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Scattering of a dyon from a dyonium

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Abstract Investigating the behaviour of a dyon moving in the field of another dyon it has been demonstrated that angular momentum for this system is modified and it carries an extra residual angular momentum Study of scattering of a dyon from a dyonium has also been undertaken and it has been demonstrated that scattering cross sections are modified from the usual scattering cross sections of quantum electrodynamics due to the presence of magnetic charge on dyon.

Keywords : Scattering, dyon, dyonium

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1. Introduction

Physicists have long been interested in existence of magnetic monopole. The early historical interest in monopoles was due to the symmetry between electric and magnetic fields in Maxwell's equation. However, due to lack of abundence of free magnetic charge compared to electric charge, they were not included in the final formulation of those equations. In 1931, Dirac [1] showed that existence of free magnetic charge (Dirac monopole) could provide reason for quantization of electric charge [2]. This work motivated renewed interest in searching for monopoles. Although there was no guidance as to the mass, size, *etc* of these monopoles, several experimental consequences were apparent. It was assumed that the monopole mass would not be very much different from other elementary particles (*e.g.* protons) and would be highly relativistic. As such, these would produce a great deal of ionization while passing through matter but none of these effects were observed casting doubt on existence of these particles. A fresh interest in the subject was enhanced when 't. Hooft [3] and Polyakov [4] demonstrated separately that monopoles exist as solutions in many non-Abelian gauge theories. The possibility of these GUT monopoles provides stimulus for much recent interest in the subject. These monopoles have enormous importance in connection with the problem of quark confinement [5] of quantum chromodynamics, C.P. violation [6], proton decay [7] and baryon number non-conservation processes [8]. Inspite of potential importance of these particles, the theories to describe them suffered from many paradoxes such as Dirac's veto and wrong connection between spin and statistics [9]. Schwinger [10] showed that some of these problems can be resolved by taking electric and magnetic charge on the same particle known as dyon. Moreover, Witten has shown that monopoles are necessarily dyons [6]. The theories to describe these particles were also clumsy and manifestly non-covariant. In order to develop a theory for these particles which will be conceptually as transparent as the usual quantum electrodynamics, we [11–13] started with the idea of two four-potentials to avoid the use of singular potential by taking generalized charge, generalized four potential and generalized four-current associated with these particles as complex quantities with their real and imaginary parts as electric and magnetic constituents. With the help of this theory, we have undertaken the study of bound states and scattering of dyon-dyon [14] and dyon-fermion [15, 16] systems and it has been demonstrated that exact solutions of bound states for these systems in relativistic framework is not possible due to presence of a term vanishing more rapidly than r^{-1} in the potential of such systems. To overcome this difficulty, we studied the Pauli equation for dyon-dyon and dyon-fermion [17] system by *ad hoc* introduction of spin in the Hamiltonian of the system and obtained bound state solutions in Abelian and non-Abelian gauge theories. We have further, studied the bound states of three and four dyons [18,19] and have demonstrated that the

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bound state solutions are modified from the bound state solutions of quantum electrodynamics due to the presence of magnetic charge on dyon. Extending this work in the present paper, we have undertaken the study of scattering of highly energetic dyon from a dyonium.

2. Generalized field associated with generalised charges

Let us introduce the generalized charge q as a complex quantity

$$q = e - ig\,, \tag{2.1}$$

where e and g are the electric and magnetic charges on dyon. Assuming the generalized field $\psi(x)$, generalized four-potential V_μ and generalized four-current J_μ as complex quantities in a similar manner, the equation of motion of generalized charge q in the generalized field may be written as [14]

$$m\ddot{x} = e\{\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{H}\} + g\{\boldsymbol{H} - \boldsymbol{v}\times\boldsymbol{E}\}, \tag{2.2}$$

where $\boldsymbol{v}$ is the velocity of particle and ψ is the generalized field defined as

$$\psi(x) = \boldsymbol{E} - i\boldsymbol{H} \tag{2.3}$$

in terms of electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields. Using the equation of motion (eq. (2.2)), we obtain the following expression for the angular momentum vector $\boldsymbol{J}$ of the j-th generalized charge q_j moving in the field of k-th generalized charge q_k which is assumed to be at rest :

$$\boldsymbol{J} = \boldsymbol{r}\times\boldsymbol{P} + \mathrm{Im}(q_j q_k)\frac{\boldsymbol{r}}{r}, \tag{2.4}$$

where $\boldsymbol{P} = m\frac{d\boldsymbol{r}}{dt}$. But this angular momentum is not acceptable in the presence of magnetic charge because it is not gauge invariant. A gauge invariant and rotationally symmetric angular momentum operator has been derived in the following form

$$\boldsymbol{J} = \boldsymbol{r}\times\left(\boldsymbol{P} - \mathrm{Re}\,q\boldsymbol{V}^T\right) + \mu_{ij}\frac{\boldsymbol{r}}{r}, \tag{2.5}$$

where $\boldsymbol{V}^T$ is the spatial part of generalized four potential $\{V_\mu\}$. The gauge invariant linear momentum operator of j-th generalized charge q_j interacting with the field of k-th generalized charge q_k has the following form

$$\boldsymbol{\pi} = \boldsymbol{P} - \mathrm{Re}\,q\boldsymbol{V}^T. \tag{2.6}$$

