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Finite field-dependent BRS [FFBRS] transformations and axial gauges

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Abstract : We need to make choice of gauge to define a gauge theory There are many choices of gauges The theories [greens functions etc] in different types gauges are difficult to relate to each other as the transformation from one gauge type to another is hard to construct explicitly Yet the physical results in various types of gauges are supposed to be gauge-independent In fact, however, examples have been found where the anomalous dimensions of physical observables have been found to differ in Lorentz and axial gauges

In this work, we develop a method for constructing an explicit field transformation between two sets of gauges This field transformation is a generalization of the BRS transformation in that the anticommuting parameter now is not infinitesimal nor field-independent [though not local]. These generalized BRS transformations, which we call "Finite Field-dependent BRS transformations", or FFBRS for short, are nilpotent and are obtained by integrating the infinitesimal field-dependent ones We show that these can be used to connect the Faddeev-Popov effective action in linear gauges with gauge parameter lambda to [i] the most general BRS-anti-BRS symmetric action in linear gauges ; [ii] the Faddeev-Popov effective action in quadratic gauges , [iii] the Faddeev-Popov effective action with another distinct gauge parameter alpha. In each case the extra terms in the latter action are shown to arise from the Jacobian for the NONLOCAL FFBRS transformation [*Phys. Rev* **D51**, (1995) 1919-1927]

The above idea is further applied to correlate axial and the Lorentz type gauges by an explicitly constructed FFBRS transformation. We show that it can be used to explicitly CONSTRUCT a prescription for light-cone and axial gauges It may further be used to correlate quantities in the two sets of gauges. [*J. Phys.* **A31** 4217 (1998)]

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1. Introduction

The known high energy physics is well represented by the standard model ; which is a non-abelian gauge theory. [1] Hence, the practical calculations in High Energy Physics [generally]

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require a choice of a gauge. There are many distinct [families of] gauges, variously useful in different situations. For example :

Lorentz Gauges . $S_{g\,f} = -1/2\lambda \int d^4x\,[\partial\cdot A]^2$

[2] Coulomb Gauge : $\nabla \bullet A = 0$;

[3] Axial Gauges : $\eta \bullet A = 0$ [Includes Light-cone, temporal etc.]

$$\text{or } S = -1/2\lambda \int d^4x\,[\eta\ A]^2,$$

[4] Planar Gauges : $S = -1/2\lambda \int d^4x\,[\eta\cdot A]\partial^2\ [\eta\ A]\ \eta^2$;

[5] Radial Gauges ; $x \bullet A = 0$;

[6] Quadratic Gauges ;

[7] in SBGT R-$_\xi$ Gauges ;

etc.

We note in passing how these gauges have been found useful under different circumstances. The use of the Coulomb gauge in QED radiation problems is well-known. The axial gauges, as commented later, are useful as they are supposed to be ghost-free. [2] A special case of these, the light-cone gauge, has been found useful in calculations involving perturbative QCD [3]. The planar gauges have advantages of the axial gauges and in addition have a simpler propagator [4]. The radial gauge has found widespread use in the context of the QCD sum-rules and operator product expansion in QCD [5]. Certain quadratic gauges have been found to simplify Feynman rules and diagram calculations in the spontaneously broken gauge theories [SBGT]. The use of the R_ξ-gauges while performing calculations in SBGT [6] and that of the Lorentz-type gauges in QCD are too well-known. [More detailed comments on the Lorentz-type and the axial-type gauges will follow in Section [6]].

Now, we expect the physical results for a physical observable P to be independent of the choice of the gauge. Indeed, gauge-independence, in a limited sense, has been proven since early days. *E.g.* in Lorentz type gauges, one establishes the λ-independence of P *etc*. Such proofs utilize the *infinitesimal gauge* transformations responsible for an infinitesimal change in gauge fixing term. Gauge independence with reference to different families of gauges has not been established. Indeed, recently discrepancy [7] has been reported in an [observable] anomalous dimension. Further, gauges such as axial gauges suffer from the prescription ambiguity [8] for the $1/\eta.q$ type singularities. Such ambiguities do not exist [or are rather easily resolved] in Lorentz type gauges. We thus expect that a field transformation from the axial gauges to Lorentz gauges will enable us to derive the correct prescription for such singularities.

