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Black hole thermodynamics

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Abstract. We review latest results on the string theoretic description of the laws of black hole thermodynamics.

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Black holes are known as classical gravitating objects which trap all objects including light. As light cannot escape them, they constitute literally 'black holes' in space-time. In 1974, S.W. Hawking, changed this definition of black holes, when he showed that they are not really 'black' but hot radiating bodies at a certain temperature. Understanding these new concepts of temperature, black hole radiation and other thermodynamic quantities like entropy which came to be associated with the black hole has been an outstanding problem of the last few decades.

Black holes arise as solutions of Einstein's theory of General Relativity. Though it is very difficult to observe black holes, today there are a dozen of black hole candidates in our universe, detected by using indirect evidences. Einstein's theory of gravity relates the curvature of space-time to matter. Through the use of this, the nature of metric and curvature of space-time for certain matter configurations, e.g., a static point mass has some very interesting properties which makes it a black hole. It can be shown that the curvature is singular at the origin of space-time, and the metric is such that coordinates which are time-like become space-like after a certain critical distance away from the singularity. This gives rise to two causally disconnected regions which cannot communicate with each other. The critical distance is called the horizon, and determines the size of the black hole. For a stationary mass, it is given by $2GM/c^2$, where G = Newton's constant, M mass of the particle, and c the velocity of light. The boundary of these two regions is a null surface located at the critical radius, called the event horizon. Since a particle

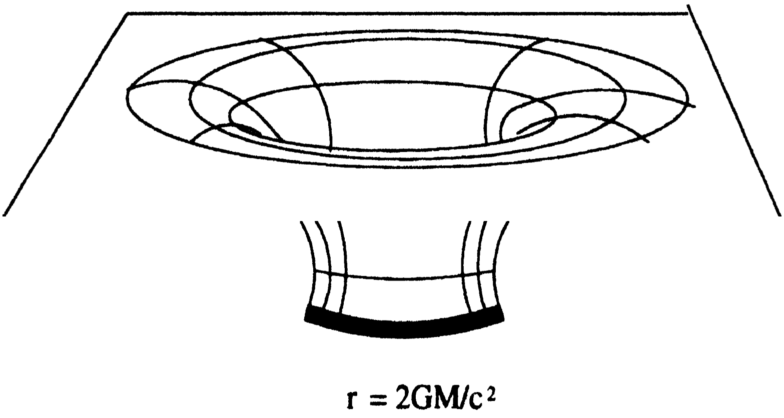

Figure 1. Time- Slice of a Schwarzschild Black Hole: A Hole in Space

which falls inside the horizon cannot communicate with an observer standing outside, it is lost to the world. By laws of gravity, the particle ends up at the singularity, where physical laws fail to describe it anymore.

This was the understanding of black holes, when it was found that there exists a special relation between the change in mass of the black hole, it's area of horizon, angular momentum and electromagnetic charge. These laws, known as the laws of black hole mechanics were derived from simple considerations of black hole processes. The set of laws were soon supplemented by the area increasing theorem, which states that in any physical process, the area of a black hole horizon always increases. Taken together these laws can be written as:

$$dM_{BH} = \frac{\kappa}{8\pi G} dA_{BH} + \Omega_{BH} dJ + \Phi_{BH} dQ \qquad (1)$$

$$dA_{BH} \geq 0 \qquad (2)$$

Where M_{BH}, J, Q are mass, angular momentum, charge of black hole, A_{BH}, area of horizon, Ω_{BH} and Φ_{BH} are angular velocity and electromagnetic potential at the horizon; κ is surface gravity. The laws resemble the first and the second laws of thermodynamics. It is easy to see that M_{BH} is like the internal energy of the black hole, and the terms $\Omega_{BH} dJ + \Phi_{BH} dQ$ are the analogs of PdV term in the first law of thermodynamics. Hence the correspondence is exact, if we identify the Area of the black hole A_{BH} as the entropy, and the surface gravity, κ with the temperature of the black hole [1]. The surface gravity of a black hole is a geometric quantity, which can be defined as the force exerted by an observer standing far away from the black hole to keep a unit

