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Strange stars with realistic quark vector interaction and density-dependent scalar potential

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Abstract We derive an equation of state (EOS) for strange matter, starting from an interquark potential which (i) has asymptotic freedom built into it, (ii) shows confinement at zero density ($\rho_B = 0$) and deconfinement at high ρ_B, and (iii) gives a stable configuration for chargeless, betastable quark matter. This EOS is then used to calculate the structure of Strange Stars, and in particular their mass-radius relation. Our present results confirm and reinforce the recent claim [7, 8] that the compact objects associated with the x-ray pulsar Her X-1, and with the x-ray burster 4U 1820-30 are strange stars.

Keywords Neutron stars, quark stars, X-ray sources

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Strange stars (SS) are compact objects which are entirely made of deconfined *u, d, s* quark matter (*strange matter*). It was conjectured [1, 2] that strange matter (SM) may be the absolute ground state of strong interacting matter rather than ^{56}Fe. Since this hypothesis was formulated, strange stars have been studied by many authors [3-6], but they remained purely theoretical entities. This situation changed in the last few years, thanks to the large amount of fresh observational data collected by the new generation of X-ray and γ-ray satellites. In fact, recent studies [7, 8] have shown that the compact objects associated with the X-ray pulsar Her X-1, and with the X-ray burster 4U 1820-30, are good strange star candidates.

Most of the previous calculations of SS properties used an equation of state (EOS) for strange matter based on the MIT bag model. In fact, to get SS configurations with a mass-radius (M-R) relation in agreement with the semiempirical M-R relations for the two SS candidates [7, 9] mentioned above, one has to use large values of the bag constant $B \_ 110\ MeV/fm^3$.

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These values are large when compared with the "standard value" $B = 56\ MeV/fm^3$ ($(144\ MeV)^4$) which is able to reproduce the mass spectrum of light hadrons and heavy mesons [10]. Large values of B are also not allowed by the requirement SM is stable in bulk [11, 12] : *e.g.* for $m_s = 150\,MeV$ the permitted values of the bag constant are in the range $(56-78)\,MeV/fm^3$ (see Figure 1 of ref. [11]). However in general B has been considered as an effective parameter and the bag model should be only regarded as a simple model with limited connection to QCD [13].

If the compact objects in Her X-1 and 4U 1820-30 are really strange stars, then there will be very deep consequences for both the physics of strong interactions and astrophysics. The existence of stable nugget of SM would also have profound implications in cosmology, in relation to the dark matter problem. Motivated by the fundamental importance of this issue, and by the criticism to the bag model mentioned above, in the present work we investigate the properties of SS, using an EOS for SM based on an alternative to the bag model.

Intuitively we know that at high density the quarks should go over from their constituent masses to their current masses, thus restoring the approximate chiral symmetry of QCD. On the other hand, the interquark interaction should be screened in the medium. The latter will give rise to deconfinement at high density. To this end we use the following Hamiltonian :

$$H = \sum_i (\alpha_i . P_i + \beta_i M_i) + \sum_{i<j} \frac{\lambda(i) . \lambda(j)}{4} V_{ij}, \tag{1}$$

where we have two potentials : a scalar and a vector. The scalar originates from the mass term. The quark mass, M_i, is taken to be density dependent and of the form :

$$M_i = m_i + (310\,MeV)\,sech\left(\nu \frac{\rho_B}{\rho_0}\right), \quad i = u, d, s, \tag{2}$$

where $\rho_B = (\rho_u + \rho_d + \rho_s)/3$ is the baryon number density, $\rho_0 = 0.17 fm^{-3}$ is the normal nuclear matter density, and $\nu(=\frac{1}{n})$ is a parameter. At high ρ_B the quark mass M_i falls from its constituent value to its current one which we take to be [14] : $m_u = 4\,MeV$, $m_d = 7\,MeV$, $m_s = 150\,MeV$. The density dependence introduces a density dependent scalar potential and more importantly restores chiral symmetry smoothly at high density.

