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Particle aspect analysis of Alfvén wave

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Abstract The particle aspect analysis has been applied to evaluate the dispersion relation and field aligned current in the presence of Alfvén wave. The applicability of theory for space plasmas has been indicated

Keywords Alfvén wave, magnetosphere, plasma sheet

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1. Introduction

The Alfvén wave is the dominant low frequency transverse mode of a magnetized plasma. The Alfvén wave propagates along the magnetic field and displays a continuous spectrum even in a bounded plasma. This is essentially due to the degeneracy of the wave characteristics, *i.e.*, the frequency (ω) is primarily determined by the wave number in the direction parallel to the ambient magnetic field ($K_{\parallel}$) and is independent of the perpendicular wave numbers. The direction along which the wave energy propagates, is identical to the ambient magnetic field lines. Therefore, the spectral structure of the Alfvén wave has a close relationship with the geometric structure of the magnetic field lines. In an inhomogeneous plasma, the Alfvén resonance ($\omega - C_A K_{\parallel} = 0$, C_A is the phase velocity of the Alfvén wave) constitutes a singularly for the defining wave equation, this results in a singular eigenfunction corresponding to the continuous spectrum.

It is of utmost interest to note that, in its simplest magnetohydrodynamic (MHD) description, the Alfvén wave has a continuous spectrum even in a spatially bounded plasma [1-3]. The interaction of ions with Alfvén waves in the solar wind has been extensively studied in space physics literature [4]. For wave observed on auroral field lines, ion trapping may be important and affects the existing models of transverse ion acceleration. The anomalous transport effects associated with the ion cyclotron turbulence can cause dissipation of the Alfvén wave, leading to an energization of the plasma and the establishment of a double layer on auroral field lines.

The present analysis of Alfvén wave is based on a physical model similar to that used by Dawson [5] in his theory of Landau damping which was further extended by Terashima [6], Mishra and Tiwari [7-9], Tiwari *et al* [10], and Varma and Tiwari [11-13] to the analysis of electrostatic and electromagnetic instabilities. In the present investigation, this well known approach has been applied for the evaluation of charged particle trajectory in the electromagnetic field of the Alfvén wave and used to evaluate the dispersion relation to explain the current drive for auroral acceleration region. The whole plasma is considered to consist of resonant and non-resonant particles. Non-resonant particles support the oscillatory motion of the waves while the resonant particles participate in the energy exchange with the wave. The Alfvén wave is assumed to start at $t = 0$ when the resonant particles are undisturbed. The trajectories of the particles are then evaluated within the framework of the linear theory. Using these particle trajectories in the presence of a Alfvén wave, the field aligned current and dispersion relation of Alfvén wave are evaluated with the help of Maxwell's equations.

2. Basic Trajectories

We consider a plasma under static magnetic field B_0, in which collisions between particles are neglected. We shall discuss the behavior of an Alfvén wave of plane polarization in the form

$$\boldsymbol{K} \parallel B_0,\ \boldsymbol{K}\cdot\boldsymbol{B} = 0 \tag{1}$$

$$\boldsymbol{K} = (0, 0, K_\parallel) \text{ and } \boldsymbol{E} = (E_x, 0, 0)$$

with
$$E_x = E_1 \cos(K_\parallel z - \omega t) \tag{2}$$

and
$$B_y = \frac{E_1 K_\parallel c}{\omega} \cos(K_\parallel z - \omega t), \tag{3}$$

where E_x and B_y are the electric and magnetic fields of the wave. Here, the frequency ω is assumed to be real and the amplitude E_1 is treated as the slowly varying function of t. c is the velocity of light.

$$\frac{1}{E_1}\frac{dE_1}{dt} << \omega .$$

B_0 is directed along the z-axis and wave propagates in the direction of ambient to the magnetic field. The wave is assumed to start at $t = 0$ when resonant particles are not yet disturbed. We begin with the equation of motion for the particles as

$$m\frac{dv}{dt} = q\left[E_1 + \frac{1}{c}v\times\left(B_0 + B_y\,\hat{y}\right)\right], \tag{4}$$

where q is the charge and m is the mass of the particle. $\hat{y}$ is the unit vector along the y direction.

