S-WAVE NEUTRON STRENGTH FUNCTION, POTENTIAL SCATTERING RADIUS AND THE OPTICAL MODEL

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ABSTRACT. The S-wave neutron strength function $\frac{\bar{\Gamma}_n^0}{D}$ and the potential scattering radius R' have been studied using spherical optical model potential with (i) pure surface absorption and (ii) combined volume and surface absorption. The numerical results have been plotted against mass numbers over the region $A = 40$ to $A = 200$ and have been compared with the experimental data.

INTRODUCTION

The nuclear optical model has been found to give satisfactory explanation in predicting the resonance behaviour of S-wave neutron strength function $\frac{\bar{\Gamma}_n^0}{D}$, the average ratio of neutron width to average level spacing, with respect to atomic mass at low energies. The neutron strength function is a measure of the average cross section for formation of the compound nucleus in the low energy phenomena. The earliest calculations (Feshbach *et al*, 1954) have been carried out with spherically symmetric complex potential of the simple square well form and the ratio shows emhanced rises near mass numbers $A \sim 55$ and $A \sim 155$ and a minimum around $A \sim 100$. Though a qualitative agreement has been obtained between the theoretical prediction and the actual measured values but the agreement unsatisfactory quantitatively since the theoretical results give more sharp and narrow reasonances and more pronounced dip of $\frac{\bar{\Gamma}_n^0}{D}$ with the variation of mass numbers when compared with the experimental findings. However, later investigations (Feshbach, 1958) with realistic, diffused potentials of Woods-Saxon type (1954), with the same form of uniform radial distribution of the real and imaginary part, have reproduced the giant resonances fairly well showing better agreement with experiment. But there still remain two major discrepancies to be explained. One of these concerns the peak near $A \sim 155$ where the experimental curve seems to be much broader, lower and more irregular than the theoretical curve and indicates the existence of two small peaks. The other discrepancy has been noticed close to the valley between the reasonances where the values of the strength function predicted by theory are considerably high in comparison with the observed data.

As regards the first difficulty it has been suggested that the non-spherical shape of target nuclei in the region $140 < A < 190$ should be taken into account in order to avoid the marked disagreement with experimental observations. Investigations by many authors (Margolis *et al* 1957; Chase *et al*, 1958 and Jain 1964) have shown successfully that the aspherical nature of nuclear structure has a tendency to break the giant resonance into a number of resonances, thereby improving the agreement with experiment.

To eliminate the second difficulty the interesting suggestion to introduce enhanced surface absorption in place of volume absorption has already been put forward and several attempts have been made to clarify the point. For low incident energies the absorption may be taken to be concentrated mainly in nuclear surface rather than distributed uniformly over the entire nucleus because of the weakened effect of Pauli exclusion principle in the surface region and this is represented by the surface peaked radial distribution of the imaginary potential (Harada *et al*; 1959)

In the present work we have investigated the detailed structure of $\frac{\bar{\Gamma}_n^0}{D}$ and potential scattering length R' taking into account (i) pure surface absorption and (ii) combined volume and surface absorption. The numerical results of our theoretical calculations which are based on the analytical expressions of $\frac{\bar{\Gamma}_n^0}{D}$ (normalised to 1 ev) and R' given in our previous note (Ganguly *et al*, 1966) are plotted against mass numbers over the region $A = 40$ to $A = 200$ for comparison with experimental data.

Our results are in agreement with the calculation of Khanna and Tang (1959), Fiedeldey and Frahn (1961) and Jain (1964) and support the argument that surface concentration of the imaginary potential lowers the value of the strength function in the region $90 < A < 130$ thereby bringing the results in closer agreement with experiment. We disagree with the conclusions of Elagin *et al* (1962) who failed to reproduce any new effect due to surface absorbing potential beyond that for pure volume absorption.

THEORY

The radial wave equation for the scattering of S-wave neutrons is given by

$$\frac{d^2u_0}{dr^2}+\left[k^2 - \frac{2m}{\hbar^2}V(r) \right] u_0(r) = 0 \qquad \ldots (1)$$

where $V(r)$ is the interaction potential and $k^2 = \frac{2mE_n}{\hbar^2}$, m and E_n being respectively the mass and energy of the incident neutron. The wave function $u = r\psi$ satisfies the necessary boundary conditions for the scattering problem at $r = 0$ and $r \to \infty$. In the asymptotic region, $r \to \infty$ the radial solution as a combination of incoming and outgoing waves is given for $l = 0$

$$u_0(r) = G[u_0^{(-)}(r) - S_0 u_0^{(+)}(r)] \quad \ldots \quad (2)$$

where G is a constant.

At sufficient low neutron energies the scattering amplitude S_0 averaged over resonances according to Feshbach, Porter and Weisskopf (1954) has the value

$$S_0 = e^{-2ikR'}\left(1 - \frac{\pi\overline{\Gamma}_n}{D}\right) \quad \ldots \quad (3)$$

where R' is a length of the order of magnitude of the nuclear radius and is a slowly varying function of the energy.