In the absence of an accurate evaluation of the potential (*e.g.* from large N_c planar diagrams) we borrow it from meson phenomenology, namely the Richardson potential [15]. It incorporates the two concepts of asymptotic freedom and linear quark confinement. The potential reproduces heavy meson spectra. It has been well tested for baryons in Fock calculations [16], [17]. The potential used for the meson and baryon is [15]

$$V_{ij} = \frac{12\pi}{27} \frac{1}{ln(1+(\mathbf{k}_i - \mathbf{k}_j)^2/\Lambda^2)} \frac{1}{(\mathbf{k}_i - \mathbf{k}_j)^2}, \tag{3}$$

with the scale parameter [14] $\Lambda = 100\,MeV$. This bare potential in a medium will be screened due to pair creation and infrared divergence. The inverse screening length, D^{-1}, to the lowest order is [18] :

$$(D^{-1})^2 \equiv \frac{2\alpha_0}{\pi} \sum_{i=u,d,s,} k_i^f \sqrt{\left(k_i^f\right)^2 + m_i^2}\ . \tag{4}$$

where k_i^f, the Fermi momentum of the i-th quark is obtained from the corresponding number density :

$$k_i^f = \left(\rho_i \pi^2\right)^{1/3} \tag{5}$$

and α_0 is the perturbative quark gluon coupling. To simplify numerical calculations, instead of summing over all the individual flavours, we have averaged over the flavours so that

$$(D^{-1})^2 \simeq \frac{3 \times 2\alpha_0}{\pi} k_{av}^f \sqrt{\left(k_{av}^f\right)^2 + m_{av}^2}\,, \tag{6}$$

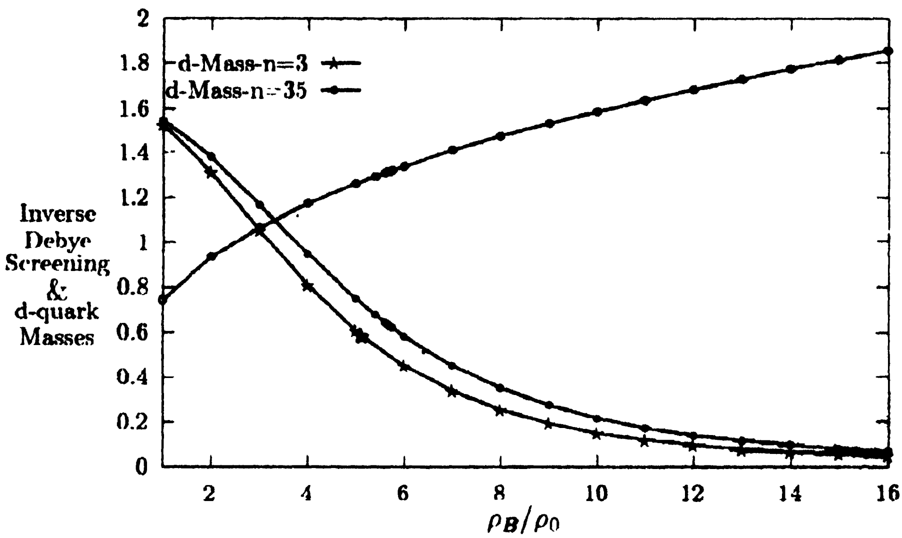

Figure 1. The Debye screening and d quark masses for n = 3.5 and 3 as a function of ratio of baryon density

where

$$k_{av}^f = \left(\pi^2 \rho_B\right)^{1/3} = \left(\frac{\left(k_u^f\right)^3 + \left(k_d^f\right)^3 + \left(k_s^f\right)^3}{3}\right)^{1/3} \tag{7}$$

$$m_{av} = \frac{m_u + m_d + m_s}{3}\,. \tag{8}$$

The model describes deconfined quarks at finite density, through the Debye screening (DS, in short). At zero density (for an isolated hadron) D^{-1} vanishes, leading to confinement. At finite density, due to *DS*, the gluon polarization acquires a non-zero value leading to deconfinement. The scalar potential also decreases with density, thus restoring chiral symmetry at high density. The resulting inverse *DS* is also plotted in the Figure 1. The energy density of *SM* can be written as

