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A splitting of the Weert superpotential

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Abstract : We show that the concept of Lanczos spintensor (*Rev. Mod Phys.* **34** 379 (1962) [1]) leads in a natural way to the splitting of $\underset{B}{T}_{ac}$ as proposed by López (*Phys. Rev.* **D17** 2004 (1978) [2]) for the bounded part of the Liénard-Wiechert electromagnetic field.

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First of all we go back to Refs. [3–9] and recall the notation and quantities employed there. When a point-charge is moving arbitrarily in Minkowski space, it generates the Liénard-Wiechert electromagnetic field whose Maxwell tensor may be split as [10]

$$T_{ac} = \underset{B}{T}_{a} + T_{a} \tag{1}$$

in its bounded $\underset{B}{T}_{ij}$ and radiative $\underset{R}{T}_{ij}$ portions, which in turn satisfy the continuity equation outside from the world line of the charge :

$$\underset{R}{T}{}^{a}{}_{c,a} = 0, \tag{2}$$

$$\underset{B}{T}{}^{a}{}_{c,a} = 0, \tag{3}$$

On the other hand, in [3,5,7–9,11,12] it was shown the non-local superpotential $\underset{R}{K}_{abc} = -\underset{R}{K}_{bac}$ for the radiative part,

$$\underset{R}{T}_{a} = \underset{R}{K}{}^{b}{}_{ac,b}, \tag{4}$$

from which (2) follows. Similarly, (3) implies the existence of the Weert superpotential [13,14] $\underset{B}{K}_{ijr}$, which has the role of a generator for the bounded part :

$$\underset{B}{T}_{ac} = \underset{B}{K}{}^{r}{}_{ac,r}. \tag{5}$$

In Ref. [8], the physical meaning of $\underset{B}{K}_{abc}$ was found and it turns out that this is the intrinsic angular momentum density of the electromagnetic field produced by the point charge.

Furthermore, a splitting of the bounded piece was suggested in [2] :

$$\underset{B}{T}_{ac} = \underset{B}{\bar{T}}_{ac} + \underset{B}{\dot{T}}_{ac} \tag{6}$$

which is quite convenient when studying the angular momentum of the Liénard-Wiechert field; however, the actual origin of (6) was not clear.

Since the potential $\underset{B}{K}_{ijr}$ has [6,8] the same algebraic and differential symmetries as that of the Lanczos spintensor [1,15–28], we will prove in this work that this fact brings naturally to the splitting

$$\underset{B}{K}_{arc} = \underset{B}{K}_{arc} + \underset{B}{K}_{arc}, \tag{7}$$

as evident from the expressions (5) and (6).

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Splitting of $\underset{B}{K}_{ijr}$:

In [4–10,12–14] one finds the expression for the bounded term of the Maxwell tensor associated to the Liénard-Wiechert case :

$$\underset{B}{T}_{bc} = q^2 w^{-4}\Big[\frac{1}{2} g_{bc} + (k_b a_c + k_c a_b)$$

$$\bar{B}(k_b v_c + k_c v_b) - w^{-2}(1-2W)k_b k_c\Big]. \tag{8}$$

Also Weert [13,14] has shown that (5) implies (8) by means of

$$\underset{B}{K}_{bjc} = \frac{q^2}{4} w^{-4}\big[w^{-1}(4W-3)\big(v_b k_j - v_j k_b\big)k_c$$

$$-4\big(a_b k_j - a_j k_b\big)k_c + g_{cj}k_b - g_{cb}k_j\big]. \tag{9}$$

The symmetries of the Weert superpotential have been studied for example in Refs. [6,8]; in fact, it is easy to verify by using (9) that they are

$$\underset{B}{K}_{bjc} = -\underset{B}{K}_{jbc}, \qquad \underset{B}{K}{}_{b}{}^{j}{}_{j} = 0,$$

$$\underset{B}{K}_{abc} + \underset{B}{K}_{bca} + \underset{B}{K}_{cab} = 0, \qquad \underset{B}{K}{}_{bj}{}^{c}{}_{,c} = 0, \tag{10}$$

which are remarkable because (10) are fulfilled also by the Lanczos spintensor K_{ijr} [1,15–28] in general relativity.

At this stage, we proceed to give the steps to generate a splitting in $\underset{B}{T}_{ac}$: we write (9) in the form (7) but carefully imposing the condition that $\underset{B}{\bar{K}}_{ijr}$ and $\underset{B}{\tilde{K}}_{ac}$ satisfy the symmetries of Lanczos (10) too. And indeed, it is not difficult to see that there is only one way to rewrite (9) fulfilling such requirement :

$$\underset{B}{K}_{bjc} = \frac{q^2}{4} w^{-4}\big[4w^{-1}W\big(v_b k_j - v_j k_b\big)k_c$$

$$-4\big(a_b k_j - a_j k_b\big)k_c - 3w^{-1}\big(v_b k_j - v_j k_b\big)k_c$$

$$+ g_{cj}k_b - g_{cb}k_j\big]. \tag{11}$$

When comparing this eq. with (7), the following superpotentials come out :

$$\underset{B}{\bar{K}}_{bjc} = q^2 w^{-4}\Big[\big(w^{-1}Wv_b - a_b\big)k_j - \big(w^{-1}Wv_j - a_j\big)k_b\Big]k_c, \tag{12}$$

$$\underset{B}{\tilde{K}}_{bjc} = \frac{q^2}{4} w^{-4}\Big[3w^{-1}\big(v_j k_b - v_b k_j\big)k_c + g_{cj}k_b - g_{cb}k_j\Big], \tag{13}$$

satisfying the properties (10). The splitting (6) proposed by López [2] appears if eqs. (7,12,13) are substituted into (5), with

$$\underset{B}{\bar{T}}_{ac} = \underset{B}{\bar{K}}{}_{a}{}^{r}{}_{c,r}, \qquad \underset{B}{\tilde{T}}_{ac} = \underset{B}{\tilde{K}}{}_{a}{}^{r}{}_{c,r}, \tag{14}$$

and one recovers the expressions (which are not necessary to display here) of the mentioned author.

In this manner, we have made clear that (6) is motivated by requiring that the Weert superpotential be the sum of two tensorial objects with the symmetries of the Lanczos spintensor.

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