Eqs. (2.5) and (2.6) lead to the following expression of gauge invariant as well as rotationally symmetric Hamiltonian operator for the interacting generalized charges q_j and q_k (see Appendix)

$$\hat{H} = \frac{\hat{\pi}^2}{2m} - \frac{\mathrm{Re}(q_j q_k^*)}{r} + \frac{\mathrm{Im}(q_j q_k^*)^2}{2mr^2} \tag{2.7}$$

where the first term corresponds to the kinetic energy term while the second and third terms give the interaction potential energy of generalized charges *i.e.*

$$V(r) = -\frac{\mathrm{Re}(q_j q_k^*)}{r} + \frac{\mathrm{Im}(q_j q_k^*)^2}{2mr^2} \tag{2.8}$$

Eq. (2.5) directly gives the scalar

$$\frac{\boldsymbol{r}\cdot\boldsymbol{J}}{r} = \mathrm{Im}(q_j q_k^*) \tag{2.9}$$

which commutes with all the observables and shows that in eq (2.5) there is a residual angular momentum

$$J_{\mathrm{res}} = \mathrm{Im}\left(q_j\, q_k^*\right)\frac{\boldsymbol{r}}{r} \tag{2.10}$$

carried by the generalized field of the generalized charge besides the orbital and spin angular momentum of each particle. This angular momentum can be identified as Wilson [20] type of angular momentum. It arises due to the rotation of the system of two dyons in the charge space around the line joining them. It may also be described as extra spin of the system which can not be associated with either particle alone. Furthermore, if the generalized angular momentum given by eq. (2.5) is quantized along the line joining the generalized charges q_j and q_k, we obtain the following quantization condition for generalized charges

$$\mathrm{Im}\left(q_j\, q_k^*\right) = n\,, \tag{2.11}$$

where n is an integers. It reduces to the following chirality quantization condition for electrodyons

$$\mathrm{Im}\left(q_j\, q_k^*\right) = \mu_{jk} = e_k g_j - g_k e_j = 0, \pm 1, \pm 2 \ldots\ldots \tag{2.12}$$

Similarly, the real part of $\left(q_j\, q_k^*\right)$ may be shown to have the following form

$$\mathrm{Real}\left(q_j\, q_k^*\right) = \alpha_{jk} = e_j e_k - g_j g_k \tag{2.13}$$

3. Scattering of a dyon from a dyonium

In order to undertake the scattering of a dyon from a dyonium we assume that the dyonium has infinite degrees of freedom, so that it can be excited during the scattering process. The incident dyon may change place with the dyon of dyonium and hence exchange effects may occur in the collision. The incident dyon produces generalized electromagnetic field which may polarize the target dyonium and hence polarization effects are also involved.

If we consider the energy of incident dyon as very high, the exchange and the polarization effects are unimportant and can be left out of consideration.

The Hamiltonian for describing the scattering of a dyon by a dyonium may be written as

$$\hat{H} = \hat{H}_0 + \hat{H}', \tag{3.1}$$

where

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\hat{\nabla}_1^2 - \frac{\hbar^2}{2m}\hat{\nabla}_2^2 - \frac{\mathrm{Re}\left(q_i\, q_j^*\right)}{r_1} + \frac{\mathrm{Im}\left(q_i\, q_j^*\right)^2}{2mr_1^2}$$

describes the internal motion of the dyonium together with the kinetic energy of the relative motion of the incident dyon and the scatterer dyonium and

$$H' = -\frac{\mathrm{Re}(q_i\, q_j^*)}{r_{12}} + \frac{\mathrm{Im}(q_i\, q_j^*)^2}{2m r_{12}^2} - \frac{\mathrm{Re}(q_i\, q_k^*)}{r_2}$$

$$\cdot \frac{\mathrm{Im}(q_i\, q_k^*)^2}{2m r_2^2} \tag{3.2}$$

represents the interaction between the incident particle and the scatterer, while q_i , q_j and q_k are the charges of dyons i, j, k respectively. If we consider the dyons as identical *i.e.* $|q_i| = |q_j| = |q_k| = |q|$ or $|e_i| = |e_j| = |e_k| = |e|$ and $g_i = g_j| = |g_k| = |g|$, the Hamiltonian (3.1) may be written as