$$\varepsilon = \varepsilon_k + \varepsilon_v\,, \tag{9}$$

where the kinetic part is given by

$$\varepsilon_k = \frac{3}{4\pi^2} \sum_{i=u,d,s} \left[k_i^f \left((k_i^f)^2 + M_i^2/2 \right) \sqrt{(k_i^f)^2 + M_i^2} - \frac{M_i^4}{2} \ln \frac{\sqrt{\left((k_i^f)^2 + M_i^2\right)} + k_i^f}{M_i} \right] \tag{10}$$

and the potential contribution is given by

$$\varepsilon_v = \frac{1}{2\pi^3} \sum_{ij} \int_{-1}^{+1} dx \int_0^{k_i^f} k_j^2 \int_0^{k_i^f} k_i^2 f(k_i, k_j, M_i, M_j, x)$$

$$V\left[D^{-1}, (k_i - k_j)^2\right] dk_j dk_i\,, \tag{11}$$

where V is the screened Richardson potential. Then we calculate the self consistent chemical potentials to satisfy the beta-equilibrium and charge neutrality conditions,

$$\mu_d = \mu_s\,, \quad \mu_d = \mu_u + \mu_e\,; \tag{12}$$

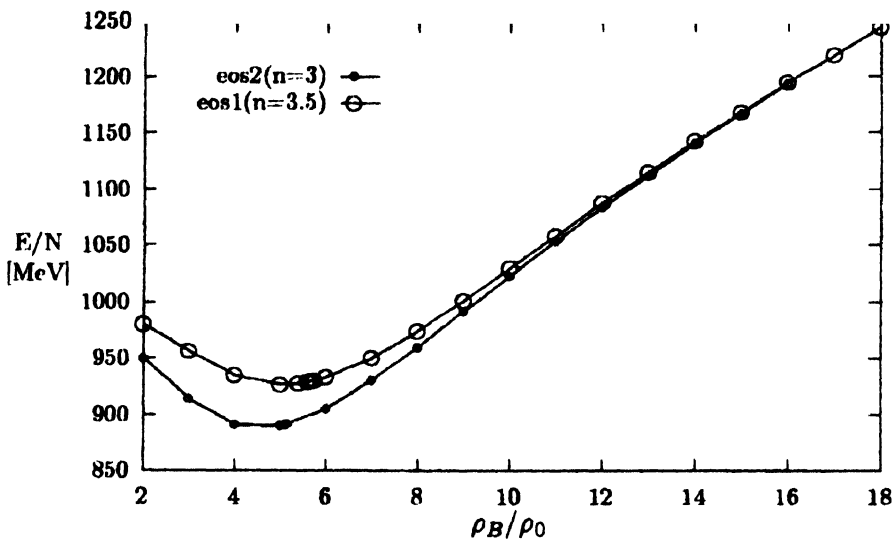

Figure 2. EOS of our model with $n = 3.5$ and $n = 3$. The minimum shifts towards lower density with low n

$$2\left(k_u^f\right)^3 - \left(k_s^f\right)^3 - \left(k_d^f\right)^3 - \left(k_e^f\right)^3 = 0, \tag{13}$$

where μ-s are the chemical potentials of u, d, s quarks and the electron, e. We assume that the neutrinos have left the system ($\mu_\nu = 0$). If m_e is the electron mass, k_i^f is obtained through

$$k_e^f = \sqrt{\mu_e^2 - m_e^2}\,. \tag{14}$$

To achieve the conditions (12, 13), we have the additional difficulties originating from the density dependence of the quark masses and DS length. The chemical potential for the *i-th* quark is given by :

$$\mu_i = \sqrt{M_i^2 + \left(k_i^f\right)^2} + (\Delta\mu_i)_M + (\Delta\mu_i)_V, \quad i = u, d, s, \tag{15}$$

where $(\Delta\mu_i)_M$ is the contribution from ε_k (10), which is evaluated straightforwardly :

$$(\Delta\mu_i)_M \equiv \frac{\partial \varepsilon_k}{\partial M_i}\frac{\partial M_i}{\partial \rho_i} = \left(\rho_s^u + \rho_s^d + \rho_s^s\right)\frac{\partial M_i}{\partial \rho_i}, \tag{16}$$

