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Non-linear vortices in planetary ionosphere

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Abstract. Effect of dynamics of dust grains in the planetary magnetospheric plasma is studied. The simulation suggests to study further to find the possibility of formation of vortices.

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1. Introduction

Dust grains or solid particles of micron to sub-micron sizes are observed in many space environments including planetary rings, comets, the interstellar medium, planetary ionosphere and magnetosphere including the case for our Earth. These dust grains, immersed in the plasma and/or radiative environment become electrically charged by different processes such as plasma current collection, photo-emissiona and secondary emission. Hence usual plasma environment is affected by this additional charge component affecting the nature of instabilities and generating some low frequency waves.

Due to greater mobility of electrons, a bare dust grain first acquires some electrons which are later compensated by by positive ions; causing the overall effect as a negative charge on the grain. Another important point to note is that the dust grains do not have a fixed charge and hence one must include formalisms for fluctuations in the grain charges due to wavelike perturbations in the plasma, generally at average dust plasma frequencies. The time rate of change of charge on the grain becomes

$$\frac{dQ_D}{dt} = I_e + I_i$$

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For a dusty plasma containing electrons, ions and negatively charged dust grains, the quasi-neutrality condition can be achieved as

$$q_d n_{d_0} + (q_i n_{i_0}) = 0$$

where, $q_{d_0} = a\psi$ = equilibrium charge on dust, a=dust grain size, ψ=unperturbed surface electric potential, n_{α_0}=unperturbed number density of particles of type $\alpha, [\alpha = e, i, d]$.

2. Dusty plasma in planetary magnetosphere

There are many exciting phenomena associated with the interaction of magnetosphere and dusts in plasma. The size of dust grains provide an important role. Smaller grains are influenced by Lorentz force while gravity plays the key role in case of longer sized grains.

To construct the basic models, some assumptions are considered for the sake of simplicity. These are, namely, a) planetary magnetic field is considered as dipole situated at the center of the planet, b) planetary rotational axis and magnetic axis coincide, c) planetary magnetosphere rigidly rotates with the planet causing no shear. The inertial system to be considered is planetocentric coordinate system.

For planetary magnetosphere of infinite conductivity that rigidly rotates with the planet with rotation rate $\vec{\Omega}$, electric field which is also corotational becomes $\vec{E} = \left(\vec{r} \times \vec{\Omega}\right) \times \vec{B}$, where $\vec{B}$ is the magnetic field and $\vec{r}$ is the radius vector of the grain. In the equatorial plane, magnetic field becomes $B=B_0(R/r)^3$ where B_0 is the magnetic field at the surface and R is the radius of the planet.

The equation of motion becomes

$$\frac{d}{dt}\left(\frac{\dot{r}^2}{2}\right) = \frac{d}{dt}\Big[\frac{r^2\dot{\phi}^2}{2} - \frac{q\phi}{r} + \frac{q\omega}{r} + \frac{GM}{r} - \frac{G\left(I_3 - I_1\right)\left(1 - 3\sin^2\lambda\right)}{2r^3}$$

$$-c_D\rho_D^{-\frac{2}{3}} m^{-\frac{1}{3}} \rho_B \dot{r}^2 r - \sum_i \frac{Gm_i}{4md_i} \ln\left|d_i^2 - r^2\right|\Big]$$

where, $q=\frac{QB_0R^3}{m}$, Q being charge of dust of mass m, R being radius of planet of mass M,

$I_1=\frac{1}{5}M\left(R^2_{equator} - R^2_{pole}\right)$, $I_3=\frac{2}{5}MR^2_{equator}$, where I_j are moment of inertia about j^{th} axis and $I_1 = I_2 \neq I_3$,

c_D=drag coefficient $\approx$ 1.00 for spherical dust grain, ρ_D=density of test dust grain, ρ_B=background environment density, d_i=distance of i^{th} moon from planet.

The effective potential energy $U(r)$ of dust grain can be obtained from Jacobi constant. $H = \varepsilon - \Omega J$, where ε provides energy and J provides angular momentum.

