Indian J. Phys. **75A** (3), 277-278 (2001)

NOTE

IJP
an international journal

An exact solution in SU(2) Yang Mills theory

Avijit K Ganguly
Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Calcutta 700 064, India

Received 15 November 2000, accepted 7 February 2001

Abstract : An exact solution of Yang Mills equation in (1+1) dimension has been found This solution shows a characteristic oscillation in time, whose frequency depends on amplitude and it varies linearly with spatial coordinates

Keywords . Yang Mills equation, exact solution

PACS No. : 12.10 -g

The solution of classical sourceless Yang-Mills (YM) equation has attracted the attention of many workers [1, 2] for a long time for obtaining more information about the structure of the QCD ground state. It is also important from the point of view of studying Quantum Chromo Dynamics (QCD) vacuum, in the back ground of such a solution and there from getting information about the stability of the system. Here, we will motivate the study of such a solution from the point of view of QCD flux tube model, introduced originally by Low [3] and Nussinov [4] to account for the observed scaling behaviour of scattering cross sections in hadron hadron collisions as well as in the nucleus nucleus collisions.

In this model, it is assumed that at high energy, when the two highly Lorentz contracted nuclei pass through each other, the partons of one nucleus interact with the partons of the other nucleus by the exchange of soft (color octet) gluons. If the fly by time of the nuclei is less than the time scale of interaction of the partons, the receding nuclei get randomly color-charged by exchange of soft gluons. These color octet partons in the receding nuclei get connected to each other by means of color flux tubes with color electric fields inside them. The purpose of this note is to show that due to the presence of the nonlinear terms in the Lagrangian, the fields interact among each other, generating an electric field that undergoes a characteristic non-linear, non-abelian oscillation in time. For this purpose, we will make certain assumptions based on the geometry of the prolem. These assumptions however do not change the qualitative nature of our observation. The first assumption is that each of the color-charged nucleus has a uniform distribution of charge in the plane transverse to the direction of motion so that there exists no gradient of the field in this direction. Our second assumption is that these color charges produce a chromoelectric field such that A_0 and A_z are the only non-zero potentials.

With these simplifying assumptions, the dynamics of the gluon fields can essentially be described in (1+1) dimensions rather than (3+1) dimensions. Therefore in order to get information about the nature of the classical gluon fields, one needs to solve the classical Yang-Mills field equations in (1+1) dimensions.

We next show that in (1+1) dimensions, the Yang-Mills equations have a solution with a sinusoidally time varying component whose frequency depends on the amplitude. We first write the sourceless Yang Mills equation in (1+1) dimensions

$$D_\mu F^{\mu\nu} = 0, \tag{1}$$

where the Greek indices μ and ν take values 0 and 1 only. The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig[A_\mu\ ], \tag{2}$$

with g as the coupling constant and $g[A_\mu\ ]$, as the commutator bracket. Since we are working with an SU(2) color symmetry, A_μ is defined as $A_\mu = A_\mu{}^a \tau_a$ where τ_a are the generators obeying the commutation rules

$$\left[\tau^a, \tau^b\right] = i\,\varepsilon_{abc}\,\tau^c\,. \tag{3}$$

The indices a,b,c takes values from one to three. Further, the only non-zero components of the vector field in this case are A_0

© 2001 IACS

and A_z, and we have chosen the axial gauge $A_z = 0$. With this choice of the gauge we get from eq. (1) for $\nu = 0$,

$$\partial^2{}_z A_a^0 = 0 \tag{4}$$

whose solution is

$$A_a^0(t) = \alpha_a(t)\, z + \beta_a \,. \tag{5}$$

Here α_a and β_a are arbitrary integration constants. In order to find out an exact solution of this equation, we take α_a to depend on time and β_a to be a constant. Eq. (1) for $\nu = 1$ gives

$$\partial_0 \partial_z A_a^0 + g\,\varepsilon_{abc} A_a^0 \partial_z A_c^0 = 0 \,. \tag{6}$$

Substituting the solution (5) in eq. (6), we arrive at

$$\dot{\alpha}_a(t) + g\,\varepsilon_{abc}\,\alpha_c\,\beta_b = 0 \,. \tag{7}$$

One can derive a conservation law from this equation, namely

$$\alpha_a(t)\alpha_a(t) = constant \,. \tag{8}$$

A summation over repeated indices is implied and α is scaled by g.