ρ_s^i being the scalar density for the *i-th* quark

$$\rho_s^i = \frac{3}{4\pi^2}\left[2M_i k_i^f \sqrt{\left(k_i^f\right)^2 + M_i^2} - 2M_i^3 ln \frac{\sqrt{\left(k_i^f\right)^2 + M_i^2} + k_i^f}{M_i}\right]. \tag{17}$$

On the other hand, the contribution from the potential part (11) of the energy density

$$(\Delta\mu_i)_V = \frac{\partial \varepsilon_v}{\partial k_i^f}\frac{\partial k_i^f}{\partial \rho_i} + \frac{\partial \varepsilon_v}{\partial M_i}\frac{\partial M_i}{\partial \rho_i} \tag{18}$$

is rather complicated and it is evaluated numerically. Eqs. (12, 13) can now be satisfied and EOS for beta-stable SM is obtained.

The β-equilibrium EOS of SM is shown in Figure 2. The minimum shows a self-sustained system, the pressure being zero at the point. Thus, there is a possibility of SM to be saturated at such a high density. This comes about from the deconfinement and chiral symmetry restoration. The saturation density shifts to a lower density for a lower n value.

With this EOS we solve the Tolman-Oppenheimer-Volkov equation to calculate the structure of non-rotating SS. The properties of the maximum mass configuration for different choices of our model parameters are summarized in Table 1.

Table 1. Properties of the maximum mass strange star configuration obtained from different EOS : M_G is the gravitational (maximum) mass, R is the corresponding radius, n_c the central number density, ρ_c the central mass density. Our EOS for different choices of the parameters are denoted as follows : (eosl) $v = 0.286$, $\alpha_0 = 0.20$; (eos 2) $v = 0.33$, $\alpha_0 = 0.20$; (eos3) $v = 0.33$, $\alpha_0 = 0.25$. The bag model EOS are denoted as follows : (B110 ; 150) $B = 110$ *MeV* / fm^3, $m_s = 150$ *MeV* ; (B110 ; 0) $B = 110$ *MeV* / fm^3, $m_s = 0$ ($\alpha_c = 0$ in both cases).

EOS	$M_G / M_\odot$	R(km)	n_c (fm^{-3})	ρ_c (10^{14} g/cm^3)
eosl	1.325	6.56	2.52	52.20
eos2	1.44	7.07	2.31	46.40
eos3	1.41	6.98	2.25	45.70
B110 : 150	1.358	7.5	1.868	42.00
B110 : 0	1.448	7.9	1.762	38.10

Our EOS is most sensitive to the parameters ν and α_0 which rule the density dependence of the quark mass and DS length respectively. However, we found that a change within 20% of these parameters, *e.g.* around the values $\nu = 0.33$, $\alpha_0 = 0.20$ (eos2, eos1), produce a change of M_{max} and of the corresponding radius which is smaller than 10%. The variation with α_0 alone (eos2 and eos3) is even less.

In the same table we also show the results we obtained using an EOS for SM based on the bag model with parameters : $B = 110\,MeV/fm^3$, $m_s = 150\,MeV$, and $B = 110\,MeV/fm^3$, $m_s = 0$, with the strong coupling constant $\alpha_c = 0$ (no gluons) in both cases. There have been other theoretical approaches to the study of SS [6, 19]. The work of Drago *et al.* [6] uses (u, d) quark masses close to 100 MeV in the colour dielectric model and gets a maximum mass ~ $1.6\,M/M_\odot$ and a radius of about 10 km. In [19], a very strong magnetic field is proposed as a mechanism for softening a bag equation of state.