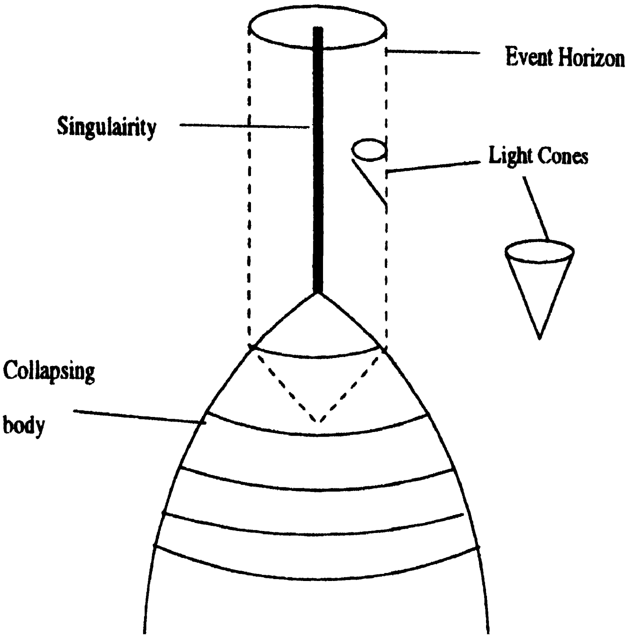

Figure 2. A Black Hole Space-time

mass test particle stationary at the horizon. It is remarkable that this force is constant over the horizon, and is determined uniquely in terms of black hole parameters. The identification of a black hole with a thermodynamic system however could not be achieved based on these evidences. The important point to be clarified was that if there exists a hot body at a certain temperature, then it should radiate too. The black hole as it was known till then, could only absorb, with the event horizon serving as a one way membrane.

It was in 1974 that Stephen Hawking [2] came with the idea of using quantum mechanics near a black hole. It was well known that, quantum gravity becomes important for gravitating objects whose Compton wavelength is of the order of it's Schwarzschild radius. This quantity called Planck's mass is very small, $\sim 10^{-5} gm$. For black holes greater than this mass, gravity can be taken as a fixed classical background, while other fields can be treated as quantum. The study of quantum fields on a fixed curved background is called the semi-classical approximation for a theory with gravity. Using the semi-classical approximation for scalar fields, Hawking showed that the scalar vacuum at past infinity evolves into a thermal state in the future. The obvious conclusion is that the black hole radiates particles in a thermal spectrum. The

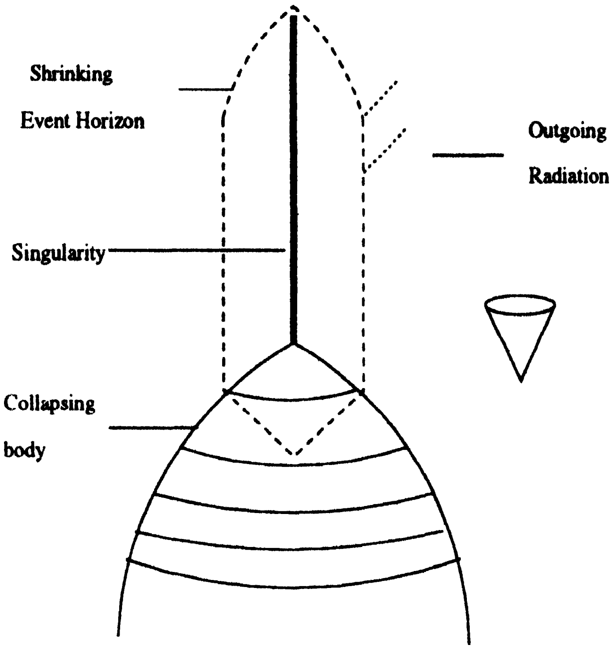

Figure 3. An evaporating black hole space-time

most interesting part of this result is that the temperature of the distribution is $\kappa/2\pi$, exactly what was required to complete the thermal description of the black hole.

Though the temperature for a solar mass black hole $\sim 10^{-8}K$, there can be black holes formed in the early universe whose Hawking temperature is much greater than the ambient temperature of the Universe. The questions which were raised by Hawking's remarkable discovery were, that

- Is the black hole a thermodynamic system with properties similar to ordinary thermodynamic systems like a black body? If so, then, is there a heat bath associated with the black hole? Is there a statistical description in terms of micro-states, which would determine entropy in terms of microscopic degeneracy of states?
- For asymptotically flat black holes, the system is unstable against radiation, and hence dissipative in nature. e.g. for a Schwarzschild black hole temperature $T_H \propto 1/M$, which means that a slight radiation increases the temperature, and hence the radiation rate. If the black hole completely evaporates due to radiation, then it implies a non-unitary evolution as initial pure states evolve to a thermal state. For a burning piece

> of coal, which macroscopically has the same behavior, we know that there is an underlying microscopic unitary process which involves excitations of molecules due to external heat. In quantum mechanics, unitary evolution is a primary requirement, and hence does the presence of black hole imply a contradiction to the laws of quantum mechanics?

