Indian J. Phys. 75A (2), 201-204 (2001)

IJP A
an international journal

On determination of phase function from phase shift

I Ahmad* and Jamal H Madani
Department of Physics, Faculty of Science, King Abdulaziz University,
Jeddah, Kingdom of Saudi Arabia

Received 19 April 2000, accepted 5 May 2000

Abstract : The accuracy of the phase function $\chi(b)$ as determined from the phase shift δ_ℓ using the relation $\chi(b) = 2\delta_\ell|_{\ell=kb-\frac{1}{2}}$ has been studied taking the example of ^{12}C–^{12}C system in which case the phase function so obtained has been used recently to determine the optical model potential by inversion in the energy range of about 0.14 – 1.0 GeV We find that the phase function as determined from the above prescription, differs significantly from the realistic one at lower side of the energy range.

Keywords : Nuclear scattering, eikonal approximation, phase function.

PACS Nos. : 24.10.Ht, 11.80.Hv

Recently, some authors [1-4] have determined the heavy-ion optical model potential (OMP) at intermediate energies by inversion using the expression for the potential in terms of the phase function $\chi(b)$ derived by Glauber [5] in the eikonal approximation. The same approach was earlier applied to study proton OMP [6] and the α-particle OMP [7,8] at intermediate energies. In these studies, the required phase function $\chi(b)$ is obtained from the diffraction model phase shift δ_ℓ using the relation $\chi(b) = 2\delta_\ell|_{\ell=kb-\frac{1}{2}}$, where k is the incident momentum in the center of mass system and ℓ is the orbital angular momentum quantum number. On the other hand, in some calculations of the heavy-ion elastic scattering, differential cross section (*e.g.* Refs. [9,10]) based on Glauber multiple scattering model but using the partial wave expression for the scattering amplitude (presumably to enlarge the angular domain of validity), the required phase shifts are obtained from the Glauber phase function from the relation $2\delta_\ell = \chi(b)|_{b=(\ell+\frac{1}{2})/k}$.

Needless to say, the procedure of determining $\chi(b)$ from the phase shift (or *vice versa*) using the correspondence $kb \leftrightarrow (\ell+\frac{1}{2})$, is approximate and is expected to be valid only at sufficiently high energy. Therefore, care need be exercised in applying the above correspondence at the lower side of the intermediate energy region, especially if the $\chi(b)$ as determined from the diffraction model phase shift is to be used for the determination of the optical potential by inversion as in Refs. [1, 3].

In this work, we calculated $\chi(b)$ and the Glauber S-function $[S(b) = e^{i\chi(b)}]$ exactly from the diffraction model phase shifts and compare them with those obtained from the correspondence $(\ell+\frac{1}{2}) \leftrightarrow kb$ for ^{12}C–^{12}C system in which case, diffraction model phase shifts are available over the wide energy range of 140-2400 MeV and have been used to determine OMP by inversion [1]. Significant differences between the exact and approximate calculations are found at lower energies.

The partial wave decomposition of the scattering amplitude $f(q)$ for the scattering of a spin-zero particle is given by

$$f(q) = \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1-e^{2i\delta_\ell}\right)P_\ell\left(1-\frac{q^2}{2k^2}\right) \tag{1}$$

where $q(=2k\sin\theta/2)$ is the momentum transfer, k is the incident momentum and P_ℓ is the Legendre polynomial of degree ℓ. Using the integral representation [11]

$$P_\ell\left(1-\frac{q^2}{2k^2}\right) = 2k\int_0^\infty J_0(qb)\, J_{2\ell+1}(2kb)\, db\,, \quad (q^2 < 4k^2) \tag{2}$$

* Corresponding Author.

and substituting it in eq. (1), the scattering amplitude may be written as

$$f(q) = i\,k \int_0^{\infty} db\; b\; J_0(qb)\, \Gamma(b)\,, \tag{3}$$

where

$$\Gamma(b) = \frac{1}{kb} \sum_{\ell=0}^{\infty} (2\ell+1) J_{2\ell+1}(2kb)\left[1 - e^{2i\delta_\ell}\right]. \tag{4}$$

Next, using the relation

$$\sum_{\ell=0}^{\infty} (2\ell+1) J_{2\ell+1}(z) = \frac{z}{2} \tag{5}$$

in eq. (4), the scattering amplitude as given by eq. (3) may be expressed as

$$f(q) = i\,k \int_0^{\infty} db\; b\; J_0(qb)\left[1 - S(b)\right], \tag{6}$$

where $S(b)\ (= e^{i\chi(b)})$ is given by

$$S(b) = \frac{1}{kb} \sum_{\ell=0}^{\infty} (2\ell+1) J_{2\ell+1}(2kb)\, e^{2i\delta_\ell}\ . \tag{7}$$

