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Random phase approximation with high orbits configuration for the low lying negative parity, $T = 0$ states in ^{16}O

R A Radhi*, Ali H Taqi** and A K Ahmad**

*Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

**Department of Physics, College of Science, Saddam University, Baghdad, Iraq

E-mail sadsci@uruklink.net

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Abstract Electroexcitations of the dominantly low-lying $T = 0$, negative parity, particle-hole states of ^{16}O are studied in the framework of the random phase approximation (RPA). These states include states with J^{π} (E_x MeV) 1^- (7.12), 3^- (6.13) and 4^- (17.79). All possible single $T = 0$ particle-hole states of allowed angular momenta, for both ground states and excited states are considered in bases including single particle states up to the 1f-2p shell. The Hamiltonian is diagonalized in this extended space in the presence of the modified surface delta interaction (MSDI). Admixture of higher configurations is considered for the ground state to allow for a large scale of a collective motion in the nucleus. Effective operators are used to account for the core polarization effect. The form factors calculated in this paper are in good agreement with the experimental data.

Keywords Nuclear reactions, form factors, random phase approximation

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1. Introduction

The simplest configuration for closed shell nuclei such as ^{16}O or ^{40}Ca is doubly closed shell. If such a system is excited through the single application of a one body number-conserving operator, then the resulting states must have a particle-hole pair. The excited collective oscillation can be described as a linear combination of particle-hole states. Such an approximation is called Tamm-Dancoff approximation (TDA) [1]. In this approximation, there is an asymmetry in the way the ground state is treated on one hand and the excited states on the other. The ground state is described in terms of one configuration only and this is in fact corresponds to independent-particle model. A system of states more general than that considered in the TDA appears when treating the ground state and the excited states more symmetrically. In that case, one allows both to have particle-hole pairs. Such approximation is referred to as random-phase approximation (RPA) [1]. The theory of the RPA is fundamental to the collective motion in nuclei.

The RPA model can be tested in two domains, first by calculating the energy eigenvalues and eigen vectors (amplitudes) through the diagonalization of the Hamiltonian, and second by comparing the calculated and measured electron scattering form factors. In this paper, we study the isoscalar transition in ^{16}O which connects the ($J^{\pi} = 0^+, T = 0$) ground state with the low-lying J^{π} : 1^-, 3^- and 4^- isoscalar states. The Hamiltonian is diagonalized in the orbits $1s$, $1p$, $2s$, $1d$, $2p$ and $1f$, in the presence of the modified surface delta interaction (MSDI) [2].

In the calculation of the form factor, the ground state is modified to include admixture from higher configurations upto the orbits $3p$–$2f$. A comparison of the electron scattering form factors calculated by using RPA model with the available experimental data for the low lying isoscalar, negative parity states are discussed.

2. Theory

2.1 Random phase approximation (RPA) :

In the RPA, the ground state and the excited state are treated symmetrically, allowing both to have particle-hole pairs. This means that the excited state can be reached either by creating or destroying particle-hole pairs in the ground state. The particles

* Corresponding Author.

and holes are labeled by a and b, respectively, each with quantum numbers $(nljm_j\ 1/2t_z)$ with $t_z = 1/2\ (-1/2)$ for a proton (a neutron).

The equations of motion are linearized and take the form [1]:

$$\sum_{a'b'}\left[A^{JT}_{ab,a'b'}X^{JT}_{a'b'} + B^{JT}_{ab,a'b'}Y^{JT}_{a'b'}\right] = \varepsilon X^{JT}_{ab}\ , \qquad (1a)$$

$$\sum\left[-A^{JT}_{ab,a'b'}Y^{JT}_{a'b'} - B^{JT}_{ab,a'b'}X^{JT}_{a'b'}\right] = \varepsilon Y^{JT}_{ab} \qquad (1b)$$

where

$$A^{JT}_{ab,a'b'} = (\varepsilon_a - \varepsilon_b)\,\delta_{ab,a'b'} + v^{JT}_{ab,a'b'}\ , \qquad (2)$$

$$B^{JT}_{ab\ a'b'} = (-1)^{1/2+1/2+T'}(-1)^{j_b - j_a - J}\,v^{JT}_{ab,a'b'}\ . \qquad (3)$$

The matrix elements for particle hole states $v^{JT}_{ab,a'b'}$, coupled to J and T are given by a sum of particle-particle matrix elements $\langle a'b|V|ab'\rangle_{J'T'}$, coupled to different values of J' and T' [2]

$$v^{JT}_{ab,a'b'} = -\sum_{J'T'}(2J'+1)(2T'+1)\begin{Bmatrix} j_a & j_{b'} & J' \\ j_{a'} & j_b & J \end{Bmatrix}$$

$$\times\begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix}\langle ab'|V|a'b\rangle_{J'T'}. \qquad (4)$$

The particle-particle residual interaction used in this work, is the modified surface delta interaction (MSDI) [2].