It is these two basic questions which we try to address subsequently. In the beginning, Hawking's results were probed for their validity within the semi-classical framework. It was investigated whether the interactions are basically unitary, and subsequent averaging leads to thermalization. This involved considering back reaction of matter particles on the gravitational field of the black hole. Later with the advent of string theoretical black holes, it was observed that black holes can be treated as ordinary thermodynamical systems, with a statistical interpretation in terms of string micro-states. Whether the above succeed in implying a unitary evolution is a question still being investigated. In the following, we describe the work done using the string approaches.

A major breakthrough came in the work of [3], where certain string theoretic black holes were described as comprised of string solitonic states. The focus then shifted to understanding string black holes and a new physics emerged.

String theory is a candidate for the theory of everything, including quantum gravity. In string theory the fundamental objects are not point particles, but strings, with a given fundamental length l_s, associated with them. The various vibrational modes of the string appear as particle excitations. One of the modes of vibrations yields a field to which the string does not couple. The objects which couple to these fields are called D – ***Branes*** (for an introduction see [4]). The D-p-branes or Dirichlet branes are so called as they are described in perturbative string theory as p dimensional hypersurfaces with open strings stuck on them. The strings thus have to satisfy Dirichlet boundary conditions at the position of the D-branes. When these extended objects are wrapped on compact spaces, in the super-gravity limit, they give rise to black hole solutions. It is these black holes which we shall call string black holes, and try to understand their thermodynamics.

In [3], the string black hole considered was made by wrapping D-1-branes and D-5 branes of type IIB String Theory on a compact manifold $K^3 \times S^1$ (K^3 stands for kahler manifold). It was shown that under certain restrictions of the number of charges, the metric can be mapped to an extremal five dimensional Reissner-Nordstorm black hole, a charged black hole solution of ordinary general relativity. The system of D-1/D-5 branes at low energies is described by a conformal field theory with a central charge. In [3] it was shown that for large no. of D-1 and D-5 branes, the log of number of states of this CFT is equal to $A_{BH}/4G$, and hence the entropy of the black hole. The result had remarkable implications in the in the attempts for a statistical description of black holes.

The other aspect of the D-brane description of black holes, is of course

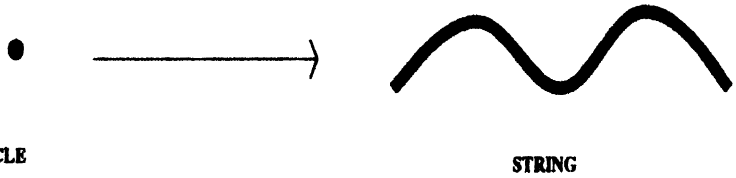

Figure 4. String theory

the study of Hawking radiation, and hence the unitarity puzzle. It was soon shown in [5, 6] that the decay rate of the collection of D-Branes into scalars, have the same radiation rate as the corresponding black hole. We examined [7] the case for fermions, for a particular four dimensional black hole obtained by compactifying M theory on $T^6 \times S^3$. The brane configuration consists of three M-5 branes intersecting along a common line . The computation of the black hole decay rate was somehow involved, as the fermion propagation on these black hole backgrounds is not exactly determined. Under certain approximations, we showed that the rate had a structure in which, there was a product of two thermal distributions, at two different characteristic temperatures T_L and T_R, such that $4\pi/\kappa = 1/T_L + 1/T_R$. One of the thermal distributions was fermionic and the other was bosonic. The interpretation of this in terms of a microscopic theory had to be done by looking at the branes. We took help of evidences of the fact that the excitations along the common intersection lines of the brane constituted a 1+1 conformal field theory. We used an interaction between the CFT modes and the bulk gravitino, which corresponds to two open strings colliding to give a closed string in the bulk for D-Branes. The gravitino with vector polarization along the compact directions gives rise to a fermion in the four non-compact directions, and our calculations reproduced the black hole emission rates. The two characteristic temperatures were the effective left and right temperatures of the 1+1 dimensional CFT.

