergence of the wave function. However, the energy difference between adjacent levels remains almost the same in all methods of quantization. As we are mostly concerned with the energy differences, even the anti-eigen values will serve the purpose.

Table 2 gives the overlap integral for the classical wave functions with  $\psi = 0$ , and surely, these wave functions are not orthogonal to each other. However, for large excited

**Table 2.** Overlap integral of classical wave functions,  $\psi = 0$  at the classical turning point

$n_j$	1	2	3	4	5	6	7	8
1	1.000	*	0.321	*	0.094	*	0.000	*
2	*	1.000	*	0.308	*	0.010	*	-0.066
3	0.321	*	1.000	*	0.288	*	-0.041	*
4	*	0.308	*	1.000	*	0.270	*	-0.072
5	0.094	*	0.288	*	1.000	*	0.256	*
6	*	0.010	*	0.270	*	1.000	*	0.243
7	0.000	*	0.041	*	0.256	*	1.000	*
8	*	0.066	*	-0.072	*	0.243	*	1.000

\*zero from symmetry

state, these wave functions assume more or less orthogonal character.

Table 3 gives the classical eigen values of hydrogen atom. In this case too, there is no correspondence between Bohr and the present methods of quantization. For  $L = 1$ , we

**Table 3.** Classical eigen values of hydrogen atoms.  $\psi = 0$  at the classical turning point. The table gives the values of  $\lambda$  where Energy =  $-R/\lambda^2$

Sl no	$L = 0$	$L = 1$	$L = 2$	$L = 3$
1	2.0262677	2.0928281	2.1868078	2.292095
2	3.305131	-	-	-
3	4.170584	-	-	-
4	5.320192	5.73695	-	-
5	6.393183	-	-	-

get only two classical levels whereas for higher values of  $L$ , there is only one classical eigen values.

We do not envisage much use for the anti-eigen values. At the most, we might think of a situation where a particle is incident on a potential barrier resembling harmonic or quartic potential, and if the energy of the particle happens to be equal to the anti-eigen value, it would be totally reflected from the potential barrier.

#### Acknowledgment

The authors acknowledge the help of Prof. K K Sharma, Department of Physics, Indian Institute of Technology for his support and help.

#### References

- [1] M A Baba and M S Ansari *Indian J. Phys.* **69B** 399 (1995)
- [2] M S Ansari and M A Baba *Indian J. Phys.* **72B** 225 (1998)
- [3] S N Biswas, K Dutta, R P Saxena, P K Srivastava and V S Varm *Phys. Rev.* **D4** 3617 (1971)
- [4] S N Biswas, K Dutta, R P Saxena, P K Srivastava and V S Verma *J. Math. Phys.* **14** 1190 (1973)
- [5] F T Hioe and E M Montroll *J. Math. Phys.* **16** 1946 (1975)
- [6] K Banerjee *Proc. Roy. Soc. London* **A364** 265 (1978)
- [7] A Hautot and M Nicolas *J. Phys.* **A16** 2953 (1983)
- [8] Francisco M Fernandez, Alejandro M Meson and Eduardo A Castro *Phys. Lett.* **104** 401 (1984)
- [9] F N Fernandez, A M Meson and F A Castro *Phys. Lett.* **A104** 410 (1984), *Phys. Lett.* **A124** 1 (1987)
- [10] R F Boshop and M F Flynn *Phys. Rev.* **A38** 2211 (1988)
- [11] R N Chaudhary and M Mondal *Pramana-J. Phys.* **37** 13 (1990)
- [12] G P Flessa *Phys. Lett.* **72A** 289 (1979); *Phys. Lett.* **81A** 17 (1981)