The present calculated masses and radii are plotted in Figure 3. There is qualitative agreement with the bag model results, and in particular for low values of the mass, M is proportional to R^3. However, the two bag model calculations, close to our curves indeed use a large bag pressure *which remains constant at all densities*. In the figure, we also compare the theoretical M–R relations with the semiempirical M-R for the two strange star candidates. The closed region in the figure, labeled 4U 1820-30 represents the semiempirical M-R relation recently extracted from observational data by Haberl and Titarchuk [9] and used in the theoretical analysis of ref. [8].

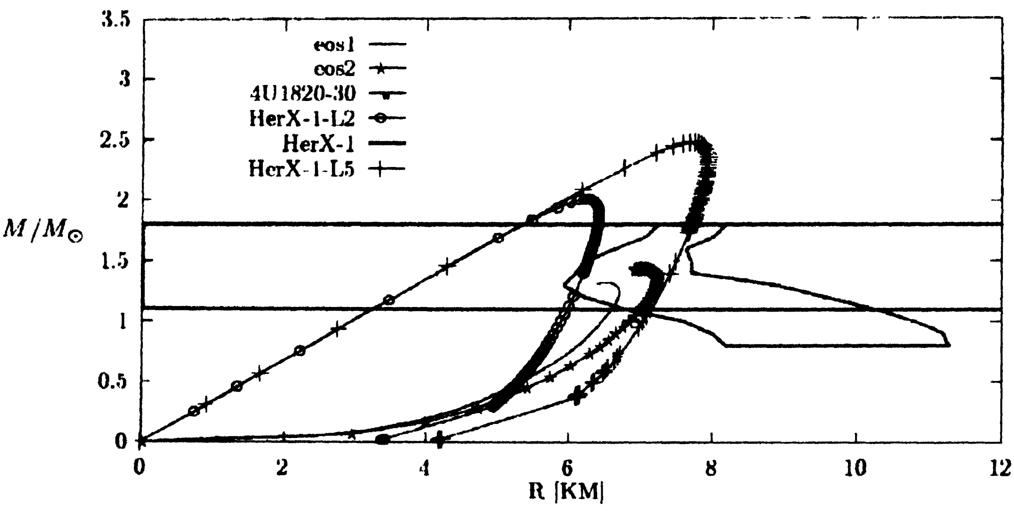

Figure 3. Comparison of our SS, n = 3.5 and 3, with empirical data.

The trapezium-like region labeled Her X-1 represents the semiempirical M-R relation for the compact object in Her X-1. We followed the analysis by Wasserman and Shapiro [20] updated with new mass and distance measures of Her X-1 as reported in ref. [21, 22]. Dashed curves a (-o—) and b (-+—) in the figure 3 denote the M-R relation for Her X-1, assuming an X-ray luminosity $L = 2.0.10^{37}$ erg/s and $L = 5.0.10^{37}$ erg/s respectively. The two values of the luminosity we used correspond to a distance of 5.0 kpc and 7.9 kpc respectively. To be aware of the sensitivity to the luminosity, we also report the M-R relation (curve c) assuming the

Eddington luminosity for spherical accretion as an upper bound. The two horizontal lines locate the measured mass of Her X-1 $(1.1 - 1.8) M_{\odot}$.

It is very important to stress that the above M-R relations have been extracted from two different types of astronomical phenomena – X-ray burst spectra (4U 1820-30) and cyclotron line data from a X-ray pulsar in a binary system (Her X-1) – and using different theoretical models to analyze the original observational data. The two semiempirical M-R relations overlap in a region of the M-R plane indicating the existence of a compact object with a radius of 6-8 km. This shows that the analysis performed by Bombaci [8] in the case of the X-ray burster 4U 1820-30 also extends to the case of Her X=1. In particular, neutron star models based on "conventional" EOS of dense matter [8] are unable to reproduce the semiempirical M-R relation for these two compact objects.

The M-R relation calculated with our EOS for SM is well within both the semiempirical M-R relations of the two SS candidates. In conclusion, our present results confirm and reinforce the recent claim [7, 8] that the compact objects associated with the X-ray pulsar Her X-1, and with the x-ray burster 4U 1820-30 are strange stars.

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