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# Fusion cross section for some reactions using different forms of M3Y force

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**Abstract** : Fusion cross sections are calculated within the double-folding model using different forms of M3Y force for the nuclear pairs $^{40}Ca + {}^{40}Ca$, $^{64}Ni + {}^{64}Ni$, $^{16}O + {}^{28}Si$, $^{165}Ho + {}^{14}N$ and $^{30}Si + {}^{170}Er$. The results are comparable with that of Skyrme SKIII force predictions and with the experimental data.

**Keywords** : Fusion cross sections, double folding model, M3Y force

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## 1. Introduction

Heavy ion fusion reaction have been the subject of experimental and theoretical investigations but a clear understanding is far from being established. In a previous work [1] an effective M3Y nucleon-nucleon interaction is used to calculate the real part of a nucleus-nucleus potential using a double folding model. The results agree with the experimental data when $\left(A_1^{1/3} + A_2^{1/3}\right) < 7$. Lilley *et al* [2] reported that the double folding model with M3Y force leads to a failure for some reactions despite its success in predicting the interaction potential between two ions. Different attempts have been made to improve this force by including explicit energy and density dependence [3,4].

To check the validity of such modifications, we compared the calculated fusion excitation function of several pairs of nuclei with the experimental data at several bombarding energies [5]. The results are also compared with the predictions of Woods-Saxon potential and the proximity potential.

It was found that the DDM3Y force is suitable for describing the two nucleon interaction in comparison with the Woods-Saxon and the proximity potential. Also, the smooth cut-off approximation is better in some cases than the sharp cut-off approximation.

In the above calculations, the exchange part of the potential is of zero-range. Several modifications to the M3Y force were made using finite range effects in the exchange part [6,7].

In the present work, calculations are made for the fusion-cross sections using different versions of M3Y force with zero range and finite range exchange potentials. A comparison was also made with the predictions of the famous Skyrme (SKIII) force.

In Section 2, we give a brief description of the formalism used. In Section 3 we present the numerical results and discussion.

## 2. The formalism

The nucleus-nucleus potential splits into two parts

$$V(R,E)=\sum_{\substack{i\in P\\ j\in T}}\langle ij|v_d|ij\rangle+\sum_{\substack{j\in P\\ j\in T}}\langle ij|v_{ex}|ji\rangle, \tag{1}$$

$|i\rangle$ and $|j\rangle$ refer to the single particle wave functions of the nucleons in the two nuclei, respectively, $v_d$ is the direct and $v_{ex}$ the exchange potential of the *NN* interaction. A commonly used procedure in calculating the exchange part is the choice of a $\delta$-function for the interaction $v_{ex}$ whereas for the direct term

$$v_d = 7999\,\frac{\exp(-4S)}{4S} - 2134\,\frac{\exp(-2.5S)}{2.5S}. \tag{2}$$

This is the normal M3Y force with zero range exchange potential [8]

$$v_d + v_{ex} = 7999\,\frac{\exp(-4S)}{4S} - 2134\,\frac{\exp(-2.5S)}{2.5S} - 262.2\,\delta(S). \tag{3}$$

For a finite range exchange force [9] $v_{ex}$ takes the form

$$v_{ex} = 4631.38\,\frac{\exp(-4S)}{4S} - 1787.13\,\frac{\exp(-2.5S)}{2.5S} - 7.8474\,\frac{\exp(-0.7072S)}{0.7072S}. \tag{4}$$

For the direct part $v_d$, the long range OPEP is exactly zero, whereas for the exchange part $v_{ex}$, the OPEP component has a finite contribution as shown in eq. (4) [6].

In our analysis, we have also used a third and a much more recent version of the M3Y force derived by the Paris group [10] which has the following form

$$v_d = 11061.6\,\frac{\exp(-4S)}{4S} - 2537.5\,\frac{\exp(-2.5S)}{2.5S}, \tag{5}$$

$$v_{ex} = -1524\,\frac{\exp(-4S)}{4S} - 518.8\,\frac{\exp(-2.5S)}{2.5S} - 7.8474\,\frac{\exp(-0.7072S)}{0.7072S}. \tag{6}$$

It is clear that for the Paris potential the exchange component is more attractive than the first version, the OPEP part is the same for both cases.

