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Nuclear surface thickness using semi-phenomenological nucleon density distribution

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stract : Surface thickness parameters t_p and t_n for proton and neutron densities respectively have been calculated for β-stable nuclei for mass nber A ≥ 12 using the semi-phenomenological nucleon density distribution It is found that in general, the calculated $t_{p(n)}$ values for odd proton utron) nuclei differ noticeably from those for even proton (neutron) nuclei in a systematic way Best fit curves describing dependence of the culated t_p and t_n on A as well as (N-Z)/A have been obtained

ywords Nuclear surface thickness, nucleon density distribution, semiphenomenological model

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Introduction

me years ago, Gambhir and Patil [1,2] proposed a simple ebraic expression for nucleon densities which incorporates rectly the asymptotic and central behavior of the density as juired from some theoretical considerations. This semi-enomenological density has only one free parameter namely half-density radius which can be fixed by demanding the odel to reproduce the measured charge rms radius. plications of the semi-phenomenological density show that gives quite satisfactory results for the charge form factor [3] d for the neutron density distributions [4].

Nuclear surface thickness, defined as the distance over which nucleon density falls from 90% to 10% of its central value, is important characteristics of nuclei. It plays an important role determining heavy-ion scattering at medium energies, which ensitive mainly to the surface region of the colliding nuclei This is evident from the success of the simple Karol model for calculating the nuclear transparency function. In this odel, the transparency function is derived using surface-malized Gaussian densities for the colliding nuclei, which fer greatly from the realistic densities in the interior region of nuclei.

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Gambhir and Patil [2] used their semi-phenomenological density model to calculate also the surface thickness parameters t_p and t_n for proton and neutron density distributions. They showed that the t_p values as predicted by the model are in satisfactory agreement with the experimental ones. For t_n, they noted that in general, it increases with the mass number. However, since the main aim of their work was to study the applicability of the semi-phenomenological density model, they considered a very limited number of nuclei and did not undertake a quantitative study of the dependence of t_p and t_n on some characteristics of nuclei like the mass number A and (N-Z)/A.

In this work, we use the semi-phenomenological density to calculate t_p and t_n for a large number of β-stable nuclei for A ≥ 12, to study their dependence on mass number A and (N-Z)/A. It turns out that the average behaviour of t_p and t_n as a function of A and (N-Z)/A can be described in terms of simple expressions, reasonably well. The present study is expected to be especially useful for calculating the transparency function for heavy-ion systems involving neutron rich nuclei.

2. Theoretical considerations

Considering general requirements to be satisfied by nuclear densities, Gambhir and Patil [1,2] proposed the following algebraic form for proton and neutron density distributions in nuclei

$$\rho_i(r) = \frac{\rho_i^0}{\left[\frac{1}{2}+\frac{1}{2}(r/R)^2\right]^{\alpha_i}\left[e^{(r-R)/a_i}+e^{-(r+R)/a_i}\right]}, \quad i=(p,n), \tag{1}$$

where p stands for proton and n for neutrons. The parameter R is the half-density radius to a very good degree of approximation and the quantities a_i and α_i are given by

$$a_i = \frac{\hbar}{\sqrt{2(2m\varepsilon_i)}} \qquad i=(p,n), \tag{2}$$

$$\alpha_i = \frac{q}{\hbar}\sqrt{\frac{m}{2\varepsilon_i}}+1, \quad i=(p,n), \tag{3}$$

where $\varepsilon_{p(n)}$ is the proton (neutron) separation energy, m is the nucleon mass and

$$q = \begin{cases} 0, & \textit{for neutrons} \\ (Z-1)e^2, & \textit{for protons} \end{cases}$$

with Z as the atomic number.

The unknown parameters in the expression for proton and neutron densities are ρ_p^0, ρ_n^0 and R. The first two are determined from the normalization conditions and the last one, namely R is fixed by requiring that the proton density, after considering finite proton charge distribution, must give the measured charge rms radius. The study of Gambhir and Patil [2] shows that the values of R so determined are well described by the relation :

$$R = r_0 N^{\frac{1}{3}}, \tag{4}$$

where r_0 = 1.31 fm and N is the neutron number. It may be mentioned that a recent study [7] shows that this dependence of R on N, to a very good approximation, is the same as shown by the diffraction radii R_d extracted from the zeros of the charge form factors of the modified Helm's model [8].

To calculate the surface thickness t_i ($i = p, n$) for the semi-phenomenological density given by eq. (1), we use the relation (4) and numerically solve the equations.

$$\rho_i(r_1) - 0.9\rho_i(0) = 0,$$

$$\rho_i(r_2) - 0.1\rho_i(0) = 0$$

for r_1 and r_2 which respectively are the radial distances at which $\rho_i(r)$ falls to 90% and 10% of its central value. The required separation energies are taken from the compilation of Audi and Wapstra [9] and the proton and neutron surface thicknesses are obtained by subtracting r_1 from r_2.

