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A temperature anisotropic instability in a two component bi-Lorentzian plasma

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Abstract : A generalized Kappa distribution function is useful for modelling plasma distributions with high energy tails In this paper, we investigate the nature of convective instability for a right circularly polarized electromagnetic wave in a plasma containing a hot and a cold electron species, both modelled by Kappa distributions The cold, temperature isotropic electrons drift with a velocity v_d with respect to the hot, anisotropic electron component. An expression for the growth rate, valid for any finite value of the spectral index κ, has been derived Growth rates were then calculated for various values of κ (4,6), temperature anisotropy $\frac{T_\perp}{T_\parallel}$, density ratios of cold to hot electrons and drift velocities.

We find that the growth rate, which is restricted to wave frequencies well below the electron gyrofrequency, increases dramatically at the lower frequency end as κ decreases or as the high energy tail becomes more pronounced While in all cases the source of instability is temperature anisotropy of the hot species, for non-zero values of drift velocity v_d instability increases with increasing cold electron density. However, when the drift velocity is zero, the instability decreases with increasing cold electron density A comparative study of this instability has also been made in bi-Lorentzian and Maxwellian plasmas

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1. Introduction

Plasmas occurring naturally in space are generally non-Maxwellian; they are often characterised by an energetic tail distribution [1,2]. A useful distribution function to model such plasmas is the generalised Lorentzian or Kappa distribution. The well known

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Maxwellian and Kappa distributions differ substantially in the high energy tail region; the drop towards zero is much more abrupt for a Maxwellian distribution when compared to that of a Kappa distribution with a low spectral index κ. This difference is however less significant as κ increases. When compared to particles with a Maxwellian distribution, such high energy particles can enhance the growth of plasma waves, especially when the phase velocity of the wave is large compared to the thermal bulk velocity of the plasma. Such conditions commonly occur in space and other magnetospheric plasmas and Kappa distributions have been used to analyse and interpret spacecraft data in the Earth's magnetospheric plasma sheet [1,2], the solar wind [3,4], Jupiter [5] and Saturn [6]. In fact it has been found that, in practice, many space plasmas can be modelled more efficiently by a superposition of Kappa distributions than by Maxwellians.

The analysis of resonant interactions with a high energy component has been investigated by a few authors [3–5]. More recently, Thorne and Summers [7] have carried out a more accurate and systematic study of the controlling influence of high energy particles on wave growth. They have compared the growth of electromagnetic waves in a Maxwellian plasma with that for a bi-Lorentzian or Kappa distribution with comparable bulk properties such as density and temperature.

The measured electron data in the solar wind has been successfully represented by two components— a core component which has been observed to be nearly Maxwellian and a halo component which is less Maxwellian in nature and more importantly, as mentioned above, a Kappa distribution has been successfully used to model the solar wind particle distributions. Besides, these two components have been observed to drift relative to each other and other constituents of the plasma along the magnetic field [3]. We have in this paper, thus studied the stability of right circularly polarised electromagnetic waves propagating parallel to the magnetic field, in a two component electron plasma. In order to simplify the algebra, we have modelled both the components using a bi-Lorentzian (or Kappa) distribution : a hot component which exhibits a temperature anisotropy while the other is temperature isotropic and drifts with a velocity v_d relative to the hot component.

Thorne and Summers [8] had considered beam driven ion right hand modes keeping the wave vector k real. We too consider the stability of right circularly polarised electromagnetic waves propagating parallel to the magnetic field. In contrast to their studies, ours is a two component electron plasma with a complex k (ω real).

Our expression for the growth/damping rate is valid for any finite value of κ. We find that temperature anisotropy of the hot species is the source of the instability, which is, however, restricted to wave frequencies well below the electron gyrofrequency. Once excited the wave growth is increased manifold due to the presence of the second, drifting component. This growth, however, decreases sharply as the spectral index κ increases or as the high energy tail becomes less pronounced. A comparison has been made with the corresponding growth rate in a similar plasma but described by Maxwellian distributions. While there is a small drop in the growth rate in Maxwellian plasmas, the other gross features of the instability are the same in both the plasmas.