By introducing the one-body density matrices $\rho_1(2)(r, r')$ of the two colliding nuclei with the diagonal terms giving the matter densities $\rho(r, r') \equiv \rho(r)$, one can write the direct and exchange nucleus-nucleus potentials in the following forms

$$V_D = \int \rho_1(r_1)\rho_2(r_2)v_D(S)dr_1dr_2, \qquad S = r_2 - r_1 + R, \tag{7}$$

where $\rho_1(r_1)$ is the Fermi-distribution of the nuclear density of the projectile and $\rho_2(r_2)$ is the same for the target,

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/a}}, \tag{8}$$

where $\rho_0$, $R_0$ and $a$ are the parameters of the Woods-Saxon distribution, $v_D(S = r_2 - r_1 + R)$ is the effective $NN$ interaction and $R$ is the separation of center of mass of the colliding nuclei.

$$V_{EX} = \int \rho_1(r_1, r_1 + S)\rho_2(r_2, r_2 - S)v_{EX}(S)\exp\left[\frac{ik(R)S}{M}\right]dr_1dr_2. \tag{9}$$

Here, $k(R)$ is the relative motion momentum given by

$$k^2(R) = \frac{2mM}{K^2}\left[E_{c.m.} - V(R) - V_C(R)\right], \tag{10}$$

where $M = A_1A_2/(A_1 + A_2)$ is the reduced mass, $E_{c.m.}$ is the center-of-mass energy in nucleus-nucleus frame, $V = V_D + V_{EX}$ and $V_C(R)$ are the total nuclear and Coulomb potential respectively.

The direct potential and the Coulomb potential are calculated using the same method as developed by Greiner *et al* [11] and as in Ref. [5] where the six-dimensional integral is reduced to only three-dimensional integral.

After some transformations the exchange integral takes the following form [6]

$$V_{EX} = 4\pi\int_0^\infty v_{EX}(s)s^2ds j_0\left(\frac{k(R)s}{M}\right)f_1(r, s)f_2(r - R, s)dr, \tag{11}$$

where $f_{1(2)}(s) = \rho_{1(2)}(s)\ \hat{j}_1\ (k_{F1(2)}(r)s)$, and the spherical Bessel functions are

$$j_0(X) = \sin X/X$$

$$\hat{j}_1(X) = 3j_1(X)/X = 3(\sin X/X^3 - \cos X/X^2),$$

and

$$k_F(r) = \left[\left(\frac{3}{2}\pi^2\rho(r)\right)^{2/3} + \frac{5C_s|\nabla\rho(r)|^2}{3\rho^2(r)} + \frac{5\nabla^2\rho(r)}{36\rho(r)}\right]^{1/2} \tag{12}$$

$C_s$ is taken here to be $\approx 1/4$.

Since the exchange potential contains the relative motion momentum $k(R)$ which in turn depends on the total HI potential, one has to solve a self-consisting problem to obtain the exchange part of the HI potential at each radial point.

In the present work, we have chosen the iterative method suggested by Chaudhuri *et al* [6] to ensure self consistency at all radial points, using $V_D$ as the starting potential to enter $j_0$ $(k(R)\ S/M)$ in the exchange integral. From the above equations it is clear that the exchange potential takes into account the density dependence of the potential through $k_{F1}$ and $k_{F2}$ in eq. (12).

Finally, on adding the Coulomb potential $V_C$ $(R)$ to the nuclear potential $V_D$ $(E, R)$ + $V_{EX}$ $(E, R)$ we get $V(R)$ which gives the height $V_B$ and position $R_B$ of the interaction barrier for each reaction. From these information we can calculate the fusion cross section above the barrier $E_{c.m.} > V_B$ which is described by the sharp cut-off model expression [5,12]

$$\sigma_{\rm fus} = \pi R_B^2\left[1 - \frac{V_B}{E_{\rm c.m.}}\right], \tag{13}$$

for the reactions considered using three different interactions of M3Y type.

## 3. Numerical results and discussion

In this section, we present our calculations of fusion barrier heights, positions and fusion cross sections for the following pairs $^{40}$Ca + $^{40}$Ca, $^{64}$Ni + $^{64}$Ni, $^{16}$O + $^{28}$Si, $^{165}$Ho + $^{14}$N and $^{30}$Si + $^{170}$Er. Our calculations are compared with the empirical data and other model calculations using Skyrme SKIII force [13]. Table 1 gives this comparison for M3Y force

**Table 1.** Comparison between the values of $R_B$ *(fm)* and $V_B$ (MeV) in our work with other theoretical predictions.