The Gaussian system of units is adopted in this paper and interactions between particles are neglected. The electric field E_1 on the right hand side is considered to be a small perturbation and $\boldsymbol{v}$ can be expressed as a sum of the unperturbed velocity $\boldsymbol{V}$ and the perturbation velocity $\boldsymbol{u}$, *i.e.*, $\boldsymbol{v} = \boldsymbol{V} + \boldsymbol{u}$, $\boldsymbol{u}$ is determined by the following set of equation for $\boldsymbol{u}$.

$$\frac{du_+}{dt} - i\Omega u_+ = \frac{q}{m}\left[1 - \frac{V_\parallel K_\parallel}{\omega}\right] E_1 \cos(K_\parallel z - \omega t),$$

$$\frac{du_{\parallel}}{dt} = \frac{q}{m} V_{\perp} \frac{E_1 K_{\parallel}}{\omega} \cos(\theta - \Omega t) \cos(K_{\parallel} z - \omega t), \tag{5}$$

where $u_+ = u_x + iu_y$ and $\Omega = \dfrac{qB_0}{mc}$

We solve eq. (5) in the approximation by replacing the coordinates of the particles on the right hand side by those of a free gyration, u_x and u_y are the perturbed velocities in the x and y directions respectively, E_1 and B_x are slowly varying quantities and treated as constants. Our method follows that of Terashima [6]. We start taking the trajectories of free gyration as

$$\begin{aligned} z &= z_0 + V_{\parallel} t, \\ V_x(t) &= V_{\perp} \cos(\theta - \Omega t), \\ V_y(t) &= V_{\perp} \sin(\theta - \Omega t), \\ V_z(t) &= V_{\parallel} = \text{Constant}, \end{aligned} \tag{6}$$

where z_0 is the initial position of the particles in the z direction, θ is the initial phase of gyration.

To find an oscillatory solution of $\boldsymbol{u}$ (t) for the resonant particles, it is necessary to take into account the initial condition of $\boldsymbol{u}$ $(t - 0)$ inferred from the assumption stated above.

$$u_x(t) = \frac{q}{m}\left[1 - \frac{V_{\parallel} K_{\parallel}}{\omega}\right] E_1 \left[\frac{\Lambda_0}{a_0^2} \sin(\Lambda_0 t + K_{\parallel} z_0) - \frac{\delta}{2(\Lambda_0 - \Omega)} \sin(K_{\parallel} z_0 + \Omega t) - \frac{\delta}{2(\Lambda_0 + \Omega)} \sin(K_{\parallel} z_0 - \Omega t)\right],$$

$$u_y(t) = \frac{q}{m}\left[1 - \frac{V_{\parallel} K_{\parallel}}{\omega}\right] E_1 \left[\frac{\Lambda_0}{a_0^2} \cos(\Lambda_0 t + K_{\parallel} z_0) - \frac{\delta}{2(\Lambda_0 - \Omega)} \cos(K_{\parallel} z_0 + \Omega t) - \frac{\delta}{2(\Lambda_0 + \Omega)} \cos(K_{\parallel} z_0 - \Omega t)\right],$$

$$u_z(t) = \frac{q}{m} \frac{V_{\perp} E_1 K_{\parallel}}{\omega} \frac{1}{2}\left[-\frac{1}{(\Lambda_0 - \Omega)} \sin(\Lambda_0 t + K_{\parallel} z_0 + \theta - \Omega t) + \frac{1}{(\Lambda_0 + \Omega)} \sin(\Lambda_0 t + K_{\parallel} z_0 - \theta + \Omega t) - \frac{\delta}{(\Lambda_0 - \Omega)} \sin(K_{\parallel} z_0 + \theta) - \frac{\delta}{(\Lambda_0 + \Omega)} \sin(K_{\parallel} z_0 - \theta)\right]. \tag{7}$$

where $\delta = 0$ for the non-resonant particle and $\delta = 1$ for resonant one and

$$\Lambda_0 = V_\| K_\| - \omega\,,\quad a_0^2 = \Lambda_0^2 - \Omega^2\,. \tag{8}$$