The potential energy can be formulated as

$$U(r) = \frac{J^2}{2r^2} + \frac{J^2}{2r^4} + \frac{JQ}{r^3} - \frac{q\omega}{r} - \frac{GM}{r} + \frac{G(I_3 - I_1)(1 - 3\sin^2\lambda)}{2r^3}$$
$$+ c_D \rho_D^{-\frac{2}{3}} m^{-\frac{1}{3}} \rho_B V_d^2 r + \sum_i \frac{Gm_i \ln|d_i^2 - r^2|}{4md_i}$$

where,

$$J = r^2 \left[\omega_k^2 - \frac{3G(I_3 - I_1)(1 - 3\sin^2\lambda)}{2r^5} - \sum_i \frac{Gm_i}{2md_i(d_i^2 - r^2)}\right]^{\frac{1}{2}} - r^2\omega_g$$

Here, $\omega_g = \frac{q}{r^3}$=dust gyro-frequency, $\omega_k = \sqrt{\frac{GM}{r^3}}$=Keplerian angular frequency, V_D=dust velocity.

The model was used to view dust interaction in Jovian magnetosphere and it revealed - Jovian satellites show jet as dust ejection as can be seen in Fig. 1 and Fig. 2. For dust grains of typical size of 1.0 μm two peculiar events occur. Negatively charged dusts of this size seem to have been ejected off the planet Jupiter as observed in Fig. 1 while positively charged dusts of this size simply rushes toward Jupiter as clearly seen in Fig. 2.

This phenomenon of dusts inspires us to find possible formation of vortices as one type of charged dust rushes towards the planet while other type move out of it. This may cause ionospheric vortex formation for planets.

Experiments reveal that ionosphere is slightly charged although basic property is determined by neutral dusts as $N_e/N_d \ll 1$ where N_α are number density of particles of type α.

The hydromagnetic equation becomes

$$\frac{du}{dt} \equiv \frac{\partial u}{\partial t} + \left(\vec{u} \times \vec{\nabla}\right)\vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\left(\vec{j} \times \vec{B}\right) - 2\left(\vec{\Omega} \times \vec{u}\right)$$

$\vec{j}$ being the current density.

For ionosphere, generalized Ohm's law can be written as

$$\vec{j} = \sigma_{\parallel}\vec{E_d}_{\parallel} + \sigma_{\perp}\vec{E_d}_{\perp} + \frac{\sigma_H}{B}\left(\vec{B} \times \vec{E_d}\right)$$

where,

$\sigma_{\parallel} = \frac{Ne^2}{m_e\nu_e}$=conductivity parallel to $\vec{B}$.

$\sigma_{\perp} = \frac{Ne^2(\nu_e\nu_{id} + \Omega_H\omega_H)\nu_{id}}{m_e(\nu_e^2\nu_{id}^2 + \Omega_H^2\omega_H^2 + \omega_H^2\nu_i d^2)}$=conductivity perpendicular to $\vec{B}$.

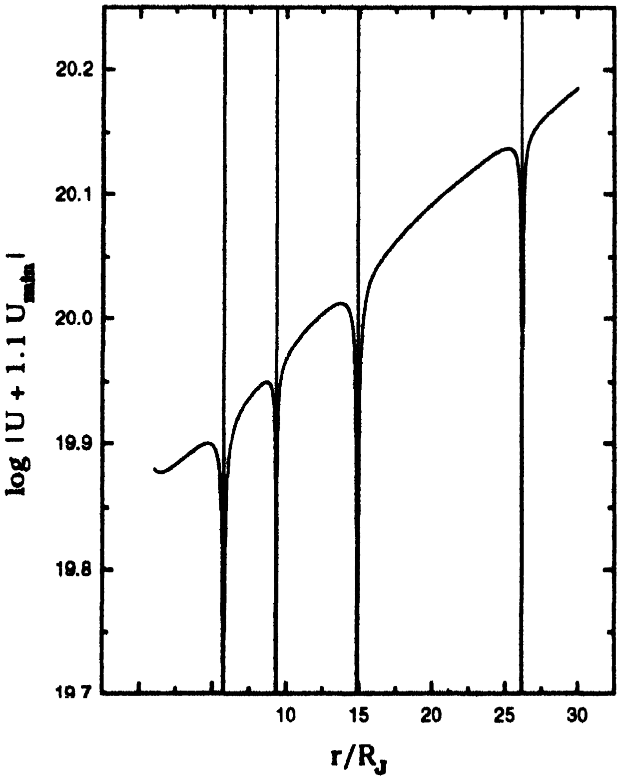

Figure 1. The effective potential energy for dust grains with the dust size of 1.0 μm having surface potential of -30V. The troughs show the gravitational pull due to satellites. The dusts seem to be ejected off the Jovian surface. This effect may help to contribute to some extent to sustain the Jovian ring.