The Fourier-Bessel representation of the scattering amplitude as given by eq. (3) is an exact expression valid throughout the whole angular domain $\theta \in (0, \pi)$. It was introduced by Blankenbecler and Goldberger [12] who took it as the starting point to study high energy diffractive scattering. Later, starting from the partial wave expansion, it was derived using the integral representation for $P_\ell(\cos\theta)$ as discussed above [13]. An exact expression for $S(b)$ in terms of δ_ℓ has been derived by Wallace also using a different approach [14].

From the properties of Bessel function [11], it follows that the first maximum of $J_{2\ell+1}(2kb)$ occurs at $\ell \approx kb - \frac{1}{2}$. For $\ell < kb - \frac{1}{2}$, $J_{2\ell+1}(2kb)$ has oscillating behaviour and assumes positive and negative values, while for $\ell > kb - \frac{1}{2}$, it assumes negligible values for $\ell >> kb$. Thus, when large number of partial waves are involved, the most significant contribution to the sum in eq. (7) is expected to come mainly from the term for which $\ell \approx kb - \frac{1}{2}$. Hence, taking $e^{2i\delta_\ell}\big|_{\ell = kb - \frac{1}{2}}$ out of the sum in eq. (7) and using eq. (5) we have

$$S(b) \approx e^{2i\delta_\ell}\Big|_{\ell = kb - \frac{1}{2}} \tag{8}$$

which is the commonly used prescription for obtaining $S(b)$ from $S_\ell = e^{2i\delta_\ell}$.

Here, we present results of our calculation of $\chi(b)$ and $S(b)$ for ^{12}C–^{12}C system using eqs. (7) and (8) at 139.5, 240 and 1016 MeV lab energies. The required phase shifts have been taken from the work of Eldebawi and Simbel [1] who made a diffraction model analysis of ^{12}C–^{12}C elastic scattering data in the energy range 139.5–2400 MeV using Ericson's parameterization of the phase-shift :

$$S_\ell = \frac{1}{1 + e^{\quad \Delta}} \tag{9}$$

where $S_\ell = e^{2i\delta_\ell}$; and ℓ_0, Δ and λ are the parameters

In Figures (1-3), we show calculated $|S(b)|$, Re $\chi(b)$, and

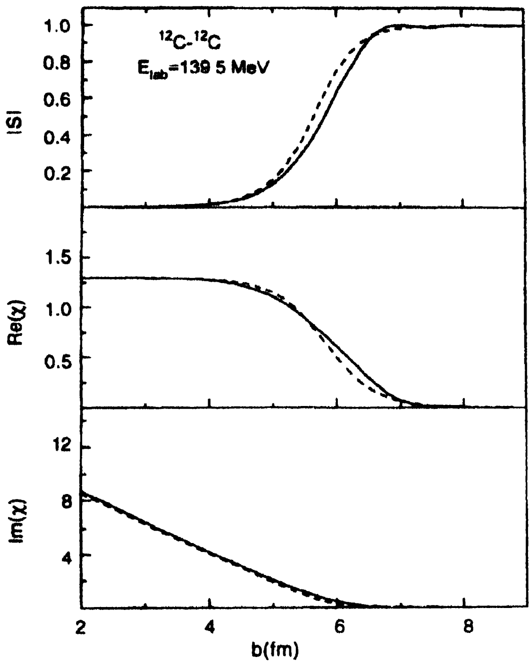

Figure 1. The phase function $\chi(b)$ and $|S(b)|$ at 139.5 MeV Continuous curve : exact calculation. Dashed curves : approximate calculation

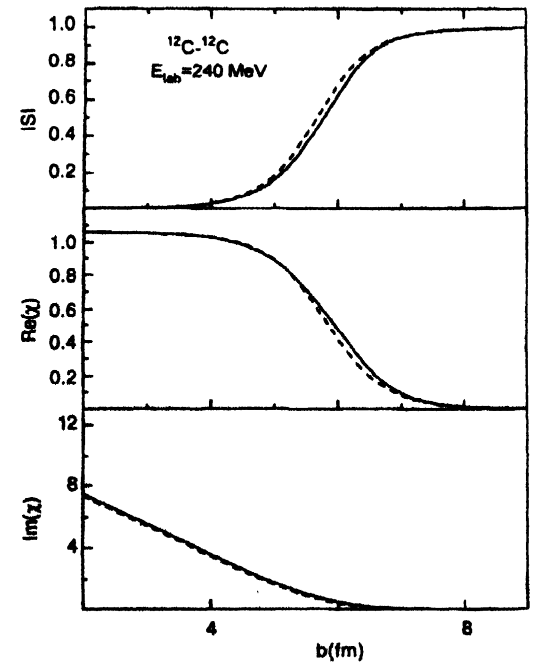

Figure 2. Same as in Figure 1 but for 240 MeV.