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\hat{\nabla}_1^2 - \frac{\hbar^2}{2m}\hat{\nabla}_2^2 - \frac{e^2+g^2}{r_1} - \frac{e^2+g^2}{r_{12}} - \frac{e^2+g^2}{r_2}. \tag{3.3}$$

The eigen functions of H_0 are specified by two parameters α and a through the equation

$$H_0\phi_{\alpha a}(r_1, r_2) = E_{\alpha a}\phi_{\alpha a}(r_1, r_2). \tag{3.4}$$

Here. α specifies the initial quantum state of the incident dyon and a specifies that of the dyonium. We can write

$$\phi_{\alpha a}(r_1, r_2) = w_a(r_1)\, \phi_\alpha(r_2), \tag{3.5}$$

where $w_a(r_1)$ is the unperturbed wave function for the dyonium and $\phi_\alpha(r_2) = \exp(ik_\alpha, r_2)$ is the free particle wave function for the incident dyon.

$$E_{\alpha a} = E_\alpha + \epsilon_a \tag{3.6}$$

where, E_α is the kinetic energy of the incident free dyon and ϵ_a is the unperturbed eigen values of the dyonium.

We can write the wave function $\psi_{\alpha a}^{(+)}$ of the total Hamiltonian $\hat{H}$ in the Born approximation as follows

$$\psi_{\alpha a}^{(+)}(r_1, r_2) = \phi_{\alpha a}(r_1, r_2) + \frac{2m}{\hbar^2}\int G_{\alpha a}^{(+)}(r_1, r_2; r_1', r_2')$$

$$H'(r_1', r_2')\psi_{\alpha a}^{(+)}(r_1', r_2')d^3r_1'd^3r_2', \tag{3.7}$$

where $G_{\alpha a}^{(+)}(r_1, r_2; r_1', r_2')$ is the Green's function for the solution (3.7).

If the velocity of the incident dyon is very high, we can use the Born Approximation (replacing $\psi_{\alpha a}^{(+)}$ inside the integral sign in (3.7) by the free particle function $\phi_{\alpha a}$ and hence, we can write the asymptotic behaviour of $\psi_{\alpha a}^{(+)}$ as follows :

$$\psi_{\alpha a}^{(+)}(r_1, r_2) - \quad \to \phi_{\alpha a}(r_1, r_2)$$

$$+\sum_k \frac{\exp(ik_\alpha r_2)}{r_2} f(k_\beta, b; k_\alpha, a) w_a(r_1),$$

where the scattering amplitude $f(k_\beta, b; k_\alpha, a)$ is given by

$$f(k_\beta, b; k_\alpha, a) = -\frac{1}{4\pi}\cdot\frac{2m}{\hbar^2}\iint \exp(ik_\beta, r_2')\, w_b^*(r_1')H'(r_1', r_2')$$

$$\times \phi_{\alpha a}(r_1', r_2')d^3r_1'd^3r'_2$$

$$\cdot\frac{2m}{4\pi\hbar^2}\iint \exp(-ik_\beta\cdot r_2')\, w_b^*(r_1')H'(r_1', r_2')$$

$$\times \exp(ik_\alpha\cdot r_2')\, w_a^*(r_1')d^3r_1'd^3r_2' \tag{3.8}$$

or

$$f(k_\beta, b; k_\alpha, a) = -\frac{2m}{4\pi\hbar^2}\iint \exp(ik.\, r_2')\, w_b^*(r_1')H'(r_1', r_2')$$

$$\times w_a^*(r_1')d^3r_1'd^3r_2', \tag{3.9}$$

where $k = (k_\alpha - k_\beta)$

or, deleting primes on the variables of integrations, this can also be written as

$$f(k_\beta, b; k_\alpha, a) = -\frac{2m}{4\pi\hbar^2}\iint \exp(ik.\, r_2')\, w_b^*(r_1')H'(r_1', r_2')$$

$$\times\, w_a(r_1')d^3r_1'd^3r_2. \tag{3.10}$$

Now for considering dyon scattering from a dyonium, we start with the following Hamiltonian

$$H'(r_1', r_2') = -\,\mathrm{Re}(qq^*) \quad \mathrm{Re}(qq^*) \tag{3.11}$$

Thus, the scattering amplitude (3.10) can be written as

$$f = -\frac{2m}{4\pi\hbar^2}\iint \exp(ik.\, r_2')\left(-\frac{\mathrm{Re}(qq^*)}{|r_2 - r_1|} - \frac{\mathrm{Re}(qq^*)}{r_2}\right.$$

$$\times\, w_b^*(r_1)\, w_a(r_1)d^3r_1d^3r_2. \tag{3.12}$$

Solving this equation in the usual way, we get

$$f = \frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2k^2}\{\int 1 - \exp(ik.\, r_1')\}\, w_b^*(r_1)\, w_a(r_1)d^3r_1. \tag{3.13}$$