It is easy to calculate the true trajectory of the particle to first order by first integrating $\boldsymbol{u}(t)$ and then adding (6) to it. It follows that $\boldsymbol{u}(t)$ with $\delta = 1$ increases linearly if $V_\| K_\| - \omega = 0$. Therefore, the resonant particle condition is given as $V_\| K_\| - \omega = 0$. If we again substitute (6) in (7), then $\boldsymbol{u}(t)$ can be written in the form $\boldsymbol{u}(\boldsymbol{r}, t)$ as

$$u_x(\boldsymbol{r},t) = \frac{q}{m}\left|1 - \frac{V_\| K_\|}{\omega} \qquad \frac{\Lambda_0}{a_0^2}\sin(K_\| z - \omega t)\right.$$

$$\frac{\delta}{2(\Lambda_0 - \Omega)}\sin(K_\| z - \omega t - \Lambda_0 t + \Omega t) - \frac{\delta}{2(\Lambda_0 + \Omega)}\sin(K_\| z - \omega t - \Lambda_0 t - \Omega t)$$

$$u_y(\boldsymbol{r},t) = \frac{q}{m}\left[1 - \frac{V_\| K_\|}{\omega}\right] E_1 \quad \frac{\Omega}{a_0^2}\cos(K_\| z - \omega t)$$

$$-\frac{\delta}{2(\Lambda_0 - \Omega)}\cos(K_\| z - \omega t - \Lambda_0 t + \Omega t) + \frac{\delta}{2(\Lambda_0 + \Omega)}\cos(K_\| z - \omega t - \Lambda_0 t - \Omega t)$$

$$u_z(\boldsymbol{r},t) = \frac{q}{m}\frac{V_\perp E_1 K_\|}{\omega}\frac{1}{2}\left[\frac{1}{(\Lambda_0 - \Omega)}\sin(K_\| z - \omega t + \theta - \Omega t)\right.$$

$$(\Lambda_0 + \Omega)\,\sin(K_\| z - \omega t - \theta + \Omega t) - \quad (\Lambda_0 - \Omega)\,\sin(K_\| z - \omega t - \Lambda_0 t + \theta)$$

$$(\Lambda_0 + \Omega)\,\sin(K_\| z - \omega t - \Lambda_0 t - \theta) \tag{9}$$

These equations represent the trajectories of charged particle in the presence of Alfvén waves. The trajectories has vast application in plasma heating processes, confinement devices and the space plasmas.

3. Density perturbation

To find the density perturbation associated with the velocity perturbation, we consider a group of the particles with the same initial condition and let its number density be

$$n(\boldsymbol{r}, t, \boldsymbol{V}) = N(\boldsymbol{V}) + n_1(\boldsymbol{r}, t, \boldsymbol{V}). \tag{10}$$

Where N is the zeroth order distribution, n_1 is perturbed density which can be derived from the equation of continuity.

The equation of continuity in the presence of Alfvén wave can be written as

$$\frac{\delta n_1}{\delta t} = -N(V)\,\nabla_z u_z \tag{11}$$

The expression on the right hand side can be expressed as function of variable t, for instance, $\nabla_z u_z$ can be written down in terms of "t" and the initial parameters with the help of equation (6). After differentiating (9) with respect to "r" and substituting in eq. (11), followed by integration we obtain the following solution for the non-resonant particle density.