| System | Present work I | | Present work II | | Present work III | | IV Ref. [13] | | Exp. | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ | $R_B$ | $V_B$ |
| $^{40}$Ca + $^{40}$Ca | 9.55 | 56.11 | 9.84 | 53.73 | 10.29 | 53.38 | 9.68 | 54.26 | 9.5 ± 0.5 [20] | 50.628 [20] |
| $^{64}$Ni + $^{64}$Ni | 10.03 | 98.79 | 11.71 | 94.00 | 11.67 | 93.61 | 10.77 | 95.69 | 8.6 [16] | 93.5 [16] |
| $^{16}$O + $^{28}$Si | 9.01 | 16.87 | 9.06 | 16.12 | 9.30 | 16.00 | 8.55 | 17.25 | 7.98 [17] | 17.23 [17] |
| $^{14}$N + $^{165}$Ho | 11.55 | 57.08 | 11.24 | 57.19 | 11.37 | 59.81 | — | — | — | — |
| $^{30}$Si + $^{170}$Er | 11.44 | 112.83 | 12.24 | 107.05 | 12.62 | 106.58 | — | — | — | — |

with zero range exchange interaction and two types of M3Y force with finite range exchange interaction in addition to the results of SKIII force and the empirical data. The barrier heights increase with increasing $Z_1Z_2$ values. This is because for larger $Z_1Z_2$ values, more nuclear interactions are needed to balance the Coulomb repulsion. Computer programmes have been established and checked to calculate the direct and exchange M3Y potentials and the Coulomb potential as explained in the previous section.

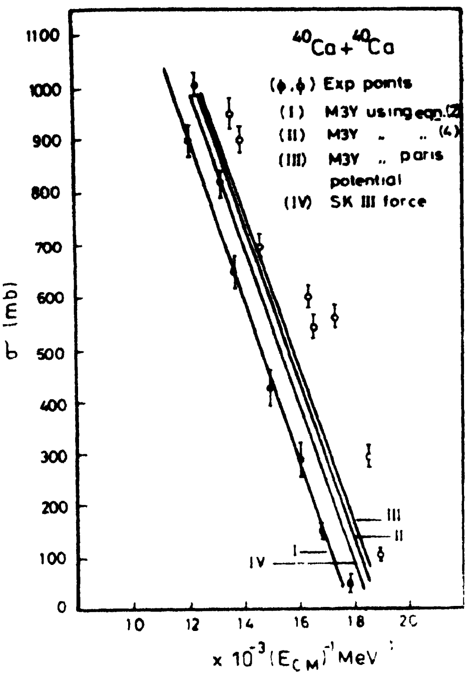


**Figure 1.** Fusion cross section for $^{40}$Ca + $^{40}$Ca pair
ɸ, ɸ experimental points Refs [14,15].
(I) M3Y with zero range exchange potential.
(II) M3Y using eq (4) for the exchange part
(III) M3Y using Paris potential.
(IV) SKIII using Skyrme force, Ref. [13].

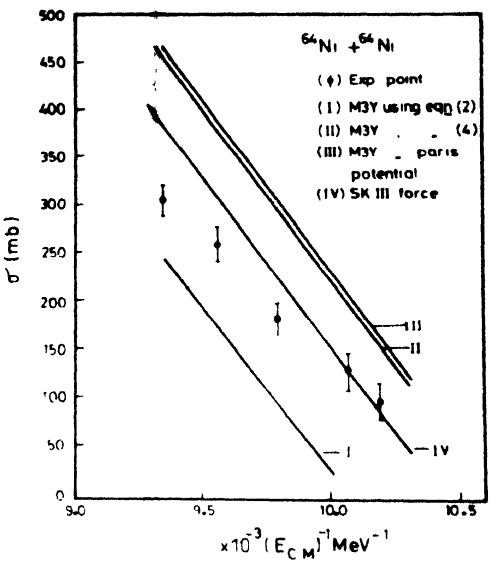


**Figure 2.** Fusion cross section for $^{64}$Ni + $^{64}$Ni pair
ɸ experimental points, Ref [16]
(I) M3Y with zero range exchange potential.
(II) M3Y using eq (4) for the exchange part.
(III) M3Y using Paris potential
(IV) SKIII using Skyrme force, Ref. [13]

The fusion cross section of all reactions studied are plotted *vs* $(E_{c\,m})^{-1}$ and are shown in Figures (1–5) along with the previous SKIII calculations [13] and the experimental data which are taken from Refs. [14–19]. The results are good in comparison with the experimental data as well as with SKIII force calculations using EDF. The different forms of M3Y force which are used here are energy and density independent. The dependence on density here is implicit. Explicit energy and density dependences may be introduced in the N–N M3Y force mentioned in the text. For such potentials a smooth cut-off approximation may be appropriate for calculating $R_B$ and $V_B$. Such work is in progress and will be reported elsewhere.

