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Octupole transitions in 1*p* shell nuclei

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Abstract : Significant differences are observed in the comparison of electron scattering data from the octupole transitions in ^{12}C, ^{15}N and ^{16}O with the results of a large basis shell model calculations A good description of the octupole data is obtained for the longitudinal from factors in ^{12}C, ^{15}N and ^{16}O when contributions from collective state are allowed in the shell model transition density. The amplitude of this admixture is identical in the three nuclei

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A detailed test of the radial shape of nuclear excited state wave functions is provided by inelastic electron scattering (e, e′) form factors. The (e, e′) form factor is a Fourier-Bessel transform of the transition density which may be then compared to a nuclear model prediction. The shell model has been reasonably successful in explaining a large body of data in 1*p*-shell nuclei and in this letter we examine radial octupole transition densities predicted by the shell model for ^{12}C, ^{15}N and ^{16}O. Octupole transitions are particularly interesting for nuclei in upper half of the 1*p*-shell since such transitions are expected to have, within the approximation of a $1\hbar\omega$ shell model, a unique radial shape of $1p_{1/2,3/2} - 1d_{5/2,3/2}$ transitions. The use of a $3\hbar\omega$ model space [1] gives little modifications to the shape of the octupole transitions. Saxon-Woods wave functions do not change the *q*-dependence in any significant way and the use of Harmonic-Oscillator (HO) wave functions, to define the radial shape of basis states, is justified.

For a stringent test of the wave functions, it is necessary to define (e, e′) form factors over as large a *q*-range as possible. The experimental data used here are from Ref. [2] for ^{12}C, Ref. [3] for ^{15}N and Ref. [4] for ^{16}O.

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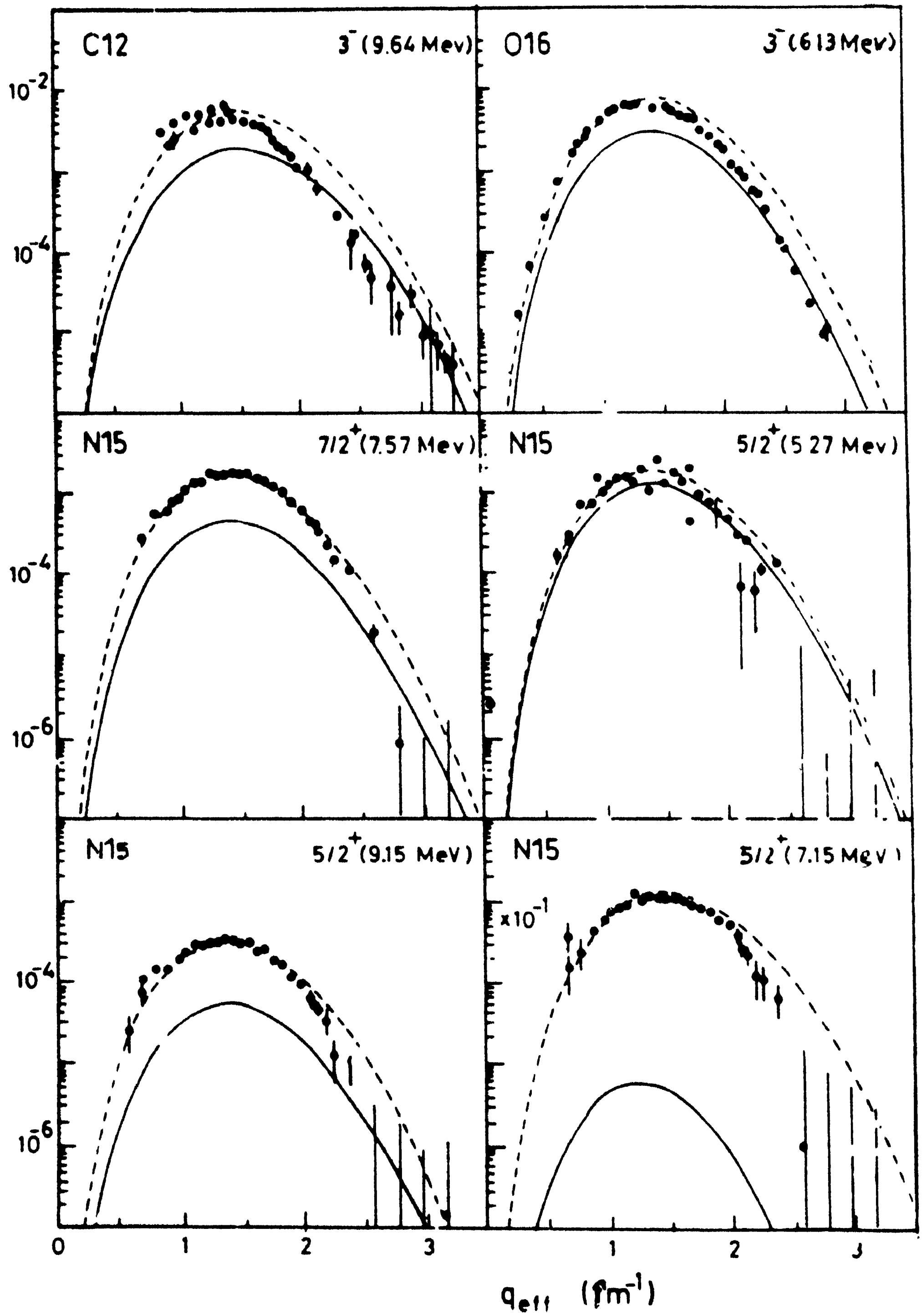