We thus summarise some of the motivations for obtaining the field transformations from one family of gauges to another :

- To obtain a method for relating the Feynman diagrams in two sets of gauges and thus establish a basis for comparison of the results in the two gauges.

- To establish the gauge-independence of a physical observable P with respect to a wider class of gauges.
- To obtain a [proven rather than ad-hoc] prescription for the 1/n.q type singularities in the axial and, in particular, the light-cone gauges.
- To address to the already existing discrepancies in the calculations for the physical observables in the axial and Lorentz type gauges.

We know explicitly the infinitesimal gauge transformation that relates two gauge functions differing infinitesimally ; *viz.*

$$F^{\alpha} \rightarrow F^{\alpha} + \delta F^{\alpha}$$

It is given by [9]

$$\delta A^{\alpha}_{\mu} = D^{\alpha\beta}_{\mu} \, M^{-1}_{\beta\gamma} \, \delta F^{\gamma}.$$

This is an example of a field-dependent infinitesimal gauge transformation. It is difficult to integrate and explicitly evaluate the finite version of this. For example, even in a simple case such as seeking the transformation from $\partial \bullet A = 0$ to $A'_0 = 0$ gauge, the explicit transformation can be formally solved for, but is difficult to evaluate.

A' is given by $A' = U[A]\,[\partial + g\,A\,]\,U\,[A]^{-1}/g$ with,

$$U[A] = T\exp\{\textstyle\int A_0(x,t')\,dt'\}.$$

The results in other cases are expected to be even more involved. We instead seek an alternate approach in which we try to integrate the *BRS transformation*. [10] This seems to be more easily managable. This property arises mainly from the facts that [1] BRS transformations are nilpotent [2] The finite BRS and infinitesimal BRS have the same form unlike the gauge transformations.

2. BRS transformations and generalizations

We shall firstly introduce what we call "finite" BRS somewhat simplistically :

Infinitesimal	"Finite"
$\delta A^{\alpha}_{\mu}(x) = D^{\alpha\beta}_{\mu} c(x)^{\beta}\,\delta\Lambda,$	$A'^{\alpha}_{\mu} = A^{\alpha}_{\mu} + D^{\alpha\beta}_{\mu} c^{\beta}(x)\,\Lambda,$
$\delta c^{\alpha} = -g/2\, f^{\alpha\beta\gamma} c^{\beta} c^{\gamma}\,\delta\Lambda,$	$c'^{\alpha}(x) = c^{\alpha}(x) - g/2\, f^{\alpha\beta\gamma} c^{\beta} c^{\gamma}\,\delta\Lambda,$
$\delta\bar{c}^{\alpha} = -[\partial\bullet A]^{\alpha}/\lambda\;\delta\Lambda,$	$\bar{c}'^{\alpha}(x) = \bar{c}^{\alpha}(x) - [\partial\bullet A]^{\alpha}/\lambda\;\Lambda,$

are symmetries in both cases : $\delta\Lambda$ infinitesimal and Λ finite. So it is easier to integrate from the infinitesimal case to the finite case.

Further, the above transformations are a symmetry even when $\delta\Lambda$ or Λ is *FIELD-DEPENDENT but x-independent*. This is so since the BRS invariance depends only on $\delta\Lambda^2 = 0$, or $\Lambda^2 = 0$.

Example :

$$\Lambda = \int \bar{c}(y)^{\alpha} \, [\partial \bullet A]^{\alpha}(y) d^4 y .$$

We note that Λ is a 'finite' field operator in the sense that if a Greens' function of Λ between vacuum and a state with a ghost and a gauge field is evaluated, it has finite [as opposed to infinitesimal] value. Yet, $\Lambda^2 = 0$ as seen below.

$$\Lambda^2 = \left\{ \int \bar{c}(y)^{\eta} \, [\partial \bullet A]^{\eta}(y) d^4 y \right\}^2 ,$$

$$\Lambda = \int \bar{c}^{\eta}(y) [\partial \bullet A]^{\eta}(y) \, \bar{c}(z)^{\xi} \, [\partial \bullet A]^{\xi}(z) d^4 y d^4 z .$$

$$\Lambda = \int c^{\eta}(y) \bar{c}(z)^{\eta} \, [\partial \bullet A]^{\xi}(y) [\partial \bullet A]^{\xi}(z) d^4 y d^4 z .$$

Now by interchanging the summation variables $(\eta, y) \rightarrow (\xi, z)$, one sees that $\Lambda^2 = -\Lambda^2 = 0$.