$$n_1(r,t) = \frac{q}{m}\frac{V_\perp E_1 K_\parallel^2}{\omega}\frac{N(V)}{2}\left[\frac{1}{(\Lambda_0-\Omega)^2}\sin(K_\parallel z-\omega t+\theta-\Omega t)\right.$$

$$\left.+\frac{1}{(\Lambda_0+\Omega)^2}\sin(K_\parallel z-\omega t-\theta+\Omega t)\right]. \tag{12}$$

Similarly, for the resonant particles

$$n_1(r,t) = -\frac{q}{m}\frac{V_\perp E_1 K_\parallel^2}{\omega}\frac{N(V)}{2}\left[\frac{1}{(\Lambda_0-\Omega)^2}\sin(K_\parallel z-\omega t+\theta-\Omega t)\right.$$

$$+\frac{1}{(\Lambda_0+\Omega)^2}\sin(K_\parallel z-\omega t-\theta+\Omega t) - \frac{1}{(\Lambda_0-\Omega)^2}\sin(K_\parallel z-\omega t-\Lambda_0 t+\theta)$$

$$-\frac{1}{(\Lambda_0+\Omega)^2}\sin(K_\parallel z-\omega t-\Lambda_0 t-\theta) - \frac{t}{(\Lambda_0-\Omega)}\cos(K_\parallel z-\omega t-\Lambda_0 t+\theta)$$

$$\left.-\frac{t}{(\Lambda_0+\Omega)}\cos(K_\parallel z-\omega t-\Lambda_0 t-\theta)\right]. \tag{13}$$

To evaluate the dispersion relation and current we can hereafter take the zeroth order distribution $N(V)$ of the form

$$N(V) = N_0 f_\perp(V_\perp)\, f_\parallel(V_\parallel), \tag{14}$$

$$f_\perp(V_\perp) = \left[\frac{m}{2\pi T_\perp}\right]\exp\left\{-mV_\perp^2/2T_\perp\right\},$$

$$f_\parallel(V_\parallel) = \left[\frac{m}{2\pi T_\parallel}\right]^{1/2}\exp\left\{-mV_\parallel^2/2T_\parallel\right\},$$

where $T_\parallel$ and $T_\perp$ are the parallel and perpendicular temperatures with respect to ambient magnetic field and N_0 is the plasma density.

4. Current density

To evaluate the perturbed current density we use the following set of equations

$$J = \int_0^{\lambda} ds \int_0^{2\pi} d\theta \int_0^{\infty} V_\perp \, dV_\perp \int_{-\infty}^{+\infty} dV_\parallel \, e\left[(N+n_1)(V+u) - NV\right] \tag{15}$$

and $J = J_i - J_e$.

With the help of equations (9), (12) and (15), we obtain the ionic current as

$$J_{zi} = -\frac{3}{4} \frac{e\,\omega_{pi}^2 \, E_1{}^2 \pi \, K_\parallel{}^2 V_{T\perp i}^2}{m_i \omega \Omega_i^4} . \tag{16}$$

Similarly, the electron current can be written as

$$J_{ze} = +\frac{3}{4} \frac{e\,\omega_{pe}^2 \, E_1{}^2 \pi \, K_\parallel{}^2 V_{T\perp e}^2}{m_e \omega \Omega_e^4} . \tag{17}$$

Substituting eq. (16) and (17) into eq. (15) we get

$$J_z = -\frac{3\pi}{4} \left(\frac{\cdot}{m_e} \right) \frac{K_\parallel^2}{\omega} \left[\frac{\omega_{pe}^2 V_{T\perp e}^2}{\Omega_e^4} + \frac{\omega_{pi}^2 V_{T\perp i}^2}{\Omega_i^4 (m_i / m_e)} \right] \tag{18}$$

and the average value of perpendicular current (*i.e.*, J_x and J_y) becomes zero. Here $q = +e$ for ions and $-e$ for electrons $\omega_{pi,e} = (4\pi N_0 e^2 / m_{i,e})^{1/2}$ is the plasma frequency. $V_{T\perp i,e} = (2T_{\perp i,e} / m_{i,e})^{1/2}$ is the perpendicular component of thermal velocity, $\Omega_{i,e}$ is the cyclotron frequency and Ks = K.r

5. Dispersion relation

To evaluate the dispersion relation, we use the Maxwell's equations as

$$\nabla(\nabla \cdot E_1) - \nabla^2 E_1 = -\frac{4\pi}{c^2} \frac{\partial J_1}{\partial t} - \frac{1}{c^2} \frac{\partial^2 E_1}{\partial t^2} , \tag{19}$$

where J_1 is the first order current density.

