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Cosmology with Sunyaev-Zeldovich effect

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Abstract. The use of Sunyaev Zeldovich effect (SZE) as a cosmological probe is briefly reviewed, with special emphasis on using SZE of clusters to constrain Ω_0. Particular attention is paid to recent and upcoming blank sky surveys. Lastly, it is shown how the combined analysis of SZE and X-Ray emission from clusters can be used to derive the Hubble Constant.

Keywords : Observational cosmology, X-Ray spectra

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1. Introduction

Sunyaev Zeldovich effect arises due to inverse Compton scattering by intra-cluster electrons of the cosmic microwave background (CMB), as the CMB photons, after decoupling ($z \sim 1100$), pass through the cluster on their way to the present epoch. The effect is perceived as an apparent change in the sky brightness towards the cluster, and is characteristic by a specific frequency dependence which makes it possible to isolate it from other distortions of the CMB. Study of the SZE has been done to probe cluster scale physics (for example, the properties of gas distribution in the clusters). However, in the recent past SZE has grown in importance due to its use in probing cosmological physics (especially, in determining and constraining the different cosmological parameters : particularly H_0 and Ω_0). In this review, we stress on using SZE as a tool in cosmology.(For an excellent review on SZE, see [3]).

2. Physics of Sunyaev Zeldovich effect

The basic physics of Sunyaev Zeldovich effect is simple ([3,12). If a cluster has a mass M contained within a radius R, then any gas in hydrostatic equilibrium in the cluster gravitational potential well, will have electron temperature T_e given by

$$k_B T_e \sim \frac{GMm_p}{2R} \tag{1}$$

Since the electrons can scatter the CMB photons, we have the scattering optical depth

$$\tau_e \sim n_e \sigma_T R \sim 10^{-2} \tag{2}$$

On average the mean change of photon energy per scattering will be given by

$$\frac{\Delta\nu}{\nu} \sim \frac{k_B T_e}{m_e c^2} \sim 10^{-2} \tag{3}$$

Therefore, the overall the change in the brightness of the microwave background from inverse Compton scattering is given by the so called *y parameter*

$$y = n_e \sigma_T R \frac{k_B T_e}{m_e c^2} \sim 10^{-4} \tag{4}$$

Note, that this is an order of magnitude larger than the anisotropy detected by *COBE*. The SZ effect manifests itself as a change in sky brightness

$$\delta i_\nu = y j_\nu(x), \tag{5}$$

towards the cluster with respect to the mean background intensity. x is a dimensionless frequency parameter defined to be $x = \frac{h\nu}{kT_o}$, h is the Planck's constant, ν is the observing frequency and T_0 is the CMB temperature at the present epoch: $T_0 = 2.7$ K. The frequency dependence of the change in sky brightness, owing to the SZ effect, is given by

$$j_\nu(x) = \frac{2(kT_o)^3}{(hc)^2} \frac{x^4 e^x}{(e^x - 1)^2} \left[\frac{x}{\tanh(x/2)} - 4\right]. \tag{6}$$

Assuming the cluster electron temperature to be about 10 keV, the net distortion observed is expected to be zero at about 222 GHz (1.35 mm wavelength), the distortion is expected to result in a decrement in the brightness temperature towards the cluster at lower frequencies and the cluster will be seen as a positive source at higher frequencies (see, Figure 1).

The flux density S_ν due to the integrated SZ effect over the sky area of a cluster is

$$S_\nu(x) = \frac{j_\nu}{D_a^2(z)} \int dV \frac{kT_e}{m_e c^2} n_e \sigma_T. \tag{7}$$

From eqn (7) it is clear that for integrated SZ effect due to a cluster, the flux density decrement, is proportional to the total hot-gas mass times the