The coefficients X and Y are the amplitudes that describe the creation or destruction of particle-hole pairs in the ground states, respectively. The quantities $\varepsilon_a - \varepsilon_b$ are the unperturbed energy of the particle-hole pair and ε are the energy eigen values for the different excited states of the given JT values. Eq. (1a) and (1b) can be written in matrix form as

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \varepsilon \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \qquad (5)$$

and the diagonalization of it will give the eigen values ε and the eigen vectors X and Y. The states of different spin-isospin (JT) values are decoupled by these calculations.

Once the secular matrix (eq. (5)) has been diagonalized and the amplitudes have been obtained, the matrix elements of the required multipole operators T_{JT} are given in terms of the single-particle matrix elements by [1]

$$\left\langle \Psi_{JT}\left|\left|\left| \hat{T}_{JT} \right|\right|\right| \Psi_{00}\right\rangle = \sum_{ab}\left[\left\langle a\left|\left|\left| \hat{T}_{JT}\right|\right|\right| b\right\rangle X^{JT}_{ab}\right.$$

$$\left. +(-1)^{1/2-1/2-T}(-1)^{j_b - j_a - 1}\left\langle b\left|\left|\left| \hat{T}_{JT}\right|\right|\right| a\right\rangle Y^{JT}_{ab}\right], \qquad (6)$$

where the triple bar denotes a reduced matrix element with respect to both angular momentum and isospin.

The single-particle matrix element reduced in both spin an[d] isospin, is written in terms of the single-particle matrix elemen[t] reduced in spin only [3]

$$a\left|\left|\left| \hat{T}_{JT} \right|\right|\right| b\Big\rangle = \sqrt{\frac{2T+1}{2}}\sum I_T(t_z)\Big\langle a\left|\left| \hat{T}_{Jt_z} \right|\right| b$$

with $I_T(t_z) = \begin{matrix} & \textit{for } T = 0 \\ (-1)^{1/2-t_z} & \textit{for } T = 1 \end{matrix}$.

2.2 *Electron scattering form factors :*

Electron scattering form factor involving angular momentum and momentum transfer q, between the initial and final nucle[ar] shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ are [3]

$$F^{\eta}_J(q)\Big| \quad \frac{4\pi}{Z^2(2J_i+1)} \sum_{T=0,1} \begin{matrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{matrix}$$

$$\Psi_{J_fT_f}\left|\left|\left| \hat{T}^{\eta}_{JT} \right|\right|\right|\Psi_{J_iT_i}\Big\rangle F_{c\,m}(q) F_{f\,s}(q)$$

with η selecting the longitudinal (L), transverse electric (E) a[nd] transverse magnetic (M) form factors, respectively. T_z is t[he] projection of the initial and final states and is given [by] $T_z = (Z - N)/2$. The finite size ($f.s$) nucleon form facto[r] $F_{f\,s}(q) = \exp(-0.43q^2/4)$ and $F_{c\,m}(q) = \exp(q^2b^2/4A)$ the correction for the lack of translational invariance in the sh[ell] model. A is the mass number, and b is the harmonic oscillat[or] size parameter.

The total longitudinal (L) and transverse (T) form factors a[re] given by

$$\left|F^L(q)\right|^2 = \sum_{J\geq 0}\left|F^L_J(q)\right|^2,$$

$$\left|F^T(q)\right|^2 = \sum_{J>0}\left\{\left|F^M_J(q)\right|^2 + \left|F^E_J(q)\right|^2\right\}. \qquad (1[0])$$

The single-particle matrix elements $\left\langle a\left|\left| \hat{T}^{\eta}_{JT} \right|\right| b\right\rangle$ for t[he] required electron scattering operators used in this work are tho[se] of Brown *et al.* [4].

3. Results and discussion

Electro-excitation of the low-lying isoscalar, negative pari[ty] states in ^{16}O are tested and compared with the availab[le] experimental data [5,6], in the framework of RPA with mod[el] space including all orbits up to the $2p$–$1f$ shell. The form facto[rs] are calculated with ground state wave function which is modifi[ed] to include higher configurations upto the orbits of $3p$–$2f$ she[ll] using mixing parameter γ that mixes the state $|nlj\rangle$ with t[he] state $|n+1lj\rangle$. The isoscalar states that are calculated in th[is] work include states with J^{π} (E MeV) : 1^- (7.12), 3^- (6.13) and (17.79).