3. Results and discussion

The results of our calculation for t_p and t_n as a function of mass number A are shown in Figure 1. The filled and open circles in Figure 1(a) show the t_n values for odd and even neutron nuclei respectively, while in Figure 1(b) they show the t_p values for odd and even proton nuclei. It is seen in Figure 1(a) that t_n

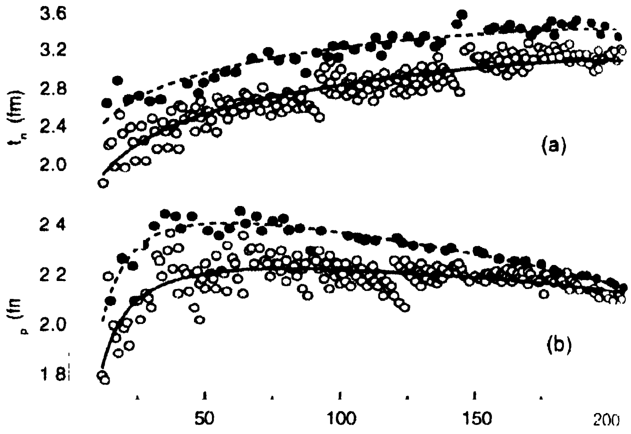

Figure 1. Proton and neutron surface thickness t_p and t_n *versus* A. Filled circles : $t_{p(n)}$ for odd proton (neutron) nuclei. Open circles : $t_{p(n)}$ for even proton (neutron) nuclei

values for odd neutron nuclei are systematically larger than those for even neutron nuclei. It is further seen that t_n values in both the cases, increase with the mass number A. The dashed and solid curves in Figure 2(a) show that the average behaviour of t_n, for odd as well as even neutron nuclei, as a function of A can be described by the relation

$$t_n = a + b \ln A, \tag{5}$$

where the best-fit parameter values are :

a = 1.51 fm, b = 0.85 fm, for odd neutron nuclei

a = 0.79 fm, b = 1.015 fm, for even neutron nuclei

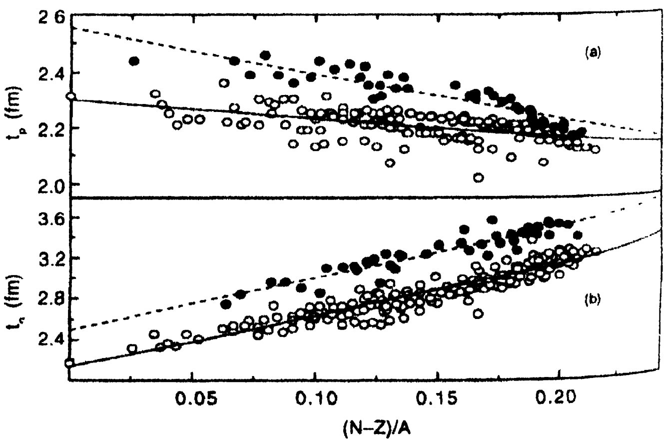

Figure 2. Proton and neutron surface thickness t_p and t_n *versus* (N-Z)/A. Filled circles : $t_{p(n)}$ for odd proton (neutron) nuclei. Open circles : $t_{p(n)}$ for even proton (neutron) nuclei.

The A-dependence of t_p is shown in Figure 1(b). As expected, we see again that the t_p values for odd proton nuclei are larger than those for even proton nuclei. In either case, t_p increases with A up to A ~ 50 and then starts decreasing slowly. However, the rate of decrease of t_p is quite small for even proton nuclei, it may be assumed essentially as constant for this mass range. The t_p values for the two cases come close together near the end of the periodic table. We find that the average A-dependence of t_p for odd as well as even proton nuclei can be well described by the relation

$$t_p = a - bA - c / A , \tag{6}$$

where the best fit parameter values are :

$a = 2.671$ fm, $b = 0.002$ fm, $c = 7.532$ fm, for odd proton nuclei;

$a = 2.394$ fm, $b = 0.001$ fm, $c = 6.600$ fm, for even proton nuclei.

Before concluding the discussion of Figure 1, it may be pointed out that the larger $t_{p(n)}$ for odd proton (neutron) nuclei as compared to the even proton (neutron) nuclei is simply a reflection of the fact that the separation energy $\varepsilon_{p(n)}$ which dominantly governs the surface diffuseness of the nucleon density through eq. (2), is smaller for odd proton (neutron) nuclei than for even proton (neutron) nuclei.

In Figure 2, we show plots of t_p and t_n as a function of (N-Z)/A. In this case also, solid circles show $t_{p(n)}$ for odd proton (neutron) nuclei and open circles even proton (neutron) nuclei. It is seen that t_p for both odd and even proton nuclei is, on the average, a linearly decreasing function of (N-Z)/A, while t_n for both odd and even neutron nuclei is a linearly increasing function of (N-Z)/A. For t_p, we find that the best fit parameter values for the linear relation

$$t_p = a - b(N - Z) / A , \tag{7}$$

$a = 2.55$ fm, $b = 1.6$ fm, for odd proton nuclei;

$a = 2.30$ fm, $b = 0.65$ fm, for even proton nuclei.