In this case, the FFBRS reads [dropping indices] :

$$A'(x) = A(x) + Dc(x) \int \bar{c}(y) [\partial \bullet A](y) \, dy ,$$

$$c'(x) = c(x) - g/2 f c c \int \bar{c}(y) [\partial \bullet A](y) \, dy ,$$

$$\bar{c}'(x) = \bar{c}(x) - [\partial \bullet A] / \lambda \int \bar{c}(y) [\partial \bullet A](y) \, dy ,$$

etc.

We note that the field transformation is 'finite' rather than infinitesimal in the sense that (A'-A) is a field operator with finite Greens' functions (as discussed earlier for Λ). The above transformations are non-local but an exact symmetry of S.

3. Construction of FFBRS by integration of an infinitesimal transformation

In order to show that the above kinds of FFBRS can interpolate between two sets of gauges, it is necessary, first of all, to obtain the Jacobian for such a field transformation. The Jacobian for the finite nonlocal transformation is by no means easy to obtain. Now, if a finite transformation can be obtained by a succession of infinitesimal ones ; in such a case, the Jacobian for the finite transformation can be obtained by integration.

We now investigate how an infinitesimal field-dependent BRS [IFBRS for short] can be integrated and show how the BRS form of the transformation is preserved during the process of integration.

We define a parameter-dependent generic field $\phi(x,\kappa)$[$= A$ or c or $\bar{c}$]; by

$$d\phi(x,\kappa)/d\kappa = \delta_{BRS} [\, \phi(x,\kappa) \,] \, \Theta' \, [\phi(y,\kappa)].$$

These stand for the three equations :

$$dA(x,\kappa)/d\kappa = Dc(x) \Theta' [A(y,\kappa), c(y,\kappa), \bar{c}(y,\kappa)],$$

$$dc(x,\kappa)/d\kappa = -g/2 f c c(x) \Theta' [A(y,\kappa), c(y,\kappa), \bar{c}(y,\kappa)],$$

$$d\bar{c}(x,\kappa)/d\kappa = -[\partial \bullet A](x)/\lambda \, \Theta' [A(y,\kappa), c(y,\kappa), \bar{c}(y,\kappa)],$$

We shall show that as these are integrated w.r.t. κ. they preserve the form and yield an FFBRS of the generic form :

$$\phi(x,\kappa) = \phi(x,0) + \delta_{BRS} [\,\phi(x,0)\,]\,\Theta\,[\phi(y,0)]$$

and setting $\kappa = 1$, and $\phi(x,1) = \phi'$; we obtain the FFBRS ;

$$\phi'(x) = \phi(x) + \delta_{BRS} [\,\phi(x)\,]\,\Theta\,[\phi(y)]\,.$$

Sketch of the proof :

Imagine integrating the equation

$$d\phi(x,\kappa)\,/\,d\kappa = \delta_{BRS} [\,\phi(x,\kappa)\,]\,\Theta\,[\phi(y,\kappa)]$$

straightforwardly. We then have

$$\phi(x,\kappa) = \phi(x,0) + \int d\kappa'\,\delta_{BRS} [\,\phi(x,\kappa')\,]\,\Theta'\,[\phi(y,\kappa')]\,.$$