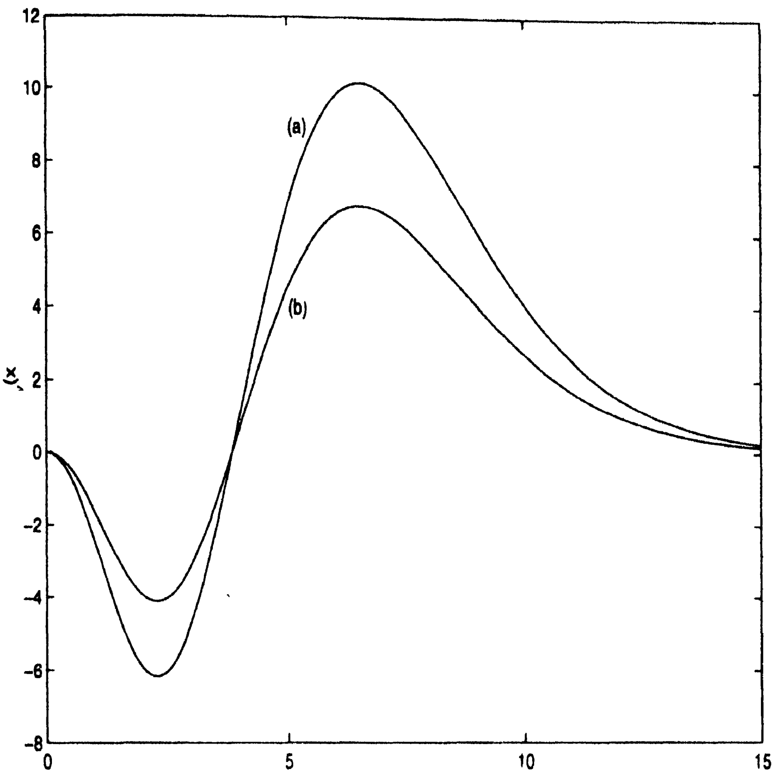

Figure 1. Frequency dependence of Sunyaev-Zeldovich effect (The amplitude are in arbitrary units), The $y-parameter$ of plot (a) is 1.5 times that of plot (b).

particle-weighted temperature. Consequently, the detection of the cluster is independent of the gas' spatial distribution (assuming the cluster is unresolved). Moreover, for SZE due to clusters, the decrement in brightness along the line of sight to clusters of galaxies has the distinct advantage of being independent of distance. The SZ flux density from a cluster will diminish with distance to the cluster as the square of the angular-size distance; this is in contrast to the X-ray flux densities from clusters which diminish as the square of the luminosity distance to the cluster. Thus the SZE does not suffer from the traditional disadvantage of $(1+z)^{-4}$ 'cosmological dimming'. Lastly, it must be mentioned that whatever discussed till now goes by the name of 'Thermal SZ effect'. Also 'Kinematic SZ effect' is possible due to bulk motion of the gas: the temperature decrease due to it is given by $\frac{\Delta T}{T} \approx -\tau_e \frac{v_z}{c}$, where v_z is the component of the peculiar velocity along the line of sight. The ratio of the decrements due to Kinematic SZ effect to that due to Thermal SZ effect is in general small for clusters.

3. Constraining Ω_o

The evolution in the abundance of clusters of galaxies is sensitive to the mean matter density Ω_o and, consequently, is a useful constraint on cosmological models. Moreover, it is these clusters that give rise to the distortion in the CMB due to SZE. Hence the mean distortion to the CMB due to SZE can be used to constrain the value of Ω_o. In the next subsection we deal briefly with both source counts and blank sky surveys used to probe Ω_o. For both the cases, we first have to relate the number density of collapsed objects (clusters in our case) to the underlying cosmological model. Under the assumption of initial Gaussian density perturbations, the mass and redshift distribution of clusters is given by the Press-Schechter mass function [11]

$$n(M,z)dM = \sqrt{\frac{2}{\pi}}\frac{\rho_{mean}}{M}\nu_c \mid \frac{d\ln\sigma(M)}{d\ln M} \quad e^{-\nu^2/2}\frac{dM}{M} \tag{8}$$

where, we have,

$$\nu_c(M,z) = \frac{\delta_c(z,\Omega,\lambda)}{\sigma(M)}\frac{D_g(0,\Omega,\lambda)}{D_g(z,\Omega,\lambda)}. \tag{9}$$

D_g is the growth function and $\delta_c = 1.68$ (see, [9]). To get $\sigma(M)$, we use

$$\sigma^2(R,z) = \int_0^\infty \frac{dk}{k}\frac{k^3}{2\pi^2}P(k)\left|\tilde{W_R}(k)\right. \tag{10}$$

where $\tilde{W_R}(k)$ is the Fourier Transform of a real space window function and $M_R = \frac{4\pi}{3}\rho_c\Omega_0 R^3$ is the mass inside the window. Finally with a suitable of choice of the Transfer Function(see [2] for example) we can get the Power Spectrum in the usual way, which is then normalized to the observed X-ray cluster abundance ([14])or the the 4 year $COBE-DMR$ data ([4])(see Figure 2).