On the other hand, for t_n, the best fit parameter values for the linear relation

$$t_n = a + b(N - Z) / A \tag{8}$$

are

$a = 2.50$ fm, $b = 5.04$ fm, for odd neutron nuclei;

$a = 2.13$ fm, $b = 5.05$ fm, for even neutron nuclei.

Coming to the A-dependence of $t_{p(n)}$ as seen in Figure 1, it has been already stated that it is a reflection of the A-dependence of the proton and neutron separation energies. To elaborate this point, it is to be noted that in the asymptotic region, the radial dependence of the density as given by eq. (1) is of the form :

$$\rho_i(r) \cdot \qquad i = (p, n) . \tag{9}$$

In fact, it was to achieve this theoretically desired asymptotic form that eq. (1) that was proposed by Gambhir and Patil [1,2]. Now in case of neutron density, $\alpha_n = 1$, hence the density fall-off is characteristically governed by the parameter a_n which depends upon the neutron separation energy ε_n through eq. (2). Since on the average, ε_n decreases with A, the parameter a_n increases as A increases. In consequence, the rate of fall of $\rho_n(r)$ with r decreases as A increases, causing t_n to increase with A. The difference in the t_n values for even and odd neutron nuclei is due to the well known fact that ε_n for even neutron nuclei is larger than for odd neutron nuclei.

On the other hand for $\rho_p(r)$, the parameter α_p, after combining eqs. (3) and (2), is given by $\alpha_p = 1 + 0.069(Z - 1)a_p$ which increases in importance as A increases. Thus unlike $\rho_n(r)$, the asymptotic behaviour of $\rho_p(r)$ is characterised by both α_p and a_p. The exponential factor in eq. (9), as in the case of $\rho_n(r)$, tends to make $\rho_p(r)$ to extend to larger r values as A increases; but this is opposed by the factor $1/r^{2\alpha_p}$ which grows in importance with A. In consequence, t_p except for lighter nuclei is essentially constant over a large mass region and slightly decreases for very large mass nuclei. The even odd effect in t_p has the similar interpretation as for t_n discussed above.

The trends seen in Figure 2 are merely a reflection of the trends seen in Figure 1 because for β-stable nuclei, the quantity (N-Z)/A in general, increases with A. Hence, t_n increases with A while t_p has a decreasing trend However, it should be noted that t_p values have a large scatter and its linear decrease with (N-Z)/A is not so well defined as linear increase for t_n values.

The present study is expected to be particularly useful for calculating heavy-ion total reaction cross sections using the modified microscopic model applied by Shen *et al.* [10], which unlike the Karol model [6] makes the provision for proton and neutron densities of the colliding nuclei to be different. In these models, the realistic densities (assumed to be described by the two-parameter Fermi distribution) are replaced by the surface normalized Gaussian densities whose parameters depend upon the half density radius and the surface thickness parameters of the respective Fermi distributions. Shen *et al.* [10], using the half-density radius parameters as given by the droplet model and taking $t_p = t_n = 2.4$ fm, find that the interaction radius parameter deduced from the total reaction cross section measurements is not reproduced. They further show that the interaction radius data could be satisfactorily accounted for if it is assumed that t_n increases linearly with (N-Z). This is in agreement with the trend of t_n found in Figure 2.

4. Conclusions

We have calculated proton and neutron surface thickness parameters t_p and t_n respectively for a large number of nuclei for $A \geq 12$, using Gambhir-Patil semi-phenomenological density. The calculated $t_p(t_n)$ values for odd proton (neutron) nuclei are found to be systematically larger than those for even proton (neutron) nuclei. It is also found that the calculated t_p and t_n values show dependence on the mass number A and (N-Z)/A which on the average, can be described by simple formulae reasonably well. These formulae could be useful for more realistic calculation of the nucleus-nucleus transparency function using the modified Karol model [10] in which the neutron and proton surface thicknesses are taken to be different.

References

[1] Y K Gambhir and S H Patil Z. *Phys* **A321** 161 (1925)

[2] Y K Gambhir and S H Patil Z *Phys.* **A324** 9 (1986)

[3] Y K Gambhir, P Ring and H de Vries *Europhys Lett* **10** 219 (1989)

[4] G A Lalazissis, C P Panos, M E Grypeos and Y K Gambhir Z *Phys* **A357** 429 (1997)

[5] S Kox *et al* *Phys. Rev.* **C35** 1678 (1987)

[6] P J Karol *Phys Rev* **C11** 1203 (1975)

[7] I Ahmad, M A Abdulmomen and J H Madani *Proc DAE Symp on Nucl. Phys.* **41B** 120 (1998)

[8] J Friedrich, N Voegler and P G Reinhard *Nucl Phys* A459 1 (1986)

[9] G Audi and A H Wapstra *Nucl Phys.* **A565** 66 (1993)

[10] W Shen *et al* *Nucl Phys.* **A491** 130 (1989)