Figure 1. The longitudinal form factors for the octupole transitions in ^{12}C, ^{16}O and ^{15}N. Experimental values are indicated by solid circles; (^{12}C, ^{15}N and ^{16}O) data are from Refs. [2–4]. Solid curves are the predictions of the shell model with no effective charges. The dashed curves are those with δe chosen to reproduce the maximum of the experimental form factor, see Table 1.

We compare the longitudinal octupole transitions in ^{12}C, ^{15}N and ^{16}O with shell model predictions of Millener in Figure 1. Two observations are obvious from this comparison : the shell model form factors do not have the correct q-dependence, tending to be too broad relative to the experimental data and the longitudinal form factors obtained in the shell model calculations are too small compared with the experimental data. This is particularly true for the so called collective transitions. Conventionally, the discrepancy between the experimental and theoretical form factors is explained by the introduction of an additive effective charge defined as :

$$e = e_i + \delta e ,$$

where e_i is the charge of the free nucleon ($e_p = 1, e_n = 0$) and δe is the effective charge. Some flexibility in the choice of different values of δe for the neutron and the proton has been found to provide a better description of the experimental data. This approach was investigated by Brown *et al* [5] for nuclei near ^{16}O and they found effective charges for the neutron $\delta e_n = 0.34$ and for proton $\delta e_p = 0.0$. For octupole transitions, it has been found that the δe_n is bigger than the δe_p, explained the experimental data very well [5,6]. Millener [9] has suggested that the octupole transitions in ^{15}N can be explained by $\delta e_n = 0.385$, $\delta e_p = 0.095$. In fact, the effective charge needed to reproduce the strength at the maximum of the form factors varies from 0.1 for the $5/2_1^+$ to 2.2. for $5/2_2^+$ in ^{15}N with other transitions requiring intermediate values. The effective charge values are presented in Table 1. In Figure 1, the solid curves represent the shell model predictions with no effective charges, while the dashed curves are those with effective charges. It should be noted that the predicted C3 form factors with the effective charges chosen to reproduce the maximum of the experimental form factor can also be obtained by choosing different values of the effective charges for protons and neutrons.

Table 1. Collective state mixing parameters.