$\text{Im}\,\chi(b)$ at 139.5, 240, and 1016 MeV respectively. The parameter values which have been used in the calculation are [1] : $\ell_0 = 26$, $\Delta = 2$, $\lambda = 2.6$, at 139.5 MeV; $\ell_0 = 34$, $\Delta = 3.02$, $\lambda = 3.63$, at 240 MeV; and $\ell_0 = 59.5$, $\Delta = 6.4$, $\lambda = 7.0$, at 1016 MeV. Calculations were made at other energies also, but are not shown for brevity.

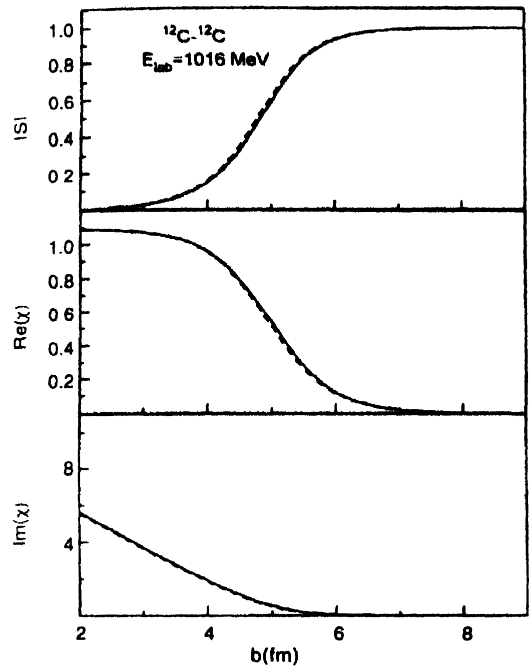

Figure 3. Same as in Figure 1 but for 1016 MeV

The solid curves in the Figures (1-3) show the results of exact calculation using eq. (7), and dashed curves show the results of approximation calculation using eq.(8) at 139.5, 240, and 1016 MeV respectively. It is seen from Figure 1 that the predictions of the approximate expression given by eq. (8) differ significantly from those of the exact expression (7) at 139.5 MeV in the region around the interaction radius R_{int} (defined by

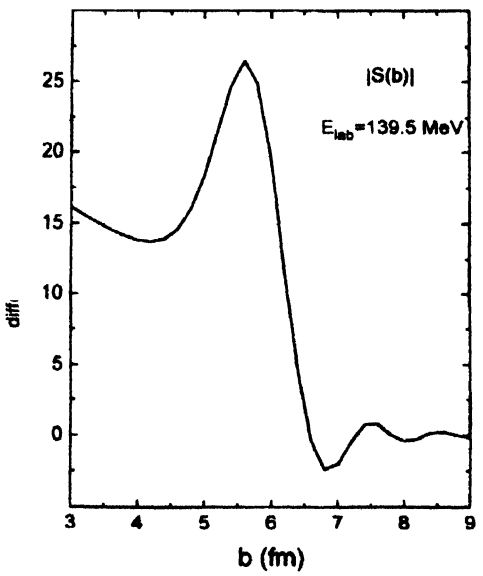

Figure 4. Percent difference between exact and approximate calculations for $|S(b)|$.

$|S(R_{\text{int}})| = \frac{1}{2}$) to which the scattering is the most sensitive. The percentage difference between the results of approximate and exact calculations are shown in Figure 4. It is seen that the maximum disagreement in $|S(b)|$ occurs around R_{int} and is about 26%.

Not unexpectedly, it is seen from Figures 2 and 3 that the situation improves as the energy increases. The maximum deviation of the approximate calculation from the exact one for $|S(b)|$ around R_{int} is about 14% at 240 MeV and less than 5% at 1016 MeV. At 1440 MeV, our calculation (not shown here) shows it to be about 1%.