2. The dispersion relation

We are interested in the stability of the right circularly polarised electromagnetic waves propagating parallel to the magnetic field. The dispersion formula for the propagation of these waves is well known and is given by [8]

$$n^2 = 1 + \frac{\pi\omega_p^2}{k\omega}\int_0^\infty dv_\perp \int_{-\infty}^{\infty}\frac{dv_\parallel}{v_\parallel - v_r} I, \tag{1}$$

where
$$I(v_\perp, v_\parallel) = -v_\perp^2\frac{\partial F}{\partial v_\perp} + \frac{k}{\omega}v_\perp^2\left[v_\parallel\frac{\partial F}{\partial v_\perp} - v_\perp\frac{\partial F}{\partial v_\parallel}\right]$$

and
$$v_r = \frac{\omega - \omega_c}{k}.$$

In (1) n is the refractive index and ω_p, the plasma frequency while ω and k are the wave frequency and wave vector respectively.

As mentioned above, the observed electron distribution in the high speed solar wind is best described in terms of two distinct components drifting relative to each other along the magnetic field. The core component is nearly Maxwellian while the halo component is less Maxwellian in nature. Since a Kappa distribution has been successfully used to model the solar wind particles we choose the distribution function to be given by

$$F = \frac{1}{\pi^{3/2}}\sum_{h,d}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\frac{1}{\theta_\perp^2\theta_\parallel}\left[1 + \frac{v_\perp^2}{\kappa\theta_\perp^2} + \frac{\left(v_\parallel - v_d\right)^2}{\kappa\theta_\parallel^2}\right]^{-(\kappa+1)} \tag{2}$$

The distribution function F is a summation over the hot (h) and drifting (d) components. In (2), κ is the spectral index and v_d, the velocity of drift along the magnetic field. For the hot anisotropic component ($\theta_\parallel \neq \theta_\perp$) and $v_d = 0$ while for the drifting component $\theta_\parallel = \theta_\perp = \theta_d$ and $v_d \neq 0$. $\theta_\parallel$ and $\theta_\perp$ are related to the mass and $T_\parallel$ and $\theta_\perp$ the temperatures parallel and perpendicular to the magnetic field respectively by

$$\theta_\parallel^2 = \left[\frac{\kappa-\frac{3}{2}}{k}\frac{2k_BT_\parallel}{m}\right] \quad \text{and } \theta_\perp^2 = \left[\frac{\kappa-\frac{3}{2}}{k}\frac{2k_BT_\perp}{m}\right]. \tag{3}$$

Substituting (2) into (1), the $dv_\parallel$ integration can be carried out using a modified form of the plasma dispersion function defined by [9]

$$Z_\kappa^*(\zeta) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\int_{-\infty}^{\infty}\frac{ds}{(s-\zeta)\left(1+\frac{s^2}{\kappa}\right)^{\kappa+1}}. \tag{4}$$

We then get the dispersion relation as

$$c^2k^2 = \omega^2 + D_h + D_d, \tag{5}$$

where $$D_h = \omega_{ph}^2 \frac{\omega}{k}\left\{\left[1 - \frac{kv_\iota A}{\omega}\right]\left[\frac{Z_\kappa^*(v_\iota / \theta_{\parallel h})}{\theta_{\parallel h}}\left(1 + \frac{v_\iota^2}{\kappa\theta_{\parallel h}^2}\right) + \frac{v_\iota\left(\kappa - \frac{1}{2}\right)}{\kappa^2\theta_{\parallel h}^2}\right] - \frac{Ak}{\omega}\right\}$$

and $$D_d = \omega_{pd}^2 \frac{\omega}{k}\left\{\left[1 - \frac{kv_d}{\omega}\right]\left[\frac{Z_\kappa^*\left(\frac{v_\iota - v_d}{\theta_d}\right)}{\theta_d}\left(1 + \frac{(v_\iota - v_d)^2}{\kappa\theta_d^2}\right) + \frac{(v_\iota - v_d)\left(\kappa - \frac{1}{2}\right)}{\kappa^2\theta_d^2}\right]\right\}.$$

The temperature anisotropy factor is defined by : $A = \left[1.0 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right]$.