This equation is difficult to evaluate because r.h.s. contains explicitly the quantities we are trying to evaluate. But here the simplification comes essentially from the nilpotency of BRS. It leads to simplifications in two places : [1] in $\delta_{BRS}[\phi(x,\kappa)]$ the κ-dependence can be dropped , so that we can formally simplify :

$$\phi(x,\kappa) = \phi(x,0) + \delta_{BRS} [\,\phi(x,0)\,] \int d\kappa'\,\Theta'\,[\phi(y,\kappa')]\,.$$

and [2] the integral, though it formally involves the unknown, can still be explicitly evaluated. To see how this works, we evaluate Θ' in terms of $\phi(0)$ by writing the differential equation for Θ':

$$d\Theta'[\kappa]\,/\,d\kappa = \delta\phi[(x,\kappa)]\,\delta\Theta'/\delta\phi$$

$$= \delta\Theta/\delta\phi\,\delta_{BRS}\,\phi[(x,\kappa)]\,\Theta'[\kappa]$$

$$\equiv f[\phi(x,\kappa)]\,\Theta'[\kappa].$$

This is solved to yield

$$\Theta'\,[\phi(\kappa)] = \Theta'\,[\phi(0)]\,\exp\{\textstyle\int f\,[\phi(y,\kappa')]\,d\kappa'\,. \tag{1}$$

As of now we still have implicit $\phi(\kappa)$ dependence. Here, nilpotency of Θ' comes to rescue. We expand $f[\phi(y,\kappa')]$ around $\kappa = 0$:

$$f[\phi(y,\kappa)] = f[\phi(y,0)] + \kappa[\,\delta f/\delta\phi][\,\delta\phi/\delta\kappa] + \ldots\ldots\ldots\ldots\ .$$

Now, each term on the right hand side except the first is proportional to $\Theta'\,[\,\phi(0)]$. In view of the fact that the right hand side contains a factor of $\Theta'[\phi(0)]$ and that $\Theta'^2[\phi(0)]. = 0$, all higher terms in the expansion of f in (1) can be dropped. Hence,

$$\Theta'[\kappa] = \Theta'[0]\exp\{\kappa\,f\,[\phi(y,0)]\}.$$

Secondly, in

$$d\phi(x,\kappa)\,/\,d\kappa = \delta_{BRS}\,[\phi(x,\kappa)]\,\Theta'[\phi(y,\kappa)]$$

a similar expansion for $\delta_{BRS}[\phi(x,\kappa)]$ can be carried out ; with a similar conclusion. Then we have

$$d\phi(x,\kappa)/d\kappa = \delta_{BRS}[\phi(x,0)]\Theta'[\phi(y,0)]\exp\{\kappa f[\phi(0)]\}.$$

This can now be easily integrated to yield :

$$\phi(x,\kappa) = \phi(x,0) + \delta_{BRS}[\phi(x,0)]\Theta'[\phi(y,0)]\{\exp\kappa f - 1\}/f$$

and, in particular, putting $\kappa = 1$, and letting $\phi[1] = \phi'$

$$\phi'(x) = \phi(x) + \delta_{BRS}[\phi(x)]\Theta[\phi]$$

which is the FFBRS in question. [We remark that only specific forms of $\Theta[\phi]$'s in the FFBRS above can be obtained from some local $\Theta'[\phi]$]. We note that this is also a non-local transformation; however, unlike finite gauge transformations, it has been evaluated explicity.

The non-local transformation, even in DR has a non-trivial Jacobian. This Jacobian, as we shall see is responsible for the differing effective actions in a pair of gauges.

4. The Jacobian

We express the vacuum to vacuum amplitude in two gauges

$$\langle 0|0\rangle = W = \int D\phi \exp\{iS^L_{eff}[\phi]\} \tag{i}$$

$$= \int D\phi' \exp\{iS'_{eff}[\phi]\} \ . \tag{ii}$$

We seek a field transformation $\phi \to \phi'$ such that W of (i) with S^L_{eff} in Lorentz gauges when expressed in terms of ϕ', becomes converted by this field transformation into W of (ii) with S'_{eff} an effective action for another gauge choice. Now, under $\phi \longrightarrow \phi'$ $W \to W = \int D\phi' J[\phi']\exp\{iS^L_{eff}[\phi']\}$, since the action S^L is invariant under FF BRS.