During the course of calculation, we noted that the goodness of the approximation being studied here, depends not only upon the energy but also upon the rate of change of $|e^{2i\delta_\ell}|$ with ℓ. To highlight this point we show in Figure 5, the results of our approximate and exact calculations of $|S(b)|$ at 360 MeV for the parameter values $\ell_0 = 39$, $\Delta = 0.9$, and $\lambda = 0$. The choice of the parameter values is such that the parameterization (9) is almost the sharp cut-off model of nuclear diffraction theory of early days. It is seen that large disagreements between the exact and approximate calculations are present [It may be pointed out that $|S(b)|$ being greater than unity as is seen to be the case for some values of b in Figures 5 and 1 is not the violation of the unitarity which applies to S_ℓ and not to $S(b)$].

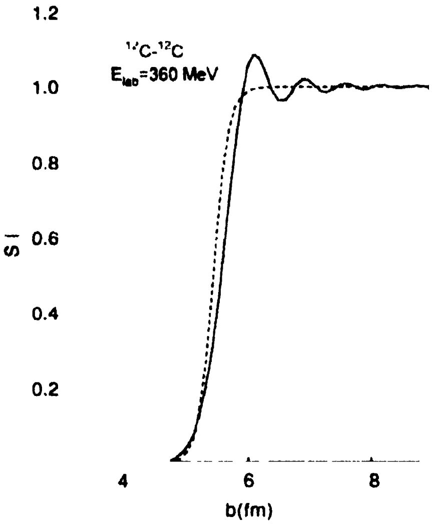

Figure 5. Absolute value of $S(b)$ at 360 MeV corresponding to the parameter values $\ell_0 = 39$, $\Delta = 0.9$ and $\lambda = 0$. Continuous curve exact calculation. Dashed curve · approximate calculation.

From above discussion, it is evident that the prescription of determining $\chi(b)$ from $2\delta_\ell$ by the correspondence $(\ell + \frac{1}{2}) \leftrightarrow kb$ does not work well at the lower side of the energy range considered in some earlier works [1, 4] for the determination of the optical potential by inversion. However, for a reasonably smoothly varying S_ℓ generally used in the nuclear diffraction model analysis, the prescription works quite well at sufficiently high energies. The present study suggests that the lower energy limit above which the approximation $\chi(b) = 2\delta_\ell\big|_{\ell=kb-\frac{1}{2}}$ could be applied with confidence, is about 85 MeV/nucleon.

In this work, we have calculated the phase function $\chi(b)$ exactly in terms of the diffraction model phase shift δ_ℓ for ^{12}C–^{12}C system in the energy range of about 140-1440 MeV. A comparison with approximate phase function calculated using the relation $\chi(b) = 2\delta_\ell\big|_{\ell=kb-\frac{1}{2}}$ shows that the approximate phase function differs significantly from the exact one at the lower side of the energy range. This suggests that the heavy-ion OMP as determined by the method on inversion using the phase shifts δ_ℓ by applying the correspondence $kb \leftrightarrow (\ell+\frac{1}{2})$ should be viewed with caution at the lower side of the intermediate energy region.

Acknowledgment

The authors are thankful to Dr. F. M. Al-Marzouki, Chairman of the Department of Physics for providing facilities to do this work.

References

[1] N M Eldebawi and M H Simbel *Phys. Rev.* **C53** 2973 (1996)

[2] H M Fyyad, T H Rihan and A M Awin *Phys Rev* C53 2334 (1996)

[3] I Ahmad, J H Madani and M A Abdulmomen *J. Phys* G24 899 (1998)

[4] N M Eldebawi *Indian J Phys.* **73A** 681 (1999)

[5] R J Glauber in *Lectures in Theoretical Physics* **Vol. 1** (ed) W E Brittin and L G Dunham (New York : Interscience) p 315 (1959)

[6] L R B Elton *Nucl Phys.* **89** 69 (1966)

[7] A Y Abul-Magd, M E El-Nadi and M H Simbel *Phys. Lett* 34B 566 (1971)

[8] W Haider and I Ahmad *J. Phys.* **A6** 1943 (1973)

[9] A Vitturi and F Zardi *Phys. Rev* **C36** 1404 (1987) ; S M Lenzi A Vitturi and F Zardi *ibid* **40** 2114 (1989)

[10] S K Chargi and S K Gupta *Phys. Rev.* **C56** 1171 (1997)

[11] M Abramowitz and I A Stegun (eds.) *Handbook of Mathematical Functions* (New York ; Dover) p 355 (1965)

[12] R Blankenbecler and M L Goldberger *Phys. Rev* **126** 766 (1962)

[13] E Perdazz *Ann Phys (NY)* **36** 228 (1966)

[14] S J Wallace *Phys. Rev.* **D8** 1846 (1973)