$$\sigma_H = \frac{Ne^2\nu_{id}^2\omega_H}{m_e\left(\nu_e^2\nu_{id}^2+\Omega_H^2\omega_H^2+\omega_H^2\nu_{id}^2\right)} = \text{Hall conductivity}$$

$N_e = N_i = N$ (say) and $\nu_e = \nu_{ei} + \nu_{ed}$, ν_{ei}=effective collision frequency of electron and ion, ν_{ed}=effective collision frequency of electron and dust, ν_{id}=effective collision frequency of ion and dust, $\omega_H = eB_0/m_ec$=electron cyclotron frequency, $\Omega_H = eB_0/m_ic$=ion cyclotron frequency, $\vec{E}_d = \vec{u} \times \vec{B}_0$=Dynamo field.

Now vortex being a local phenomenon requires some local coordinate system. Assuming x, y, z as local coordinates dust velocity can be transformed as $u_x = -\frac{\partial\psi}{\partial y}$ and $u_y = \frac{\partial\psi}{\partial x}$, where ψ is streamer function.

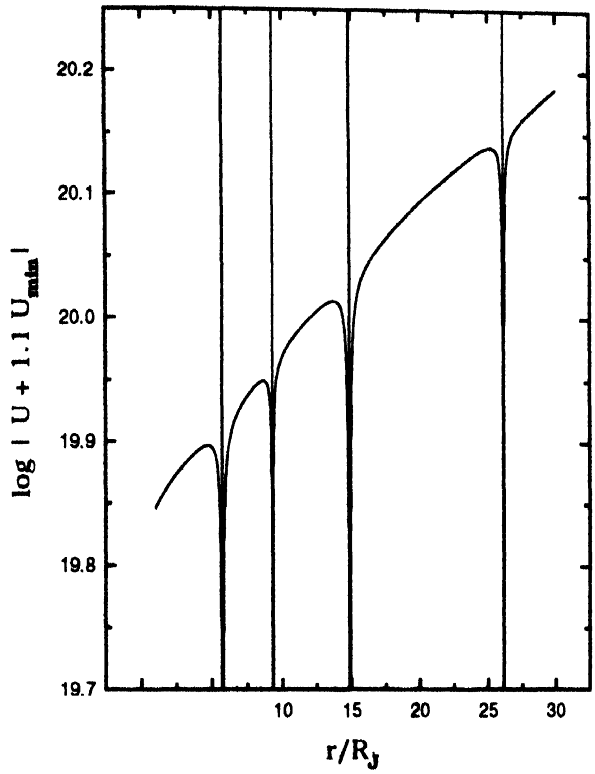

Figure 2. The effective potential energy for dust grains with the dust size of 1.0 μm having surface potential of +5V. The troughs show the gravitational pull due to satellites. The dusts seem to be attracted towards the Jovian surface.

After some mathematical formalism one may write,

$$\frac{\partial}{\partial t}(\Delta\psi) + [\psi, \Delta\psi] + \left[\psi, \tilde{\Omega}\right] + \frac{\partial\Lambda_{\parallel}}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial\Lambda_{\perp}}{\partial x}\frac{\partial\psi}{\partial x} + \Lambda_{\parallel}\frac{\partial^2\psi}{\partial y^2} + \Lambda_{\perp}\frac{\partial^2\psi}{\partial x^2} = 0$$

This is the general form of non-linear vortex forming in the planetary ionosphere.

Here,

$\tilde{\Omega} = 2\Omega_z + \Lambda_H$, $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

$\Lambda_{\parallel} = \sigma_{\parallel}B^2/h$, $\Lambda_{\perp} = \sigma_{\perp}B_z^2/h$, $\Lambda_H = \sigma_H B B_z/h$.

Experimental facts reveal that vertical velocity is much less than horizontal velocity components.

3. Summary

Kaladze *et al* [4] worked on the nature of formalism of vortices in case of our Earth's ionosphere. They have shown that for Earth's ionosphere D-layer solitary Rossby waves are observed. In this layer effect of charged particles is unimportant while neutral dusts play the key role.

Our work just shows an introduction to the study of possible formation of vortices in planetary ionosphere in general. Study regarding Jovian ionosphere is continued to examine the possibility of formation of vortices.

References

1. M Horányi et al. *J. Geophys. Res.* **98** 21245 (1993)
2. M Horányi et al. *Nature* **363** 144 (1993)
3. L Schaffer et al., *J. Geophys. Res.* **99** 17211 (1994)
4. T D Kaladze et al. *Physics Letters A* **232** 269 (1997)