In the case of plane polarized Alfvén wave, the Maxwell's equation becomes

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{4\pi}{c^2} \cdot \frac{\partial J_x}{\partial t} + \frac{1}{c^2} \cdot \frac{\partial^2 E_x}{\partial t^2} . \tag{20}$$

Substituting the value of J_x in terms of perturbed velocity u_x, the dispersion relation for the Alfvén wave is evaluated as

$$\frac{\omega^2}{K_\parallel^2} = \left(V_A^2 - V_{Ti}^2\right)\left(1 - \frac{V_A^2}{C^2}\right)^{-1} ,$$

$$\frac{\omega^2}{K_{\parallel}^2} = \left(V_A^2 - V_{T\parallel}^2\right)\left(1 + \frac{V_A^2}{C^2}\right) \tag{21}$$

which reduces to well known expression of dispersion relation $\omega = K_{\parallel} V_A$ for Alfvén waves when $V_{T\parallel} = 0$ and $V_A << C$. The thermal correction due to term $V_{T\parallel}$ is the finding of our particle aspect analysis theory. Here $V_A = B_0 / (4\pi N_0 m_i)^{1/2}$ is the Alfvén velocity and $V_{T\parallel} = (2T_{\parallel e}/m_e)^{1/2}$ is the thermal velocity of electrons parallel to ambient magnetic field.

Following the particle aspect approach and considering an anisotropic plasma, we have obtained the total current density carried by both electrons and ions, and evaluated the dispersion relation. In this model the wave frequency ω has been considered real and the principal part of plasma dispersion function is used. However, considering imaginary parts of either asymptotic series or power series of plasma dispersion function, the dispersion relation would have been complex resulting the growth/damping of the wave. The present model considers the real part only to evaluate the charged particle trajectories and other parameters and the growth/damping rates are estimated by energy exchange method [10, 13] which is not the aim of the present paper.

In this paper, we have not considered the coupling of compressional mode to the Alfvén wave. The pressure variation along the direction of wave propagation has not been considered. The Alfvén waves propagate along the direction of magnetic field by tilting the magnetic lines of force similar to the sound waves in a stretched string. However, in the case of perpendicular propagation, the magnetic field is compressed and rarefied resulting the pressure variation and hence coupling with the ion acoustic waves. Thus, in case of $B = 0$, the dispersion relation reduces to imaginary ω, a non-propagating mode.

6. Results and discussion

We have evaluated the dispersion relation and current density using the following parameters for auroral acceleration region [14]

$$B_0 = 4300\,nT,\ \Omega_i = 412\,s^{-1},\ \omega_{pi}^2 / \Omega_i^2 = 100,$$

$$E_\perp = 50\,\text{mV/m},\ V_{T\perp i} = 3.5\times10^4\ \text{m/sec}.$$

Expressions (18) and (21) have been evaluated numerically and results are presented in Figures 1 and 2. Although many theoretical attempts have been made to apply the Alfvén wave propagation to magnetospheric plasmas, most of them lack of a careful check as to the applicability of the theory. The Alfvén waves have vast application to heat a Tokamak-type plasma to the thermonuclear temperatures.

Figure 1 shows the variation of real frequency ω with $K_{\parallel}$ for different values of $V_{T\parallel}$. Here we notice that wave frequency increases linearly with $K_{\parallel}$ but decreases with the increase of $V_{T\parallel}$. It is also clear that the slope of curve will give the phase velocity of wave and phase velocity decreases with $V_{T\parallel}$. These curves show that the phase velocity is less in hot plasmas.