For $^{40}$Ca + $^{40}$Ca our calculations fit the experimental data giving their average behaviour using M3Y potential with zero range exchange force or M3Y with finite range exchange force. One can also notice that for $^{64}$Ni + $^{64}$Ni and $^{30}$Si + $^{170}$Er there is no great difference for the quality of fit between the different forces used in the present work. On the

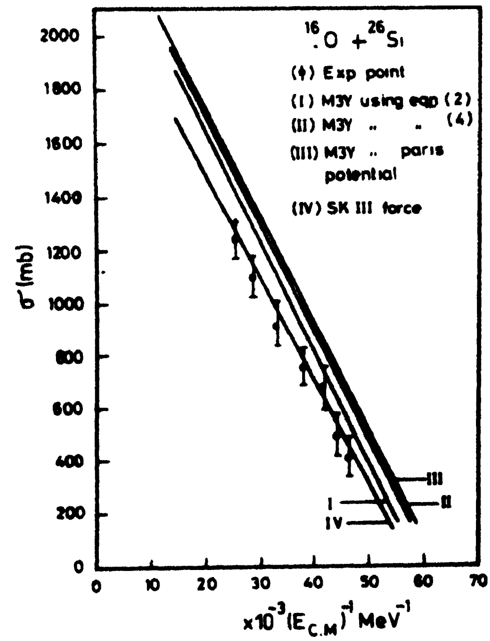


**Figure 3.** Same as Figure (2) but for $^{16}O + {}^{28}Si$
ɸ experimental points, Ref. [17].

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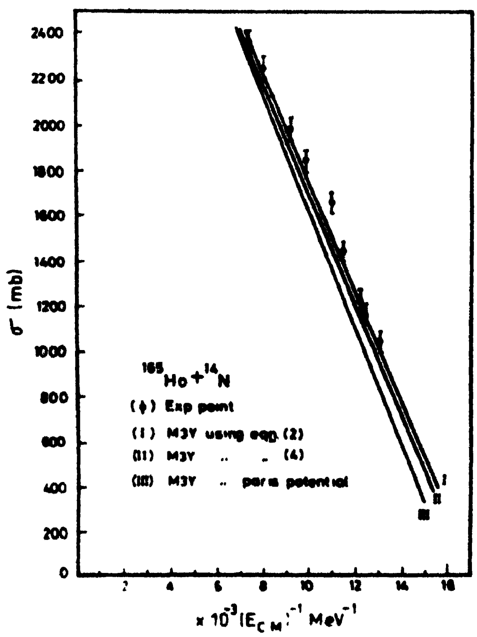


**Figure 4.** Fusion cross section for $^{14}N + {}^{165}Ho$.
ɸ experimental points, Ref. [18].
(I) M3Y with zero range exchange potential.
(II) M3Y using eq. (4) for the exchange part.
(III) M3Y using Paris potential.

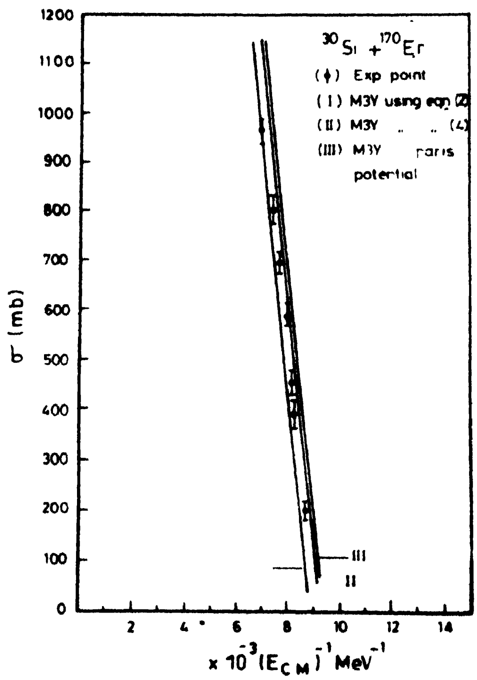


**Figure 5.** Same as Figure (4) but for $^{30}Si + {}^{170}Er$.
ɸ experimental points, Ref. [19].

contrary for $^{165}$Ho + $^{14}$N and $^{16}$O + $^{28}$Si we find that M3Y force with zero range exchange force gives a better fit than M3Y force with finite range exchange force. We conclude that the fusion excitation function for the pairs of nuclei considered here using M3Y forces are comparable with the experimental data as well as with the theoretical perdictions of Skyrme SKIII force.

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