3.1. *Using source counts*

Since SZE does not suffer from cosmological dimming, it can be used as a tracer for clusters, as unresolved clusters similar in mass and temperature will give similar decrements, independent of their redshifts. This brings in the use of future planned satellite missions (like MAP and PLANCK,see respective homepages for details) to detect and count clusters through their SZ effects. Unlike current observations this calls for 'non-targeted' surveys. Planned surveys are sensitive to both the underlying cosmology and the gas distribution within the clusters(in case they are resolved). The redshift distribution gives the epoch of 'freeze-out' of the structures, which is governed by Ω_o and/or Ω_Λ, if there is significant contribution from vacuum energy. The thing to be noted is that for low Ω_o clusters are formed earlier in time and also the total number of clusters is more than that for a flat $\Omega_o = 1.0$ universe. The future

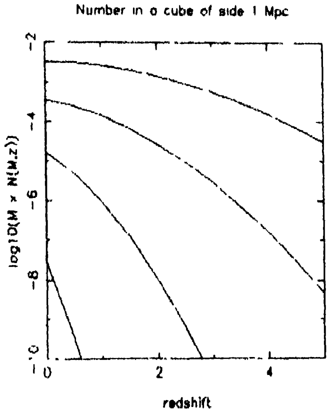

Figure 2. Plot of number density of objects versus redshift for an open universe ($\Omega_o = 0.4$) model. The 4 plots from top to bottom are for masses $10^{12}M_\odot$, $10^{13}M_\odot$, $10^{14}M_\odot$, $10^{15}M_\odot$.

satellite CMB experiments will be used to make SZ cluster surveys, providing us with a large catalog of clusters. Thus we will have an excellent information about the local abundance of clusters (though much may not be known about their redshift evolution). The redshift distribution of clusters brighter than a certain threshold can be obtained by integrating the Press-Schechter mass function. The corresponding source count is given by

$$\frac{dN(>S_\nu)}{d\Omega} \quad \int dz \frac{dV}{dzd\Omega} \int_{M_{min}} dM \frac{dn}{dM} \qquad (11)$$

where S_ν and M is connected by

$$S_\nu = (8mJyh^{2.7}) j_\nu(x) f_B \Omega_o{}^{1/3} M_{15}{}^{5/3} [\frac{\Delta(z)}{178}](1+z) D^{-2}(z) \qquad (12)$$

In eqn (12), M_{15} is the mass of the cluster in units of $10^{15}M_\odot$, f_B is the baryonic fraction,D is the dimensionless part of the angular distance and j_ν is the characteristic SZ frequency dependent part. A simple calculation using eqn(11) and eqn(12) shows that there are significant differences for open and closed models of the universe (for example: for a threshold of about $50mJy$, the expected threshold for PLANCK, there is increase of 10 clusters per degree patch of the sky if one goes from a flat $\Omega_o = 1.0$ to an open $\Omega_o = 0.2$ universe (for details see, [5] and references therein).

3.2. *Using blank SZ surveys*

The present blank SZ surveys undertaken(for example by ATCA, see [1]), aim at observing the CMB sky at arcmin patches to get the arcmin scale temperature anisotropy. These surveys can be used to compare the observed rms anisotropy of the CMB at the arcmin scale with simulated temperature anisotropy maps of the sky due to SZE for different cosmological models, taking cosmological distribution of clusters into account. There have been reports in literature of the detection of radio decrements (thought to be due to the SZ effect) in sensitive images made of 'blank' sky fields [6];however, sensitive observations of several other fields with arcmin resolution have failed to detect any decrements or CMB anisotropies [13]. These surveys have given put limits to the Q_{rms} at cluster scales, which have been compared with simulations ([7]) to constrain the matter content of the universe. To simulate square patches of the sky, arcmin scale patches have been broken down into smaller pixels,the cluster number density calculated from Press-Schechter formalism and at each redshift slice the clusters are randomly Poisson distributed. The random distribution of masses generates random images of the sky as should be seen by a telescope in a blank sky survey, an ensemble of which is needed for this study. For resolved clusters, the density profile of the cluster is given by the isothermal β model, where the core radius and the cluster gas temperature is assumed to follow simple scaling laws. The SZ decrement at each pixel is simply given by $\frac{\Delta T}{T} = -2y$. Once the cumulative decrement at each pixel is obtained for a distribution of clusters, the variance is calculated, both before and after convolution with the ATCA beam. A typical simulated image is shown in Figure 3. A comparison with the ATCA observations seem to disfavour the standard CDM model with $\Omega_o = 1.0$ and lends support to a low density universe.