For many cases, the Jacobian for the non-local transformation can effectively be replaced by $\exp\{iS_1[\phi']\}$ with $S_1[\phi']$ a local action. In such cases, the transformation takes you from the effective action in Lorentz gauges to that in some other family of gauges. Then the Jacobian explains the difference between the two effective actions.

The mathematical condition for the effective replacement $J \to \exp\{iS_1\}$ is formulated in terms of the Jacobian for the infinitesimal transformation as :

$$0 = \int D\phi\{[1/J][dJ/d\kappa] - idS_1[\phi(x,\kappa),\kappa]/d\kappa)\exp\{i[S^L_{eff} + S_1]\}.$$

5. General prescription and examples

The general prescription for constructing an FFBRS from one family of gauges to another is then :

[a] Establish a continuous route of interpolating gauges [if necessary] from one family to another ;

[b] Postulate an infinitesimal field-dependent BRS. The form of the infinitesimal gauge transformation serves as a preliminary hint.

[c] Using the form for the interpolating S_{eff} if necessary guess a form for S_1 [ϕ, κ].

[d] Evaluate the Jacobian for an IFBRS in step [b]. This is easily evaluated compared to that for an FFBRS.

[e] Impose the condition meant for validity of the replacement $J \to \exp\{iS_1\}$.

This condition leads to constraints on Θ' and the coefficients and the form for S_1. If then this condition can be fulfilled ; one has found the FFBRS under consideration.

Example

Gauge 1 : Lorentz type gauges

Gauge 2 : The general BRS-anti-BRS effective action of Baulieu and Thierry-Mieg. The S is given by :

$$S'_{eff} = S_0 - \int d^4 \times 1/2\lambda\, [\partial \bullet A]2 + S_G$$

$$S_G = \int d^4x [(1-\alpha/2)\partial\bar{c}\, Dc + (\alpha/2) D\bar{c}\,\partial c + \alpha/2(1-\alpha/2)\lambda g^2 (f\bar{c}c)(f\bar{c}c)$$

We note that gauge 1 is a special case of gauge 2 [$\alpha = 0$] and hence there is no need to construct an interpolating gauge. An inspection of the infinitesimal gauge transformation

$[\alpha : 0 \to \delta\alpha]$ suggests the form for Θ'. It is given by

$$\Theta'[\phi(y,\kappa)] = i\beta \int f^{\alpha\beta\gamma} c^{-\alpha}(y,\kappa)\, c^{-\beta}(y,\kappa)\, c^{\gamma}(y,\kappa)\, dy.$$

We note the kind of terms present in the second effective action. We make an *ansatz* for S_1 :

$$S_1[\phi(x,\kappa);\kappa] = \{\beta\kappa/2 - \xi\beta\kappa^2\} g \int f\bar{c}\bar{c}\; f c c - 2\beta\kappa/\lambda \int f\partial \bullet A\bar{c}c$$

The imposition of Jacobian condition leads to $\beta = -\xi\lambda g$. Setting $\alpha = 4\,\xi\,\kappa$,

we obtain,

$$S_{eff} + S_1 \equiv S'_{eff}$$

of Baulieu and Thierry-Mieg.

An identical discussion applies to connecting FPEA [λ] → FPEA [λ'] and FPEA [λ] → FPEA [Quad. Gauge]. Here FPEA [λ] stands for the Faddeev-Popov effective action with the gauge parameter λ [11].

We now turn to the application to the Axial Gauge problems.

6. Axial gauges

Lorentz gauges have been used widely in Standard model calculations principally because of simplicity of Feynman rules, Lorentz covariance, and availability of a gauge parameter to simplify calculations and check gauge independence. They do however need Faddeev-Popov ghosts and these complicate Feynman diagram calculations, OPE *etc*. Hence another class of gauges have found favour in calculations : the axial gauges ;

$\eta.A = 0$; where η is a 4-vector.