For elastic scatting, the initial and final state of the dyonium are same *i.e.* $w_a \equiv w_b$ and $|k_\alpha| = |k_\beta|$ follows from the energy conservation. Therefore, we may write the scattering amplitude as

$$f_{el} = \frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2k^2}\int\{1 - \exp(ik.\, r_1)\}\, w_a^*(r_1)\, w_a(r_1)d^3r_1. \tag{3.14}$$

If we consider the ground state of dyonium, the scattering cross section is given as

$$f_{el}(\theta) = \frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2k^2}\; 1 - \left|\, 1 + \frac{1}{\,}a_0^2k^2\right.^{-2} \tag{3.15}$$

In the high energy case where Born approximation is valid, the scattering amplitude is

$$f_{el} \xrightarrow[K_{\text{large}}]{} = \frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2 k^2}. \tag{3.16}$$

For inelastic scattering, w_a is different from w_b and hence the first term in eq. (3.13) is zero because of the orthogonality of the unperturbed states of dyonium; so we have

$$f_{inel} = -\frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2 k^2} \int \exp(i\boldsymbol{k}.\boldsymbol{r}_1)\, w_b^*(\boldsymbol{r}_1)\, w_a(\boldsymbol{r}_1)\, d^3 r_1 . \tag{3.17}$$

Let us consider the case when the ground state of dyonium ($1S$ state) is excited to the state $2S$ (first excitation) due to collision with an incident dyon. The scattering amplitude for this scattering is given by

$$f_{inel}(1S \to 2S) = -\frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2 k^2} \int \exp(i\boldsymbol{k}.\boldsymbol{r}_1)\, w_{2S}^*(\boldsymbol{r}_1)$$

$$\times\, w_{1S}(\boldsymbol{r}_1)\, d^3 r_1 . \tag{3.18}$$

For a dyonium, we have [21]

$$\begin{aligned} w_{1S} &= \frac{1}{\sqrt{\pi a_0^3}} \exp\left(-\frac{r}{a_0}\right) \\ w_{2S} &= \frac{1}{4\sqrt{2\pi a_0^3}} \left(2 - \frac{r}{a_0}\right) \exp\left(-\frac{r}{2a_0}\right) \end{aligned} \tag{3.19}$$

where $a_0 = \dfrac{}{m\,\mathrm{Re}(qq^*)}$

Therefore,

$$f_{inel}(1S \to 2S) = -\frac{2}{a_0 k^2} \cdot \frac{1}{a_0^3} \cdot \frac{1}{4\pi\sqrt{2}} \int \exp(i\boldsymbol{k}.\boldsymbol{r}) \left[2 - \frac{r}{\cdot\cdot} \right.$$

$$\left. \cdot \exp\left(-\frac{3}{2}\frac{r}{a_0}\right) d^3 r . \right. \tag{3.20}$$

The above integral gives us the following value of scattering amplitude for the case of inelastic scattering

$$f_{inel} = -\frac{}{\left(\frac{9}{4} + k^2 a_0^3\right)^3} \tag{3.21}$$

4. Exchange scattering of dyon from a dyonium

We have to consider this possibility due to indistinguishability of dyons *i.e.* it is not possible to distinguish the two dyons after scattering, or in other words, we can not say whether the dyon which is scattered, is the initial incident dyon or it is the dyon of dyonium.

Let us label the incident dyon as q_1 and the dyon of dyonium q_2. If the dyon q_1 is scattered, then the scattering is called direct scattering; however, if dyon q_1 replaces dyon q_2 and q_2 is scattered the scattering is known as exchange scattering. In this process, the coordinates of incident dyon $\boldsymbol{r}_1$ (as shown in Figure 1) and the coordinates of dyon of dyonium are interchanged after scattering, and hence the final state $\phi_{\beta b}$ for the exchange scattering can be written as

$$\phi_{\beta b}^{\text{exch}}(\boldsymbol{r}_1 \,.\, \boldsymbol{r}_2') = \exp\left(i\boldsymbol{k}_\beta . \boldsymbol{r}_1\right) w_b^*(\boldsymbol{r}_2) . \tag{4.1}$$

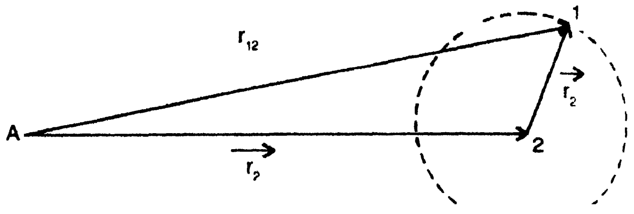

Figure 1. Indices 1 and 2 refer to two dyons of scatterer dyonium and index A refers to the incident dyon