We solve this set of coupled first order linear differential equations by Euler's method; *i.e.* we choose a solution of the form

$$\alpha_a(t) = a\_e^{pt} \,. \tag{9}$$

Substituting eq. (9) in eq. (7), we obtain a set of coupled algebraic equations whose solution is of the form

$$\alpha_1 = \beta_1 + \beta_1\beta_3\left[e^{i\omega t} + e^{-i\omega t}\right] - i\omega\beta_2\left[e^{i\omega t} - e^{-i\omega t}\right], \tag{10}$$

$$\alpha_2 = \beta_2 + \beta_2\beta_3\left[e^{i\omega t} + e^{-i\omega t}\right] + i\omega\beta_1\left[e^{i\omega t} - e^{-i\omega t}\right], \tag{11}$$

$$\alpha_3 = \beta_3 + \beta_3^2\left[e^{i\omega t} + e^{-i\omega t}\right] - \omega^2\left[e^{i\omega t} + e^{-i\omega t}\right]. \tag{12}$$

Here, $\omega = \left[(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2\right]^{\frac{1}{2}}$ and the constants of integrations are set equal to one.

Once the α's are known, one gets the solution for the A_0's by substituting eqs. (10), (11) and (12) in eq. (5). Without giving the unnecessary mathematical details, the final expression is

$$A_0^1 = \left[\beta_1 + \beta_1\beta_3\left[e^{i\omega t} + e^{-i\omega t}\right] - i\omega\beta_2\left[e^{i\omega t} - e^{-i\omega t}\right]\right]z + \beta_1 \,, \tag{13}$$

$$A_0^2 = \left[\beta_2 + \beta_2\beta_3\left[e^{i\omega t} + e^{-i\omega t}\right] + i\omega\beta_1\left[e^{i\omega t} - e^{-i\omega t}\right]\right]z + \beta_2 \,, \tag{14}$$

$$A_0^3 = \left[\beta_3 + \beta_3^2\left[e^{i\omega t} + e^{-i\omega t}\right] - \omega^2\left[e^{i\omega t} + e^{-i\omega t}\right]\right]z + \beta_3 \,. \tag{15}$$

Thus from the solution, it is clear that the electric field inside a chromo-electric flux tube oscillates with frequency $\omega = \left[(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2\right]^{\frac{1}{2}}$, which depends on the amplitude of oscillation. It may be worth, pointing out several features of this solution, namely the time independent piece corresponds to the covariantly constant field case. The time dependent oscillating piece is due to the self interaction of the fields, that shows dependence of amplitude on the frequency. It may also be pointed out that there exists [2] an exact time-dependent vacuum solution of (SU(2)) Yang-Mills equations of the type

$$A_\mu^\alpha = \left(0, H\delta_1^\alpha, H\delta_2^\alpha, H\delta_3^\alpha\right), \tag{16}$$

where

$$H = \frac{B}{\sqrt{g}} cn\left[\sqrt{2g}B(t - t_0)\right] \tag{17}$$

with frequency, dependent on amplitude. In eq. (16), μ (= 0,1,2,3) is the Lorentz index, α (= 1,2,3) is the color index and δ_i^α is the Kronecker delta. In eq. (17), cn represents the Jacobi elliptic function and B is a constant determining the amplitude of the oscillating field. Physically, these solutions represent a non-linear collective oscillation of gluons with a characteristic amplitude dependent time period $\sim\left(\sqrt{2gB}\right)^{-1}$, which is a manifestation of the intrinsic nonlinearity present in the system.

This is worth mentioning that earlier there had been attempts to study the break down of QCD vacuum, in the context of RHIC, in the background of a covariantly constant, homogeneous Chromo Electric field, as well as a sinusoidally oscillating Chromo Electric field [5]. In the light of those results, we feel, it will be interesting to look for the same effect in the presence of the type of background obtained in this note. Some work along this direction is currently under progress.

Acknowledgment

The author would like to thank Prof. P. K. Kaw for suggestions and advices provided during the course of this work.

References

[1] S Coleman *Phys. Lett.* **70B** 59 (1977) ; A Actor *Rev. Mod. Phys* **51** 461 (1979)

[2] S G Matinyan, G K Savidy and N G Ter-Arutyunyan-Savvidy *Sov Phys. JETP* **53** 421 (1981)

[3] F E Low *Phys Rev.* **D12** 163 (1975)

[4] S Nussinov *Phys. Rev. Lett.* **34** 1286 (1975) ; H Nueberger *Phys Rev.* **D20** 2936 (1979) ; C B Chiv and S Nussinov *Phys. Rev.* **D20** 945 (1979)

[5] Avijit Kr Ganguly, Predhiman K Kaw and Jitendra C Parikh *Phys Rev* **D46** R2983 (1993)