Transitions	Excitation energy (MeV)	δe	α
$^{12}C, 3^-$	9.64	0.24	0.031 ± 0.002
$^{16}O, 3^-$	6.13	0.16	0.032 ± 0.002
$^{15}N, 5/2^+$	5.27	0.1	0.004 ± 0.0004
$^{15}N, 5/2^+$	7.15	2.2	0.006 ± 0.0006
$^{15}N, 7/2^+$	7.57	0.27	0.013 ± 0.001
$^{15}N, 5/2^+$	9.15	1.2	0.006 ± 0.0006
			$\Sigma = 0.029 \pm 0.002$

The major inadequacy of the shell model is the neglect of mixing with configurations outside the restricted model space. However, significant extension of the shell model basis requires knowledge of many more two-body interaction matrix elements and even if these were known, the size of such a calculation would rapidly become prohibitive. Although the shell model description of the C3 transitions is not consistently

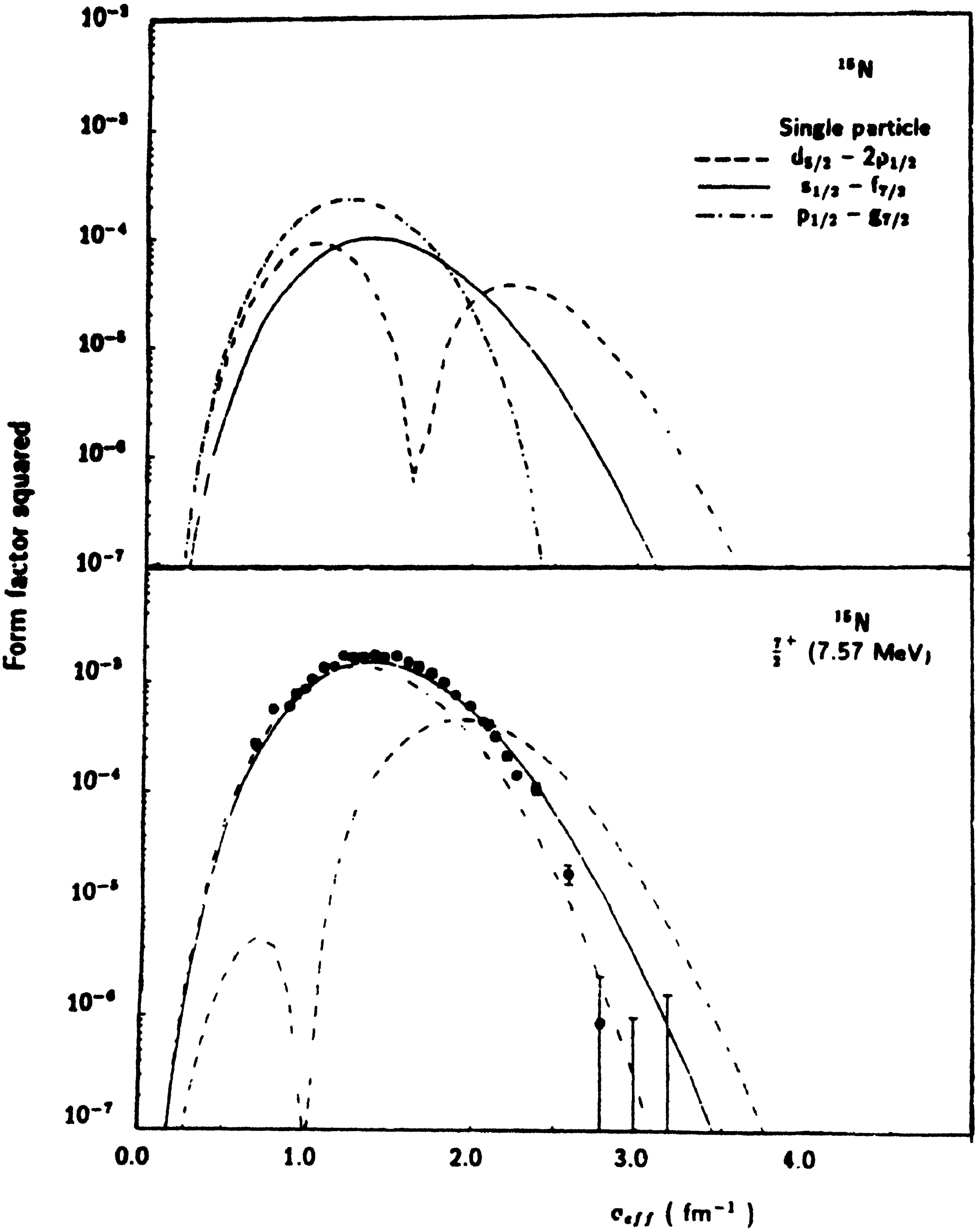

Figure 2. Possible C3 form factors calculated from different configurations, in the upper curves. In the lower curves, comparison between data and theory for the 7.57 MeV level. The curves are predictions of the shell model mixed with the configurations shown above as defined in eq. 1.

good in terms of the predicted strength, from the comparison, it would appear that the model wave function is missing only a small contribution from configurations not included in the shell model space.

It is possible to utilize the experimental data to obtain some feeling for the reasons of this contribution by writing the physical transition density as follows :

$$\rho_{tr}(r) = \rho_{SM}(r) + \alpha\rho_{add}(r). \quad (1)$$

Here, α is an adjustable parameter and $\rho_{add}(r)$ is the contribution from configurations which are outside of the space considered by the shell model. Such configurations will in general, be expected to have a q-dependence which is different from that of the shell model form factors. With this model space, it was possible to give a reasonable description of the octupole transitions [8]. However, the shapes of these configurations are displayed in figure 2, with comparison between data and theory for the 7.57 MeV level, as an example.