Hence for $\quad kR' << 1$

$$\frac{\overline{\Gamma}_n{}^0}{D} = \frac{1}{\pi} \operatorname{Re}(1 - S_0) \quad \ldots \quad (4)$$

$$kR' = \tfrac{1}{2} \operatorname{Im}(1 - S_0) \quad \ldots \quad (5)$$

The averaged cross sections of scattering and absorption are related to $\frac{\overline{\Gamma}_n{}^0}{D}$ and R' as follow :

$$\overline{\sigma}_{se} = \frac{\pi}{k^2} \,|1 - S_0|^2 \simeq 4\pi R'^2 \quad (6a)$$

$$\overline{\sigma}_a = \frac{\pi}{k^2}\left(1 - |S_0|^2\right) \simeq \frac{2\pi^2}{k^2} \frac{\overline{\Gamma}_n}{D} \quad \ldots \quad (6b)$$

We choose the neutron-nucleus potential to be of the form

$$V(r) = -\left[(V_0 + iW_0)\frac{1}{1 + e^{\frac{(r-R)}{a}}} + 4iW_1 \frac{e^{\frac{(r-R)}{a}}}{\left[1 + e^{\frac{(r-R)}{a}}\right]^2}\right] \quad \ldots \quad (7)$$

where a is the diffusivity parameter and the radius parameter R' is assumed as $R = 1.25A^{1/3}$. Corresponding to this form of nuclear potential which takes care of both volume and surface absorption we have derived (1966) the following expression for S-wave neutron strength function normalised to 1 ev and potential scattering radius R'

$$\frac{\overline{\Gamma}_n{}^0}{D} = -\frac{2k_0 a}{\pi} \operatorname{Im}(Q) \quad \ldots \quad (8)$$

$$R' = R + 2\gamma a + a\operatorname{Re}(Q) \quad \ldots \quad (9)$$

where γ is Euler's constant and the complex quantity Q is given by

$$Q = [\psi(\lambda_0+\mu)+\psi(-\lambda_0+\mu)]+\pi.$$

$$\frac{\left[\left(\frac{b}{1+b}\right)^{\lambda_0} F\left(\lambda_0+\mu, 1+\lambda_0-\mu, 1+2\lambda_0 ; \frac{b}{1+b}\right)\frac{\Gamma(1-2\lambda_0)}{\Gamma(1-\lambda_0-\mu)\Gamma(-\lambda_0+\mu)}. \cot.\{\pi(\lambda_0+\mu)\} - \left(\frac{b}{1+b}\right)^{-\lambda_0} F\left(-\lambda_0+\mu, 1-\lambda_0-\mu, 1-2\lambda_0 ; \frac{b}{1+b}\right) \frac{\Gamma(1+2\lambda_0)}{\Gamma(1+\lambda_0-\mu)\Gamma(\lambda_0+\mu)} \cot.\{\pi(-\lambda_0+\mu)\}\right]}{\left[\left(\frac{b}{1+b}\right)^{\lambda_0} F\left(\lambda_0+\mu, 1+\lambda_0-\mu, 1+2\lambda_0 ; \frac{b}{1+b}\right)\frac{\Gamma(1-2\lambda_0)}{\Gamma(1-\lambda_0-\mu)\Gamma(-\lambda_0+\mu)} - \left(\frac{b}{1+b}\right)^{-\lambda_0} F\left(-\lambda_0+\mu, 1-\lambda_0-\mu, 1-2\lambda_0 ; \frac{b}{1+b}\right) \frac{\Gamma(1+2\lambda_0)}{\Gamma(1+\lambda_0-\mu)\Gamma(\lambda_0+\mu)}\right]}$$

in which $\lambda = \pm[-a^2(k^2+p^2)]^{\frac{1}{2}}, \quad \mu = \frac{1}{2}\pm\frac{1}{2}[1+4a^2q^2]^{\frac{1}{2}}$

$p^2 = \frac{2m}{\hbar^2}(V_0+iW_0), q^2 = \frac{2m}{\hbar^2}(iW_1), \ b = \exp(-R/a)$ and

λ_0 is the value of λ at $k = 0$ F denoting hypergeometric functionsl.

For numerical computation we note that $b \ll 1$ in the range of mass numbers we are interested so that the hypergeometric functions reduce to 1 and

$$\left(\frac{b}{1+b}\right)^{\pm\lambda_0} \approx e^{\mp ipR}$$

RESULTS AND DISCUSSION

Fig. 1 and 2 give the calculated values of $\frac{\bar{\Gamma}_n{}^0}{D}$ and R' respectively plotted against mass numbers in the region $A = 40$ to $A = 200$ for different shapes of the imaginary potential, for comparison the corresponding experimental data (as given in the paper of Perey *et al*, 1962) are also shown. The values of our parameters are the same as given by Jain (1964) viz.

$$V_0 = 52 \text{ Mev}, \ a = 0.52 \text{ fm} \ \text{ and } \ R = 1.25A^{1/3}\text{fm}.$$

W has been suitably chosen by us as $W = 3.5$ Mev for pure surface absorption and $W_0 = W_1 = 2.30$ Mev for volume plus surface absorption. W have taken the same diffusivity parameter for both the real and imaginary parts of the potential.