The dispersion relation (5) has, in general, complex solutions. We shall study the case of convective instability and keep ω real and fixed. The wave vector will then in general be complex having the form $k = k_r + ik_i$.

3. Dispersion formulae

3.1. Kappa distribution function :

As mentioned above, we are interested in the parallel propagation of electromagnetic waves The wave vector k is therefore large; consequently the argument of the modified plasma dispersion function is small. We thus need the power series expansion for Z_κ However, unlike the plasma dispersion function of Fried and Conte [10], the power series expansion for Z_κ^* varies with the spectral index κ and these are given for kappa = 1 to 6 in [9] Generalising these, we have, the power series expansion for any finite κ as

$$Z_\kappa^*(\zeta) = \frac{\kappa!\sqrt{\pi}i}{\kappa^{3/2}\Gamma\left(\kappa - \frac{1}{2}\right)}\left[1 - \frac{(\kappa+1)}{\kappa}\zeta^2 + \frac{(\kappa+1)(\kappa+2)}{2\kappa^2}\zeta^4\right]$$
$$- \frac{(2\kappa-1)(2\kappa+1)}{2\kappa^2}\zeta\left[1 - \frac{(2\kappa+3)}{3\kappa}\zeta^2 + \frac{(2\kappa+3)}{3\kappa}\frac{(2\kappa+5)}{5\kappa}\zeta^4\right]. \quad (6)$$

Substituting (6) into (5) and remembering that k is complex allows us to separate the dispersion relation (5) into its real and imaginary components. The relevant expressions are

$$\frac{c^2k_r^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2}\left\{\varepsilon\left[\left[\frac{\omega}{k_r\theta_d} - \frac{v_d}{\theta_d}\right]\left[-\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma\left(\kappa - \frac{1}{2}\right)}2\zeta_{rd}\zeta_{id}\left(1 - \frac{\kappa+1}{\kappa}\zeta_{id}^2\right)\right.\right.\right.$$
$$\left.\left. - \frac{2\kappa-1}{\kappa}\zeta_{rd}\left[1 - \frac{2\kappa+1}{3\kappa}\zeta_{rd}^2 + \frac{(2\kappa+3)(2\kappa+1)}{15\kappa^2}\zeta_{rd}^4\right]\right]\right.$$
$$\left. + \frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma\left(\kappa - \frac{1}{2}\right)}\frac{\omega}{k_r\theta_d}\frac{k_i}{k_r}\left[1 - \zeta_{rd}^2 + \frac{\kappa+1}{2\kappa}\zeta_{rd}^4\right]\right]$$

$$+\left[\frac{\omega}{k_r\theta_\parallel} - A\zeta_r\right]\left[\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\frac{k_i}{k_r}\left[1 - 3\zeta_r^2 + \frac{5}{2}\frac{(\kappa+1)}{\kappa}\zeta_i^4\right]\right.$$

$$\left.\left. - \frac{(2\kappa-1)}{\kappa}\zeta_r\left[1 - \frac{(2\kappa+1)}{3\kappa}\zeta_r^2 + \frac{(2\kappa+1)(2\kappa+3)}{15\kappa^2}\zeta_i^4\right]\right] - A\right\}$$

and
$$\frac{k_i}{k_r} = \frac{X}{Y}, \tag{7}$$

where
$$X = \varepsilon\frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\left[\frac{\omega}{k_r\theta_d} - \frac{v_d}{\theta_d}\right]\left(1 - \zeta_{rd}^2 + \frac{(\kappa+1)}{2\kappa}\zeta_{id}^4\right)$$

$$+ \frac{\kappa!\sqrt{\pi}}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\left[\frac{\omega}{k_r\theta_\parallel} - A\zeta_r\right]\left(1 - \zeta_i^2 + \frac{(\kappa+1)}{2\kappa}\zeta_i^4\right)$$