An alternative choice for $\rho_{add}(r)$ as that of a collective vibrational state, however, can provide an overall satisfactory agreement for all six states with very significant improvements in both the shape and strength of the predicted form factors. This procedure has some theoretical [10,11] and experimental [12,13] justification even though the application to explain octupole transition data in this way has not been attempted so far. However, the existence of low-lying octupole collective states is well established and considerable evidence for the existence of a giant octupole resonance (GOR) at an excitation energy of $\sim 110/A^{1/3}$ MeV [14] has accumulated in recent years.

The radial shape of the collective state to be used in eq. (1) was chosen as

$$\rho_{add}(r) = Nr\rho'_{g.s}(r), \quad (2)$$

where N is adjusted to give the strength required by the sum rule [10] and $\rho'_{g.s}(r)$ is the radial derivative of the ground state charge distribution of the nucleus which was obtained from [15] in terms of a Fourier-Bessel series

$$\rho_{g.s}(r) = \sum_{\mu} A_{\mu} J_0(q_{\mu} r), \quad r < R_c$$
$$= 0, \quad r \geq R_c .$$

The values of $q_\mu R_c$ gives the μ-th zero of the spherical Bessel function : $J_0(q_\mu R_c) = 0$, where R_c = 7 fm, is the cut-off radius and A_μ are the Fourier-Bessel coefficients [15]. The description of $\rho_{add}(r)$ in eq. (2) was then used in eq. (1) and α was treated as an adjustable parameter for each transition studied. In $\rho_{SM}(r)$, no effective charges were used. The results are presented in Table 1, and Figure 3. The admixture is always such that the strength of the transition is enhanced. Considerable improvement in the q-dependence of all the studied transitions is observed for values of α which are needed to reproduce the experimentally observed form factor maxima.