In the simple shell model calculations, the ground state of ^{16}O is assumed to form closed $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$ shells. According to RPA, the ground states as well as the excited states are treated on the same footing, and all possible configurations for the ground state and the excited states are constructed by removing a particle from the closed shells and promoting it to higher shells leaving a hole state within the closed shells. Experimentally, the states 1^-, 3^- and 4^- are found at 7.12, 6.13 and 17.79, respectively [5]. Our RPA calculations predict the values 8.34 MeV, 10.83 MeV and 17.19 MeV, respectively.

The longitudinal form factor for the 1^- (7.12 MeV) state is shown in Figure 1 as a solid curve for comparison with the experimental data. The data do not show any diffraction minimum and cannot be reproduced through all the momentum transfer values. Same observations were made by Vincent and Vinh Mau

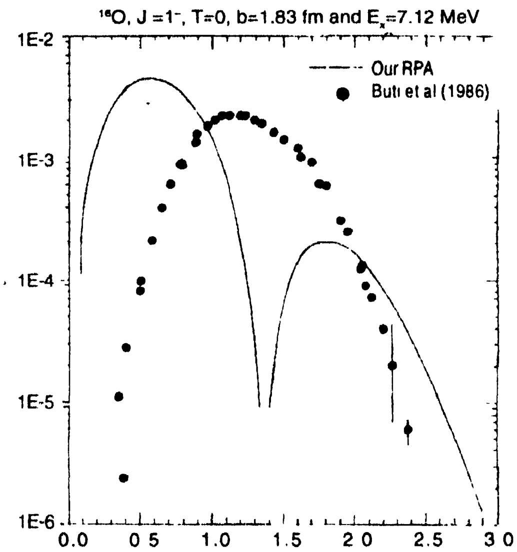

Figure 1. Longitudinal form factor for the 1^- spurious state

[7]. RPA calculation based on quantum hydrodynamics which is commonly referred to as mean field theory (MFT) [8] predicted that the first state is a spurious state and gives a spurious excitation energy of 0.5*i* MeV which is indeed zero within the numerical accuracy of their calculation. The RPA based on relativistic Hartree approximation (RHA), also predicted a spurious state but with higher energy than that of MFT [8]. MFT calculation predicts the second state which is the non-spurious state at 8.47 MeV, while the RHA calculation predicts this state at around 13 MeV. Their result for this state, agrees in shape with longitudinal form factor data, but underestimates them. In our work, the non-spurious state is at 15.58 MeV, which is close to the RHA results [8]. The calculated longitudinal form factor for this state is shown in Figure 2 and compared with the experimental data [5], where the data are fitted for medium range of q only. In our calculation, we use effective charge equal to 1.35e and 0.45e for the proton and neutron, respectively, to account for the core polarization effect for the longitudinal form factor [9]. Also admixture of higher orbits are taken into consideration with γ equal to 0.9. The value of the size parameter b of the HO potential for the single particle wave function used in this state is 1.83 fm which is consistent with that of Ref. [5] The deviation of the calculated energy levels and the longitudinal form factor for this state reflects the fact that enormous degrees of collectivity are required for proper treatment. The present as well as all previous results reveal that this state needs more investigation from the theoretical point of view.

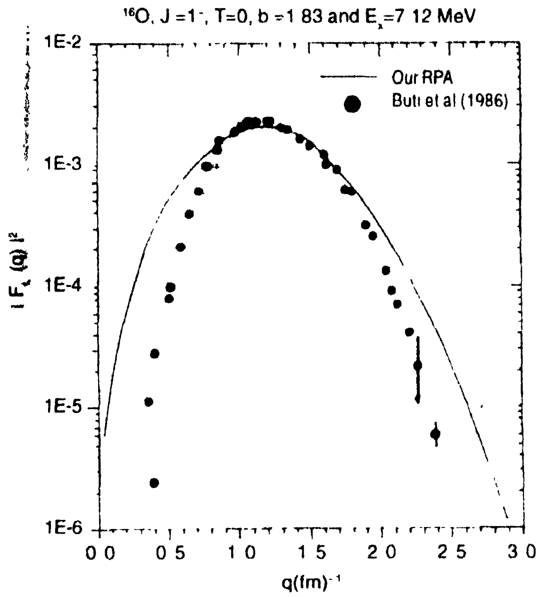

Figure 2. Longitudinal form factor for the 1^- non-spurious state

The 3^- (6.13 MeV) state is calculated to be equal to 10.83 MeV. Our RPA result is consistent with the RHA-RPA results of Ref. [8] (~ 10 MeV), where their MFT result is 5.99 MeV, which is close to the experimental value. Our RPA calculations for the longitudinal form factor is shown in Figure 3, which is based on