Also, the asymptotic behaviour of $\psi_{\alpha a}^{(+)}(\boldsymbol{r}_1 \,.\, \boldsymbol{r}_2)$ is given by

$$\psi_{\alpha a}^{(+)}(\boldsymbol{r}_1 \,.\, \boldsymbol{r}_2) \xrightarrow[r_2 \to \infty]{} \exp\left(i\boldsymbol{k}_\beta . \boldsymbol{r}_1\right)$$

$$\times\, g(\boldsymbol{k}_\beta, b; \boldsymbol{k}_\alpha, a)\, w_b(\boldsymbol{r}_2) . \tag{4.2}$$

There is no scattered part here, because the dyon q_1 is captured in the dyonium. The scattering amplitude $g(\boldsymbol{k}_\beta, b; \boldsymbol{k}_\alpha, a)$ in the exchange scattering is given by

$$g(\boldsymbol{k}_\beta, b; \boldsymbol{k}_\alpha, a) = -\frac{2m}{4\pi\hbar^2} \iint \phi_{\beta b}^{\text{exch}\,*}(\boldsymbol{r}_1 \,,\, \boldsymbol{r}_2)\, H'(\boldsymbol{r}_1 \,,\, \boldsymbol{r}_2)$$

$$\times\, \psi_{\alpha a}^{(+)}(\boldsymbol{r}_1 \,,\, \boldsymbol{r}_2)\, d^3 r_1\, d^3 r_2$$

$$-\frac{2m}{4\pi\hbar^2} \iint \exp\left(i\boldsymbol{k}_\beta . \boldsymbol{r}_1\right) w_a^*(\boldsymbol{r}_2)\, H'(\boldsymbol{r}_1, \boldsymbol{r}_2)\, \psi_{\alpha a}^{(+)}(\boldsymbol{r}_1, \boldsymbol{r}_2)\, d^3 r_1\, d^3 r_2 \tag{4.3}$$

Now due to identical nature of dyons, it is not possible to label them and according to Pauli's exclusion principle, the total wavefunction $\psi^{(+)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{s}_1, \boldsymbol{s}_2)$ should be properly symmetrized. Since we are assuming here that the incident dyon is a fermion, so the total wave function must be antisymmetric. In non-relativistic limit, the wave function can be written as

$$\psi^{(+)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{s}_1, \boldsymbol{s}_2) = \psi^{(+)}(\boldsymbol{r}_1, \boldsymbol{r}_2)\, \chi^{(+)}(\boldsymbol{s}_1, \boldsymbol{s}_2) , \tag{4.4}$$

where $\psi^{(+)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is the space part and $\chi^{(+)}(\boldsymbol{s}_1, \boldsymbol{s}_2)$ is the spin part of the wave function. Since $\chi^{(+)}(\boldsymbol{s}_1, \boldsymbol{s}_2)$ describes two spin

- 1/2 dyons, we can either have a singlet state or a triplet state. The wave function for the singlet state is

$$\chi^{(+)}_{\sin}(s_1, s_2) = \frac{1}{\sqrt{2}}\{\alpha(1)\beta(2) - \alpha(2)\beta(1)\} \tag{4.5}$$

which is antisymmetric. Thus to make total wave function antisymmetric, we symmetrize the space part as

$$\psi^{(+)}_{\rm sym}(r_1, r_2) = \psi^{(+)}_{\alpha a}(r_1, r_2) + \psi^{(+)}_{\alpha a}(r_2, r_1). \tag{4.6}$$

The first function on the right hand side corresponds to the direct scattering and second one corresponds to exchange scattering. Asymptotic behaviour of the symmetrized wave function (4.6) can be written as

$$\psi^{(+)}_{\rm sym}(r_1, r_2) \xrightarrow[r_1 \to \infty]{} \phi_{\alpha a}(r_1, r_2)$$

$$+\sum_b \frac{\exp(ik_\beta r_1)}{r_1}(f+g)w_b(r_2). \tag{4.7}$$

Thus, the scattering amplitude in the singlet state is the sum of the direct and the exchange scattering amplitudes and hence the scattering cross section for singlet state is given by

$$\sigma_{\sin g} = \frac{\kappa_\beta}{k_\alpha}|f+g|^2 \tag{4.8}$$

The triplet state is given by

$$\chi^{(+)}_{\rm trip}(s_1, s_2) = \begin{cases} \alpha(1)\alpha(2) \\ \beta(1)\beta(2) \\ \frac{1}{\sqrt{2}}\{\alpha(1)\beta(2) - \alpha(2)\beta(1)\} \end{cases} \tag{4.9}$$

i.e. all the triplet states are symmetric. Thus to make the wave function anti-symmetric, we write

$$\psi^{(+)}_{\rm antisy}(r_1, r_2) = \psi^{(+)}_{\alpha a}(r_1, r_2) - \psi^{(+)}_{\alpha a}(r_2, r_1). \tag{4.10}$$