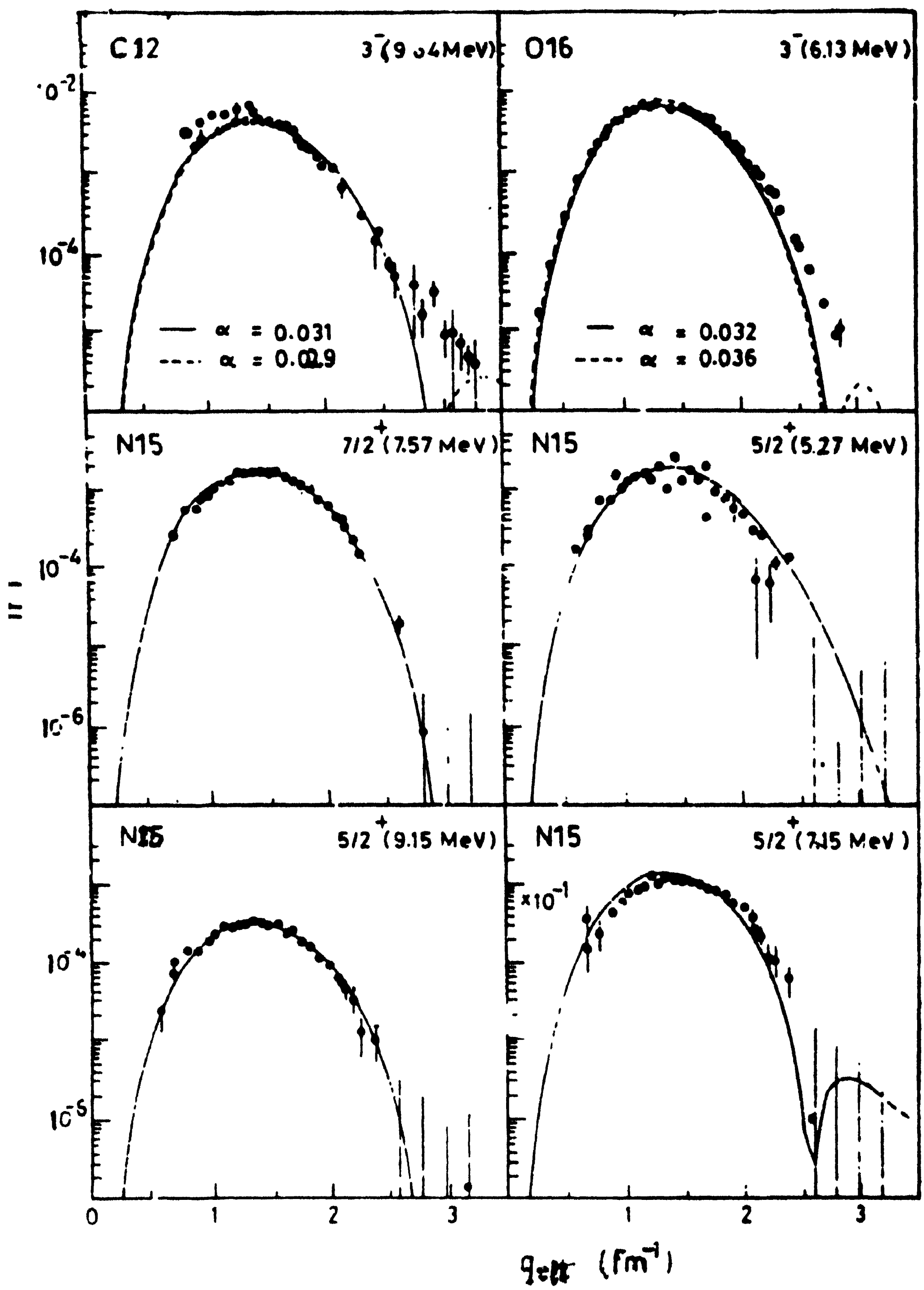

Figure 3. The experimental form factors as in Figure 1. The solid curves are the predictions of the model defined in eqs. 1 and 2. The values of α are listed in Table 1. The dashed curves show the sensitivity of the admixture α.

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