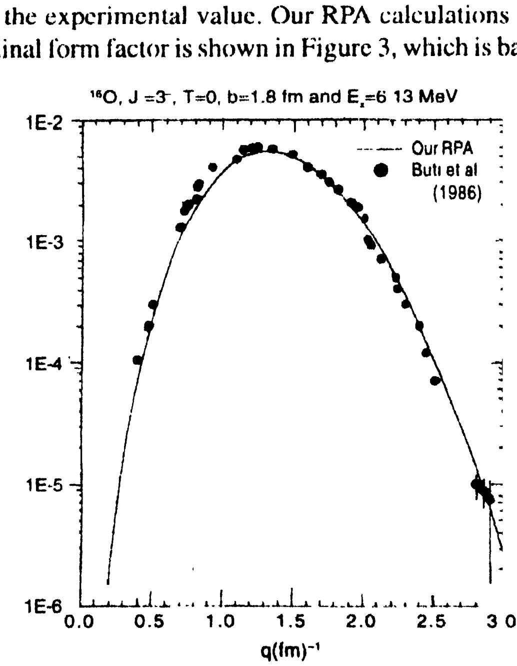

Figure 3. Longitudinal form factor for the 3^- (6.13 MeV) state.

the single-particle wave functions of the HO potential with size parameter $b = 1.8$ fm and with effective charges $e_p = 1.1e$ and $e_n = 0.15e$, for the proton and neutron, respectively. Admixture of higher orbits in the ground state for the 3^- (6.13 MeV) state is less important from that in the 1^- (7.12 MeV) state and needs small effective charge to describe the data. Our result for this state agrees in shape and magnitude quite well with the experimental data for all momentum transfer values.

Our last example is the 4^- (17.79 MeV) state, where only the transverse M4 multipole contributes to the scattering. Our RPA result predicts the lowest 4^- state at 17.19 MeV, which is consistent with the measured value at 17.79 MeV. Our calculation for the form factor is shown in Figure 4 as a dashed curve. The calculation incorporates the single particle wave function of the HO potential with $b = 1.76$ fm. The calculation agrees in shape,

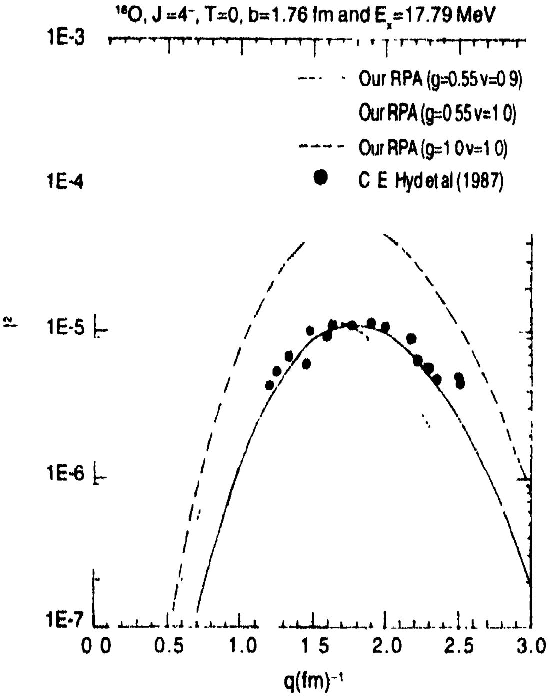

Figure 4. Transverse magnetic form factor for the 4 (17.79 MeV) state

but overestimates the data by a factor of ~ 10. Same sort of results is obtained by RHA [10]. Core polarization calculations [11,12] yield values for the g-factors reduced from the free values, which are called effective g-factors. Effective M1 g-factors equal to 0.8 of the free nucleon values are found adequate to describe the magnetic moment and the electron scattering data, while 0.6 of the free nucleon g-factors are needed to describe the M3 electron scattering data [12]. The effective g-factors used in this work to describe the M4 form factor are equal to 0.55 of the free-nucleon g-factors. Core polarization effect is taken into account to suppress the form factor as shown by the dotted curve in Figure 4. Admixture of higher configuration are also included in the ground state wave function with $\gamma = 0.9$, which is very important to add more degree of collectivity. An overall agreement is obtained with the experimental data with g-factor = 0.55 and $\gamma = 0.9$, as shown by the solid curve in Figure 4

4. Conclusions

When the space of wave functions is extended to include orbit upto 2p–1f shells, RPA results give reasonable description of the data for the electron scattering results. An improvement obtained when we allow higher shells upto the 3p–2f shell. Also including core polarization effects through the effective charge model for the longitudinal scattering and effective g factors for the transverse scattering, improve the agreement with the experimental data and gave good description for all available momentum transfer data. Such ingredients are necessary in the model to account for a large scale of collective motion in nuclei.

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