and
$$Y = \frac{2c^2k_r^2}{\omega_{ph}^2} - \varepsilon\left[\frac{\omega}{k_r\theta_d} - \frac{vd}{\theta_d}\right]\left[\frac{(2\kappa-1)}{\kappa}V_{cid}\left(1 - \frac{(2\kappa+1)}{\kappa}\zeta_{id}^2\right.\right.$$

$$\left.\left.+ \frac{(2\kappa+1)(2\kappa+3)}{3\kappa^2}\zeta_{rd}^4\right)\right] - \frac{(2\kappa-1)}{\kappa}\varepsilon\frac{\omega}{k_i\theta_d}\zeta_{id}\left(1 - \frac{(2\kappa+1)}{3\kappa}\zeta_{rd}^2\right.$$

$$\left.+ \frac{(2\kappa+1)(2\kappa+3)}{15\kappa^2}\zeta_{rd}^4\right) - \frac{2(2\kappa-1)}{\kappa}\zeta_i\left[\frac{\omega}{k_r\theta_\parallel} - A\zeta_i\right]$$

$$\times\left[1 - \frac{2}{3}\frac{(2\kappa+1)}{\kappa}\zeta_r^2 + \frac{1}{5}\frac{(2\kappa+1)(2\kappa+3)}{\kappa^2}\zeta_i^4\right]$$

with
$$V_{crd} = \frac{\omega - \omega_c}{k_r\theta_d}, \qquad \zeta_r = \frac{\omega - \omega_c}{k_r\theta_\parallel},$$

$$\zeta_{rd} = V_{crd} - (v_d/\theta_d) \text{ and } \zeta_{id} = V_{cid}k_i/k_r,$$

while $\varepsilon = n_d/n_h$ is the density ratio of drifting to hot electrons.

Expression (7) for the real and imaginary parts of the dispersion relation are very general in nature and can be used for any finite value of κ. Our studies for a complex k, thus complement the earlier studies for a complex ω.

3.2 *Anisotropic Maxwellian distribution :*

A popular and until now mathematically more convenient distribution function that has been frequently used to study temperature anisotropy driven instabilities is the anisotropic Maxwellian distribution. Thus for purposes of a comparative study, we consider in this section, the dispersion relation for right circularly polarised electromagnetic waves in Maxwellian plasmas. Our plasma now consists of a hot, anisotropic component and a colder, isotropic drifting component; both species being modelled by Maxwellian distributions. The distribution is thus given by

$$F = \frac{1}{\pi^{3/2}} \sum_{h,d} \frac{1}{\theta_\perp^2 \theta_\parallel} \exp\left[- \frac{v_\perp^2}{\theta_\perp^2} - \frac{(v_\parallel - v_d)^2}{\theta_\parallel^2} \right], \tag{8}$$

where $\theta_\perp^2 = \left[\frac{2k_B T_\perp}{m}\right]$ and $\theta_\parallel^2 = \left[\frac{2k_B T_\parallel}{m}\right]$.

In (8) as in the case of Kappa distribution, $\theta_\parallel \neq \theta_\perp$ and $v_d = 0$ for the hot, anisotropic component while for the drifting component $\theta_\perp = \theta_\parallel = \theta_d$ and $v_d \neq 0$.

Substituting (8) into the dispersion formula (1) and carrying out the $dv_\parallel$ and $dv_\perp$ integrations, we get the final dispersion relation as

$$c^2k^2 = \omega^2 + D_h + D_d, \tag{9}$$

where $$D_h = \omega_{ph}^2 \frac{\omega}{k}\left[\left[1 - \frac{kv_c A}{\omega}\right]\left[\frac{Z(v_c/\theta_\parallel)}{\theta_\parallel}\right] - \frac{Ak}{\omega}\right]$$

and $$D_d = \omega_{pd}^2 \frac{\omega}{k}\left[1 - \frac{kv_d}{\omega}\right]\left[\frac{Z\left(\frac{v_c - v_d}{\theta_d}\right)}{\theta_d}\right]$$.

where Z is the plasma dispersion function of Fried and Conte [10]; the other terms having the same meaning as in relation (5).