In this case, scattering amplitude will be $|f-g|$ and hence the scattering cross section in triplet state is given by

$$\sigma_{\rm trip} = \frac{\kappa_\beta}{k_\alpha}|f-g|^2. \tag{4.11}$$

Eqs. (4.8) and (4.11) give the differential scattering cross section including exchange effect. Total differential scattering cross section is the sum of $s_{\rm sing}$ and $\sigma_{\rm trip}$ with their proper statistical weight factor

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm tot} = \frac{1}{4}\sigma_{\rm sing} + \frac{3}{4}\sigma_{\rm trip}. \tag{4.12}$$

In order to evaluate eq. (4.12), we should calculate f and g. The direct scattering amplitude f has already been calculated and is given by eq. (3.13) while g is given by

$$g = -\frac{2m}{4\pi\hbar^2}\iint \exp\left(-ik_{\beta'}.r_1\right)w_b^*(r_2)H'(r_1, r_2)$$

$$\times\psi^{(+)}_{\alpha a}(r_1, r_2)d^3r_1d^3r_2. \tag{4.13}$$

We use Born-Oppenheimer approximation. In this approximation, we replace $\psi^{(+)}_{\alpha a}(r_1, r_2)$ by the state $\phi_{\alpha a}(r_1, r_2)$, the unperturbed wave function before settering. Hence,

$$g = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\iint \exp\left(-ik_\beta.r_1\right)w_b^*(r_2)H'(r_1, r_2)$$

$$\times\exp(ik_\alpha r_2)w_a(r_1)d^3r_1d^3r_2. \tag{4.14}$$

The initial and the final states of the dyonium are not orthogonal to each other. Due to this, a number of defects are incorporated into Born-Oppenheimer approximation. To overcome these difficulties, we use another approximation which is due to Ochkur. In this approximation, g is expanded in the inverse powers of k_α and only the leading term is retained. We write

$$g = g_{ne} + g_{ee}, \tag{4.15}$$

where g_{ee} is the contribution due to dyon-dyon interaction and g_{ne} is the contribution of another dyon of dyonium. We have

$$g_{ee} = -2m\frac{\mathrm{Re}(qq^*)}{4\pi\hbar^2}\iint\exp\left\{-ik_\beta.r_1\right\}w_b^*(r_2)\frac{1}{r_{12}}$$

$$\times\exp\left(-ik_\alpha.r_2\right)w_a(r_1)d^3r_1d^3r_2.$$

Now

$$\frac{1}{r_{12}} = \frac{1}{2\pi^2}\int\frac{\exp(iS(r_2-r_1))}{S^2}d^3S,$$

and hence,

$$g_{ee} = \frac{2m\,\mathrm{Re}(qq^*)}{8\pi^3\hbar^2}\iiint\frac{1}{S^2}\exp\{iS.(r_2-r_1)+ik_\alpha(r_2-r_1)$$

$$+ik_\alpha r_1 - ik_\beta r_1\}w_b^*(r_2)w_a(r_1)d^3r_1d^3r_2d^3S$$

$$= -\frac{1}{8\pi^3}\cdot\frac{2m\,\mathrm{Re}(qq^*)}{\hbar^2}\iiint\frac{1}{S^2}\exp\{i(S+k_\alpha).(r_2-r_1)+ik.r_1\}$$

$$\times w_b^*(r_2)w_a(r_1)d^3r_1d^3r_2d^3S. \tag{4.16}$$

Solving eq. (4.16), we get

$$g_{\rm (Ochkur)} = -\frac{2m\mathrm{Re}(qq^*)}{\hbar^2}\frac{1}{\nu^2}\int\exp(ik.r_2)$$

$$\times w_b^*(r_2)w_a(r_2)d^3r_2. \tag{4.17}$$

With the help of this equation from the direct scattering amplitude (3.13), we get

$$g_{(\text{Ochkur})} = \frac{k^2}{k_\alpha^2}\left(f - \frac{2m\,\text{Re}\,(qq^*)}{\hbar^2 k^2}\delta_{ba}\right). \tag{4.18}$$

For inelastic scattering $a \neq b$. Therefore, $\delta_{ab} = 0$ and hence we have

$$g_{(\text{Ochkur})} = \frac{k^2}{K_\alpha^2}\cdot f\,. \tag{4.19}$$

For elastic scattering, $a = b$ and hence $\delta_{ba} = 1$. Therefore,

$$g_{(\text{Ochkur})} = \frac{K^2}{K_\alpha^2}\left(f - 2m\frac{\text{Re}\,(qq^*)}{\hbar^2 k^2}\right).$$