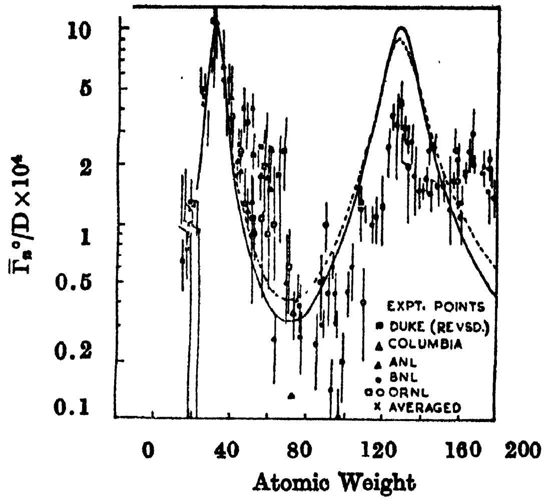

Fig. 1. Calculated values of S-wave neutron strength function, $\bar{\Gamma}_n^0/D$ as a function of mass number, compared with experimental data. The solid curve is for pure surface absordtion and the dotted curve corresponds to combined volume and surface absorption.

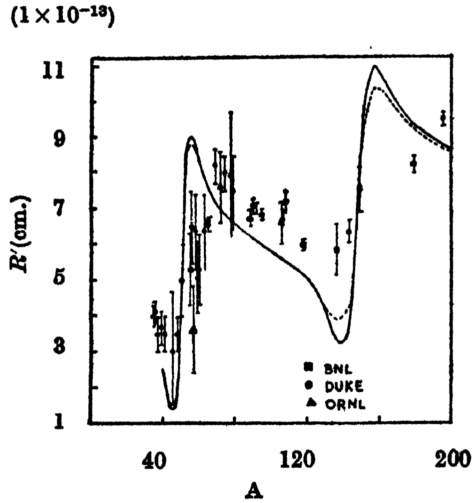

Fig. 2. Calculated potential scattering radius R' as a function of mass number compared with experiment. The solid curve and dotted curve correspond to pure surface absorption and combined volume and surface absorption respectively.

In order to show the effect of surface absorption on the strength function we compare cases of pure surface absorption and combined volume and surface absorption, the real part of the potential being taken to be of Woods—Saxon type with the same parameters for all the calculations. Neutron strength function in the case of pure volume absrption with uniform radial distribution of Woods-Saxon type has been investigated by many authors (Feshbach, 1958 : Ghosh *et al*, 1961) and found to be in fair agreement with experimental data near the giant resonance $A \sim 50$ while at the minimum the theoretical values are significantly higher than the measured values.

In fig. 1, we have kept the peak heights at $A \sim 50$ almost the same in both the cases of pure surface absorption and volume plus surface absorption by adjusting the strength of the imaginary potential while all other potential parameters left unchanged. We have found that the diminution of the strength of the imaginary potential lowers the value of minima and raises the heights of the peaks simultaneously while the opposite results are obtained when the strength is increased. The main effect of surface absorption has been shown in the vicinity of the minimum at $A \sim 91$ where the calculated values of $\frac{\bar{\Gamma}_n^0}{D}$ are smaller in the case of pure surface absorption compared with that in the case of combined volume and surface absorption. The two maxima of $\frac{\bar{\Gamma}_n^0}{D}$ in the case of pure surface absorption are equal in height on account of the periodicity of our expression for $\frac{\bar{\Gamma}_n^0}{D}$ in R unlike the case for volume plus surface absorption (also for pure volume absorption). As regards the second maximum at $A \sim 148$ we notice that unlike the theoretical curve the experimental curve indicates the existence of two small peaks rather than a smooth peak and the measured points are more irregular, lower and scattered. This disagreement may be explained by taking into account the deformed structure of nuclei in that region.

The fig. 2 gives the results of theoretical calculations for R', the potential scattering radius, compared with the experimental data (as quoted by Perey *et al*, 1962). We see that the theoretical curve for pure surface absorption gives deeper minima and higher maxima that that for combined volume and surface absorption. The agreement with experiment is fairly good in the region $A = 40$ to 150. The results of calculations with combined volume and surface absorption give somewhat better agreement with experiment than those with pure surface absorption. Above $A = 150$ the theoretical curves deviate much from experimental findings, the discrepancy may be due to aspherical shape of nuclei in that region.

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