Expressions similar to (7) can be derived for the real and imaginary parts of the dispersion relation (9) using the power series expansion of Z [10]. However, since their derivation is straight forward we shall not give them here.

4. Results

We consider model parameters of the solar wind, namely $B_0 = 7.3\ \gamma$, $T_{\parallel h} = 0.53 * 10^6$ °K, $\theta_{\parallel d} = 0.6 * \theta_{\parallel h}$ and $n_h = 4.0$ [3,7].

Relations (7) were then solved self-consistently in an iterative manner and values obtained for k_r and k_i/k_r, the real and normalised imaginary parts of the wave vector k. A negative k_i indicates an instability and wave growth; a positive k_i represents wave damping.

Figure 1 is thus a plot of k_i/k_r *versus* ω/ω_c as a function of the anisotropy factor $T_\perp / T_\parallel = 2.5$, 5.0 and 10.0 in a hot electron plasma with the spectral index $\kappa = 4$. We find that for low values of temperature anisotropy (2.5 and 5.0) the wave is partially unstable in the region of interest (k_i/k_r is negative and indicated by dotted lines in the figure). For $T_\perp / T_\parallel = 10.0$, the wave is fully unstable in this range and as is evident from the figure the range of the instability and the growth rate increases with increasing temperature anisotropy. Thus, the instability is temperature-anisotropy driven in a one component plasma.

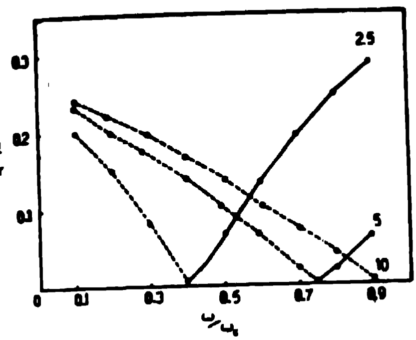

Figure 1. Plot of k_i/k_r *versus* ω/ω_c as a function of temperature anisotropy $T_\perp/T_\parallel$ = 2.5, 5.0 and 10.0 in a one component plasma. The dotted lines indicate wave instability

Figure 2 is a plot similar to Figure 1 but in a two component plasma with $T_\perp / T_\parallel$ = 10 and $\kappa = 4$ as a function of the density ratio (ε) and drift velocity v_d. Figure 2(a) thus depicts the variation of k_i/k_r with frequency for two values of ε, namely 0 (also depicted in Figure 1) and 5 but with $v_d = 0.0$. We find that the instability of Figure 1 is suppressed by

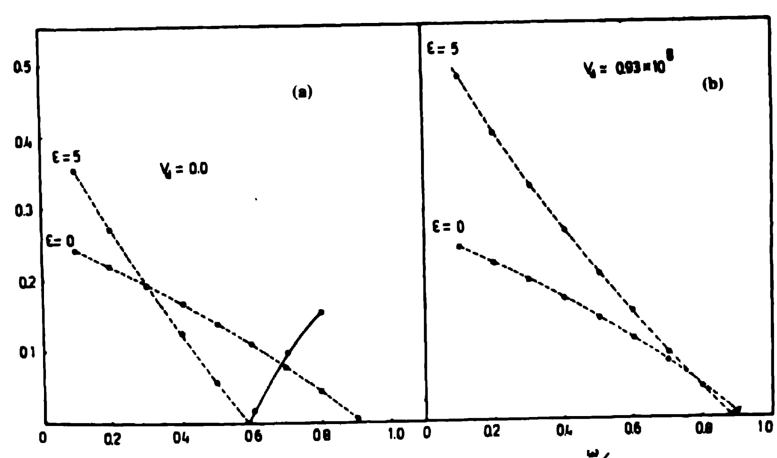

Figure 2. Plot of k_i/k_r *versus* ω/ω_c for $T_\perp/T_\parallel$ = 10.0 as a function of the drifting to hot electron density ratio (ε = 0 and 5.0). (a) depicts the wave instability (indicated by dotted lines) when the cold component is stationary, (b) depicts the situation when $v_d = 0.93 * 10^8$ cms^{-1}.

the higher density, isotropic Lorentzian component; this damping increasing with ε. This is probably due to the Landau damping by the temperature-isotropic, colder component.