Figure 2 shows the variation of current with $K_{\parallel}$ for different values of $V_{T\parallel}$. It is seen that current increases with $K_{\parallel}$ as well as $V_{T\parallel}$. It is also clear that the current decreases with phase velocity of Alfvén wave.

In the present paper, we have investigated the effect of thermal velocity on the dispersion relation and current driven by the Alfvén wave. In the past, the Alfvén waves are mostly analyzed for the cold plasmas ignoring the thermal corrections. The frequency of micropulsations noted at the sub-storm times in the magnetospheric-ionospheric coupling system and field-aligned current may depend upon the thermal particles streaming along the auroral field lines. Further the current driven by Alfvén waves in Tokamak plasmas may be also affected by the thermal velocity. The trajectory of charged particle so derived may be useful for ion-pickup processes in interplanetary plasmas [4].

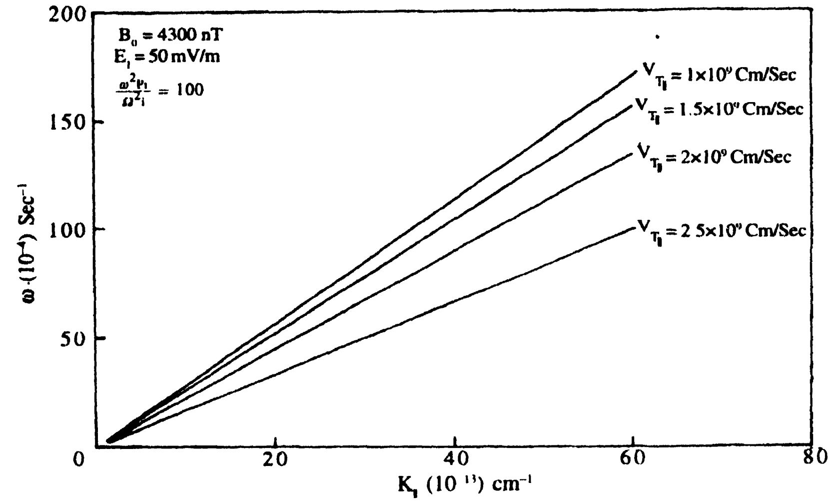

Figure 1. $K_{\parallel}$ *Versus* ω for different $V_{t\parallel}$

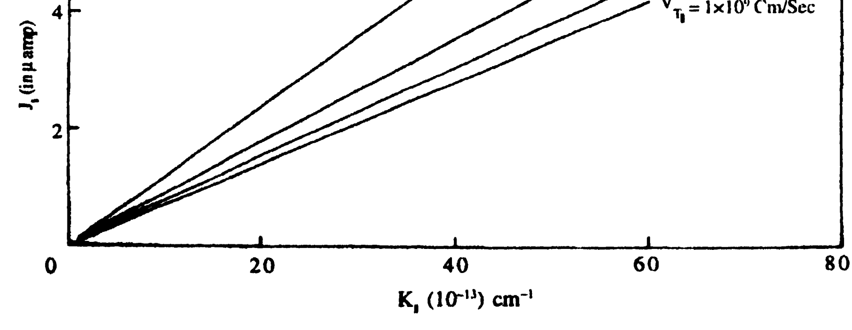

Figure 2. $K_{\parallel}$ *Versus* $J_{\parallel}$ for different $V_{T\parallel}$.

Fundamentally, great interest of study is in the magnetospheric plasmas and in the auroral acceleration region where field aligned current can be estimated through the use of our study. In the present analysis, we have investigated the variation of current density in the presence of Alfvén wave in hot plasmas.

The theory may be useful for the plasma heating processes, confinement devices and space plasmas. The Alfvén wave is fundamentally important for supplementary heating of Tokamak-type plasmas. With the help of these study, one can explain the formation of the stable auroral arcs as well as viscous interaction between the solar wind and the magnetosphere.

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