Now for elastic scattering of dyon with a dyonium,

$$f = 2m\frac{\text{Re}\,(qq^*)}{k^2\hbar^2}\left[1 - \left(1 + \frac{1}{4}a_0^2 k^2\right)^{-2}\right]$$

$$= \frac{2a_0\left(8 + k^2 a_0^2\right)}{\left(4 + k^2 a_0^2\right)^2}. \tag{4.20}$$

Therefore,

$$g_{(\text{Ochkur})} = -\frac{32}{K_\alpha^2\left(4 + k^2 a_0^2\right)^2}. \tag{4.21}$$

Hence, the elastic differential scattering cross section for dyon-dyonium scattering in the exchange effect is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{tot}} = \frac{1}{4}\left|f + g_{\text{Ochkur}}\right|^2 + \frac{3}{4}\left|f - g_{\text{Ochkur}}\right|^2. \tag{4.22}$$

5. Discussion

Eq. (2.1) gives the generalized charge of dyon. Eq. (2.2) is the equation of motion of dyon in the GEM field of another dyon. Eq. (2.4) describes the angular momentum of dyon moving in the field of another dyon which is neither rotationally nor gauge invariant. Eq. (2.5) is the rotationally as well as gauge invariant angular momentum operator. Eq. (2.6) is the equation of linear momentum associated with a dyon moving in the generalized electromagnetic field of another dyon. Eq. (2.7) is the Hamiltonian for a dyon moving in the generalized electromagnetic field while eq. (2.8) is the interaction potential of the system. Eq. (2.12) is the chirality quantization condition for a system of dyons. Hamiltonian (3.1) describes the dynamics for scattering of a dyon by a dyonium which reduces to eq. (3.3) when identical dyons are considered. Eq. (3.7) is the wave function for the total Hamiltonian under Born approximation and scattering amplitude for this case is given by eq. (3.10). Eq. (3.11) describes the Hamiltonian for scattering of a dyon by dyonium and eq. (3.16) describes the scattering amplitude under Born approximation for elastic scattering. The scattering amplitude for inelastic scattering is given by eq. (3.17). Eqs. (4.8) and (4.11) give the differential scattering cross section including exchange effect *i.e.* where dyon of dyonium is replaced by the incident dyon in the scattering process. Total differential scattering cross section is given by eq. (4.12). Eq. (4.22) describes the total scattering cross section for scattering of a dyon by a dyonium with inclusion of exchange effect. All of these scattering cross sections are modified from the usual scattering cross section of scattering of a fermion from the atom due to the presence of magnetic charge on dyon. These scattering cross sections reduce to usual scattering cross sections of scattering of a fermion from an atom in the absence of magnetic charge

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Appendix

(a) Derivation of angular momentum operator for dyons [14]

In order to construct a suitable angular momentum operator for a dyon in generalized electromagnetic field, we consider *i*-th generalized charge $q_i\,(e_i - ig_i)$ in the field of another generalized charge $q_j\,(e_j - ig_j)$ which is assumed to be at rest. In the absence of any charge, the angular momentum vector of mass particle would be given by $\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{P}$, where $\boldsymbol{P}$ is the linear momentum of the particle. But in the presence of charges, such an angular momentum vector is unacceptable because the linear momentum is not gauge-invariant. The gauge-invariant angular momentum vector in the presence of particles carrying the generalized charges, may be written as [11]

$$\boldsymbol{J} = \boldsymbol{r} \times \left(\boldsymbol{P} - \mu_{ij}\,\boldsymbol{V}^T\right), \tag{A.1}$$

where

$$\mu_{ij} = \left(e_j g_i - e_i g_j\right) \tag{A.2}$$

is the magnetic coupling parameter for two generalized charges and $\boldsymbol{V}^1$ is the transverse part of the generalized four-potential of dyons. The angular momentum vector given by eq. (A.1) is not rotationally symmetric because it leads to the following commutation relation :

$$\left[\hat{J}_k, \hat{J}_l\right] = i\varepsilon_{klm}\left(\hat{J}_m - \mu_{ij}\hat{n}\right), \tag{A.3}$$

where ε_{klm} is the usual Levi-Civita Symbol and $\hat{n} = \boldsymbol{r}/|\boldsymbol{r}|$; but for the system endowed with rotational symmetry, we must have

$$\left[\hat{J}_k, \hat{J}_l\right] = i\varepsilon_{klm}\,\hat{J}_m\,. \tag{A.4}$$

Therefore, we can write the gauge-invariant and rotationally symmetric angular momentum for a dyon in the generalized field as [11]

$$\boldsymbol{J} = \boldsymbol{r} \times \left(\boldsymbol{P} - \mu_{ij}\boldsymbol{V}^T\right) + \mu_{ij}\frac{\boldsymbol{r}}{r}. \tag{A.5}$$

(b) Derivation of eq. (2) [14] :