The main advantage of axial gauges is that they are formally ghost-free and hence this reduces the number and simplifies Feynman diagram calculations. In fact, some of the very first calculations in QCD were done in this gauge. There are however two main disadvantages : [1] the lack of manifest covariance [2] more importantly, the problem of interpreting the $1/\eta.q$ type singularities. The second problem does not exist in Lorentz type gauges ; The $1/q2$ type singularities are correctly dealt with by adding $i\epsilon$ to $q2$ Ad hoc prescriptions have been given : [a] The Principle Value Prescription [PVP][2] [b] The Mandelstam-Leibbrandt Prescription [MLP] [14]. But these inevitably run into difficulties of various kinds [15,16]. One prescription for a special kind of axial gauge $A_1 + \lambda A_3 = 0$ has been derived [17] in canonical quantization framework.

We expect that since the Lorentz type gauges have no prescription problem, if a transformation to axial gauges were available, we could *derive* the correct prescription for axial gauges. The FFBRS transformation could also enable us to express axial gauge Greens' functions in terms of *Lorentz* type greens' functions. This could in particular be used to remove the reported anomaly [7] in the anomalous dimensions n the two gauges. We follow the same procedure as outlined earlier. First we understand the $\eta.A = 0$ gauge as the $\lambda \to 0$ limit of the gauge with

$$S_{gf} = -1/(2\lambda)\int [\eta.A]^2 d^4x$$

[together with the corresponding ghost term].

Then we construct an intermediate gauge-fixing term

$$S = -1/(2\lambda)\int[(1-\beta)\partial.A + \beta\eta.A]^2 d^4x$$

[+ corresponding Ghost term].

From these, we make an *ansatz* for Θ' and S [18] :

$$\Theta' = i\gamma\int d^4y\bar{c}^{\eta}(y)(\partial.A - \eta.A)^{\eta}(y)$$

$$S_1[\phi(\kappa),\kappa] = \xi_1(\kappa)(\partial.A)^2 + \xi_2(\kappa)(\eta.A)^2 + \xi_3(\kappa)(\partial.A)(\eta.A)$$

$$+\xi_4(\kappa)\bar{c}Mc + \xi_5(x)\bar{c}\tilde{M}c$$

(All fields are here functions of κ: $A = A(x, \kappa)$ etc.)

and impose the Jacobian condition in Section 4. We then obtain as one possible solution the following :

$$\xi_1(\kappa) = [1-(1-\kappa)^2]/2\lambda \qquad ;\gamma = 1$$

$$\xi_2(\kappa) = -\kappa^2/2\lambda$$

$$\xi_3(\kappa) = \kappa(\kappa-1)/\lambda$$

$$\xi_4(\kappa) = \kappa = -\xi_5(\kappa)$$

This allows us to construct an explicit FFBRS from the Lorentz type gauges to the axial type gauges. We shall now outline one of the applications of the result ; *viz.* towards deriving the prescription for the $1/\eta.q$ type singularities in axial gauges. To exhibit our procedure [19], we note that the $1/q^2$ singularities in the Landau type gauges is correctly dealt with by the effective replacement $q^2 \rightarrow q^2 + i\varepsilon$. This amounts, in practice, to an addition of a term $-i\varepsilon A_\mu A^\mu / 2 + i\varepsilon \bar{c}c$ to the $\tilde{S}^L_{eff}$. This takes care of the singularity problem in the Lorentz type gauges. We then start with :

$$\tilde{S}^L_{eff} = S^L_{eff} - i\varepsilon A_\mu A^\mu / 2 + i\varepsilon \bar{c}c$$

and perform the FFBRS that has been constructed. Of course, S^L_{eff} will be invariant under this transformation. But the added terms in $\tilde{S}^L_{eff}$ will now generate new non-trivial and nonlocal terms $\varepsilon\delta S$ [of order ε]. Thus, taking account of the Jacobian terms $\tilde{S}^L_{eff}$ $[\phi]$ transforms into $\tilde{S}^A_{eff}$ $[\phi']$ where

$$\tilde{S}^A_{eff}[\phi'] = S^A_{eff}[\phi'] - i\varepsilon A'_\mu A'^\mu /2 + i\varepsilon \bar{c}'c' + \varepsilon\delta S.$$

The effect of the newly found term on the propagator is to be evaluated. We expect this term to give the manner in which the poles in the propagator are shifted away from the real axis. This work is in progress [19].

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