As described above, the string black holes, have a microscopic description in terms of a Conformal field theory, constituted by the excitations of the D-Branes or M-Brane configurations. This fact has support from the evidence that the degeneracy of states of this CFT and the decay rate into scalars, fermions, etc match the black hole radiation rates. But, the question is, is this sufficient evidence for a proof? One of the major shortcomings of the above approach is that the description of the Branes exists when the background metric is flat. When the gravitational coupling and the interaction between the branes became large, the branes collapses to give a black hole, and it remains to be answered whether under this collapse process, the degeneracy and the

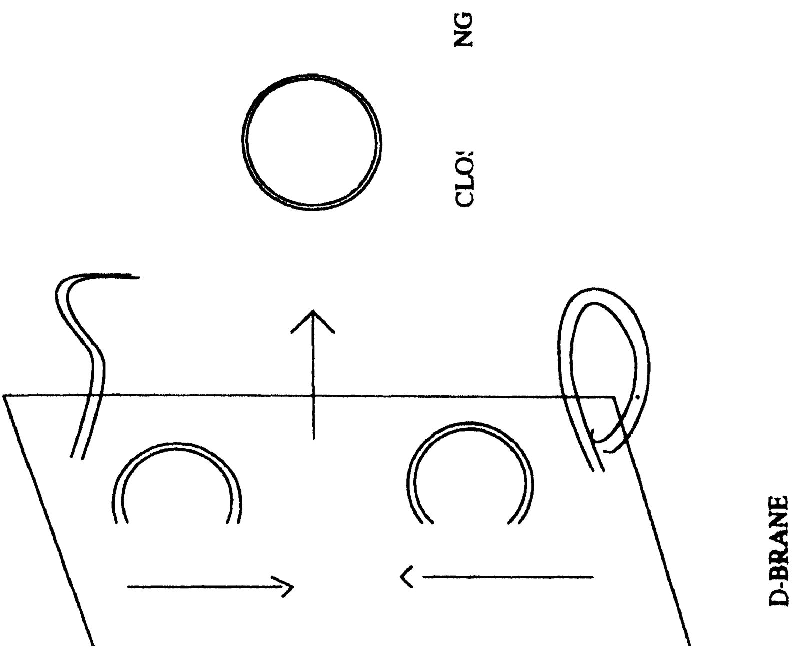

Figure 5. D-Brane decay

radiation rate do not undergo renormalization. Due to this problem another crucial question remains unanswered, and that is of the location of degrees of freedom of this microscopic theory in the black hole background. In the microscopic description of any thermodynamic system, e.g. a gas, we identify the molecules of the gas as the microscopic degrees of freedom. However, in presence of the black hole where concepts of horizon arise, the brane degrees of freedom become obscure. In trying to investigate this question it was found in [8], that the near horizon geometries of all these black holes, have the same isometries as anti-sitter space, and the product of a compact manifold. This led to a conjecture that string theory on anti-de Sitter space is dual to a conformal field theory which lives on it's boundary [8, 9].

The near horizon geometries of some 5 and 4 dimensional black holes are found to be 3 dimensional anti-de Sitter space with some global identifications. This space-time has two horizons, and is a black hole solution called the BTZ black hole [10]. The most interesting aspect about the BTZ black hole is that it's entropy can be reproduced from a CFT [11], and has the exact value as the higher dimensional black hole. We showed that the Hawking emission rates for massless fermions which are non-minimally coupled to gauge fields in higher dimension have a radiation rate in the BTZ space-time, which is same as the higher dimensional black hole [12]. These results confirm and enrich the conjecture that the microscopic degrees of freedom of the particular black holes considered are the degrees of freedom of a Conformal Field Theory, and they lie on the boundary of the near horizon geometry, obeying a 'holographic' principle.

It is undoubtedly true that the description of certain supergravity black holes in terms of microscopic theory is a major step in understanding black hole thermodynamics. However, this result excludes a class of black holes, including the Schwarzschild black hole. The black holes which are expected to be found in nature belong precisely to the above. Hence much work needs to be done in this direction. Also, though we understand the thermodynamics of the string black holes, the crucial issue of non-unitary evolution in Hawking radiation is not understood very well. It still remains to be seen whether we can think of a black hole just like a burning piece of charcoal.

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