Figure 2(b) shows that when these electrons have a finite drift velocity ($v_d = 0.93 * 10^8$ cms^{-1}), the growth rate of the waves increases dramatically in the region of interest. For a given value of the drift velocity, the instability increases with increasing ε. It starts from large values, drops to zero at a critical frequency depending on ε and thereafter the wave is damped.

Figures 3 also depicts k_i/k_r against frequency for $T_\perp/T_\parallel = 10$, $\varepsilon = 10$ as a function of v_d (= $0.83 * 10^8$ and $0.93 * 10^8$ cms^{-1}). It is intended for a comparative study of the instability in plasmas described by the Kappa and anisotropic Maxwellian distributions and

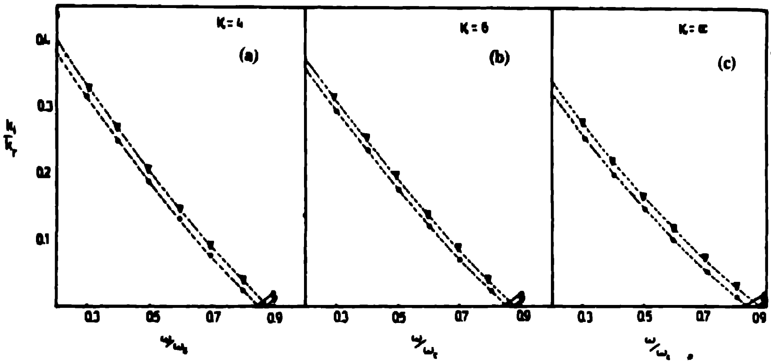

Figure 3. Plot of k_i/k_r *versus* ω/ω_L for $T_\perp/T_\parallel = 10.0$, $\varepsilon = 10.0$ for two values of v_d (= $0.83 * 10^8$ and $0.93 * 10^8$ cms^{-1}) in a plasma described by a bi-Lorentzian (plots (a) and (b), $\kappa = 4.0$ and 6.0 respectively) and Maxwellian (plot (c)) distribution ($v_d = 0.83 * 10^8$ cms^{-1} indicated by • and $v_d = 0.93 * 10^8$ cms^{-1} by ∇)

is a plot of the normalised imaginary parts of the dispersion relations (5) and (9). Figure 3(a) thus corresponds to $\kappa = 4$, 3(b) to $\kappa = 6$ and Figure 3(c) to the anisotropic Maxwellian distribution. A comparison of the three figures show that the instability increases with increasing velocities of the drifting component More importantly, however, the growth rate decreases with increasing spectral index κ or as the high energy tail becomes less pronounced. This result is in agreement with the complex ω case dealt with by Thorne and Summers [7] and abnormal levels of wave activity at very low frequencies has the simple and attractive explanation of being due to particle distributions that deviate significantly from the anisotropic, Maxwellian distribution.

As mentioned above, relations (7) are valid for any finite value of κ and hence the computations were carried out for a wide range of the spectral index. We find that the growth rate has a tendency to saturate, it being almost insensitive to κ beyond 20.

5. Conclusions

We have, in this paper, studied the convective instability of electromagnetic waves in a two component bi-Lorentzian plasma. The instability which is restricted to frequencies below the electron gyrofrequency, is driven by temperature anisotropy in a one component plasma. A stationary, second electron component suppresses this instability; however when this component has a finite drift velocity, the wave growth is increased. The wave growth drops sharply when the spectral index κ is increased; it, however, tends to saturate when κ is increased beyond a certain value. Also except for this decrease in the growth rate, the other features of the instability are nearly the same in both the Lorentzian and Maxwellian plasmas.

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