4. Using SZE of a cluster to determine H_0

If a cluster is nearby(and hence resolved),then its substructure can be probed by both SZE on the CMb and also by X-ray observations and a joint analysis taking both into account can be then be used to get an estimate of the Hubble Constant. For a resolved cluster, we have , along the line of sight, the scattering optical depth, the Comptonization parameter and the X-ray spectral surface brightness, given by

$$\tau_e = \int n_e(\mathbf{r})\sigma_T dl \tag{13}$$

$$y = \int n_e(\mathbf{r})\sigma_T \frac{k_B T_e(\mathbf{r})}{m_e c^2} dl \tag{14}$$

$$b_X(E) = \frac{1}{4\pi(1+z)^3} \int n_e(\mathbf{r})^2 \Lambda(E, T_e) dl \tag{15}$$

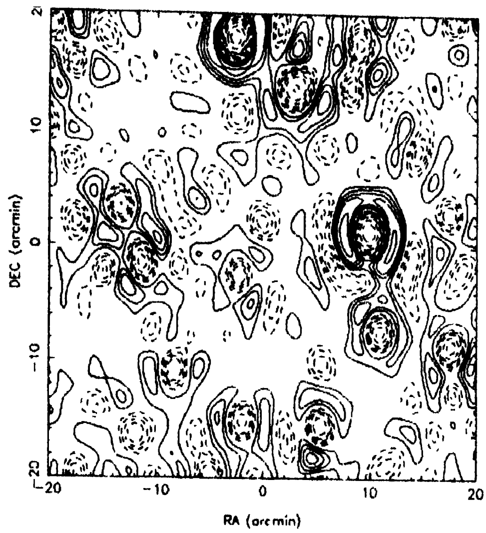

Figure 3. An ATCA beam convolved image with contours at $50\mu Jy$/beam × (-16,-12,-8,-6,-4,-2,-1,1,2,3,4,6)

In the above equations $n_e(\mathbf{r})$ is the intracluster electron distribution, which is popularly represented by the form $n_e(\mathbf{r}) = n_o(1 + r^2/r_c^2)^{-3\beta/2}$. In the rest of the text we take $\beta = 2/3$ for convenience. Then, with the assumption of spherical symmetry for a cluster, we have

$$\tau_c(\theta) = \tau_o\left(1 + \theta^2/\theta_c^2\right)^{-1/2} \tag{16}$$

$$y(\theta) = y_o\left(1 + \theta^2/\theta_c^2\right)^{-1/2} \tag{17}$$

$$b_X(\theta) = b_{X_o}\left(1 + \theta^2/\theta_c^2\right)^{-3/2} \tag{18},$$

where the central values are given by $\tau_o = \pi n_o \sigma_T r_c$; $y_o = \tau_o \frac{k_B T_e}{m_e c^2}$ and $b_{X_o} = \frac{1}{8(1+z)^3} n_o^2 \Lambda(E, T_e) r_c$. In eqns(16-18), θ is the angle from the centre of the cluster to the direction of the line of sight, $r_c = \theta/D_A$ is the angular core radius, and D_A is the angular diameter distance to the cluster given by(see Weinberg)

$$D_A = \frac{c}{H_o q_o^2} \frac{(q_o z + (q_o - 1)(\sqrt{1 + 2q_o z} - 1))}{(1+z)^2} \tag{19}$$

where q_0 is the deceleration parameter and H_0 is the Hubble Constant. Observationally, from fits to data,the central parameters can be detected, and then from joint fits to SZ and X-ray data, the value of the Hubble Constant can be determined with some specific assumption about the underlying cosmology ($i.e\, q_0 = \Omega_0/2 - \Omega_\Lambda$). The point to be noted here is that this is a *direct* method of determining the Hubble Constant, and hence can be applied to large cosmological distances, without the use of the standard 'distance estimators' and hence is devoid of usual 'distance-ladder uncertainties'.

5. Conclusion

Constraining cosmological parameters by present and future ground based and satellite missions using Sunyaev-Zeldovich effect have been discussed. It is shown that the non-targeted surveys have the prospect of constraining the value of Ω_0 through SZ source counts. Also, it has been pointed out that the present limits on arcmin fluctuations of CMB, when compared with simulations tends to favour low density open universe models. Thus measurement of the secondary temperature anisotropy due to SZE provides a measure of the underlying cosmology, independent of the primary anisotropies. Moreover, for low redshift objects SZE studies of clusters along with accompanying X-ray studies give a direct way of measuring the Hubble Constant.

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