The Hamiltonian of the dyon of mass m_1 carrying electric and magnetic charges e_i and g_i respectively, in the field of dyon of mass m_2 and charges e_j and g_j may be written as

$$\hat{H} = \frac{1}{2m_1}\left[\boldsymbol{P}_1 - \left(e_i g_j - g_i e_j\right)\boldsymbol{V}^T\right]^2 + \frac{1}{2m_2}\left[\boldsymbol{P}_2 - \left(e_j g_i - g_j e_i\right)\boldsymbol{V}^T\right]^2$$

$$\frac{\left(e_i g_j - g_i e_j\right)}{\boldsymbol{r}_1 - \boldsymbol{r}_2} \quad V\left(|\boldsymbol{r}| - |\boldsymbol{r}_2|\right), \tag{A.6}$$

where $V(r)$ is an arbitrary additional potential interaction. The justification for this Hamiltonian is that for the Heisenberg equation of motion for $\boldsymbol{r}_1$ and $\hat{\pi}_i = \left[\boldsymbol{P}_i - \left(e_i g_j - g_i e_j\right)\boldsymbol{V}^T\right]$ (if i,j = 1,2) it yields the Newtonian equation of motion

$$\dot{\boldsymbol{r}} = \frac{T_i}{m_i} \tag{A.7}$$

Upon introducing the usual relative and center-of-mass coordinates

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2\,, \qquad \boldsymbol{R} \;\; \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_1 + m_2}$$

$$\boldsymbol{p} = \frac{m_1\boldsymbol{P}_1 + m_2\boldsymbol{P}_2}{m_1 + m_2} \qquad \boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2\,, \tag{A.8}$$

one obtains

$$\hat{H} = \frac{1}{2m}\left[\hat{p}^2\right] + \frac{1}{2M}\left[\boldsymbol{P} - \mu_{ij}\boldsymbol{V}^T\right]^2 + \frac{\alpha_{12}}{r^2} + V(r), \tag{A.9}$$

where r is the separation between two dyons, $M = m_1 + m_2$ and $m = \left(m_1^{-1} + m_2^{-1}\right)^{-1}$ are the total and reduced mass respectively, and

$$\alpha_{12} = -\left(e_1 e_2 + g_1 g_2\right) = -\operatorname{Re}\left(q_1\, q_2^*\right),$$

$$\mu_{12} = -\left(e_1 g_2 - g_1 e_2\right) = \operatorname{Im}\left(q_1\, q_2^*\right). \tag{A.10}$$

From now onwards, we set the total momentum $\boldsymbol{P}$ equal to zero and express the Hamiltonian in eq. (A.6) in the following manifestly gauge invariant and rotationally symmetric form

$$\boldsymbol{H} = \frac{\pi^2}{2m} + \frac{\alpha_{12}}{r} + V(r), \tag{A.11}$$

where

$$\pi = \left[\boldsymbol{P} - \mu_{12}\boldsymbol{V}^T\right].$$

It is quite reasonable to expect the additional potential term $V(r)$ with its form and magnitude to be described by the symmetry requirement of the system. Using eq. (A.5), the value of $\boldsymbol{J}^2$ may be calculated in the following manner :

$$\boldsymbol{J}^2 = \boldsymbol{J}.\boldsymbol{J} = \left[(\boldsymbol{r} \times \hat{\pi}) + \mu_{ij}\hat{r}\right].\left[(\boldsymbol{r} \times \hat{\pi}) + \mu_{ij}\hat{r}\right] \tag{A.12a}$$

$$= (\boldsymbol{r} \times \hat{\pi}).(\boldsymbol{r} \times \hat{\pi}) + \mu_{ij}^{\,2} + 2\mu_{ij}\hat{r}.(\boldsymbol{r} \times \hat{\pi}). \tag{A.12b}$$

The third term in eq. (A.12b) is zero because of the reason $\hat{r}.(\boldsymbol{r} \times \hat{\pi}) = (\boldsymbol{r} \times \boldsymbol{r}).\hat{\pi}$ and hence we may write

$$\frac{\pi^2}{2m} \quad \frac{}{2mr^2} \quad \frac{\mu_{12}^2}{2mr^2} + \frac{P_r^2}{2m} \tag{A.13}$$

It is quite obvious that the Hamiltonian given by eq. (A.11) possesses the same higher symmetry as the pure Coulomb Hamiltonian, provided the additional potential $V(r)$ in eq. (A.13) takes the following scalar form

$$V(r) = \frac{\mu_{12}^2}{2Mr^2}. \tag{A.14}$$

Thus, the Hamiltonian given in eq. (A.11) may be written as

$$\hat{H} = \frac{\hat{\pi}^2}{2m} - \frac{\alpha_{12}}{r} + \frac{\mu_{12}^2}{2mr^2} \tag{A.15}$$

which is the required Hamiltonian.