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Research in radiation physics using energetic electrons and photons — some recent trends

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Abstract: Studies on interaction of photons and electrons with matter have been pursued vigorously over the last few decades and have contributed immensely to our understanding on the structure of matter. The results obtained from these studies are also finding important applications in industry. The availability of high resolution semiconductor detectors and sophisticated electronics in the recent past have facilitated conducting precision experiments on several important aspects of the interaction of photons with matter. Consequently, accurate data on cross sections for various processes have been obtained. On the theoretical side, the availability of fast computers and relevant software techniques have made it possible to compute the cross sections with a higher degree of accuracy. Some of these developments are discussed in this paper with a particular reference to atomic form factors and incoherent scattering functions based on the studies conducted in our laboratory.

In order to facilitate undertaking advanced level research and development programs in radiation physics using energetic electrons and photons, a variable energy microtron that accelerates electrons in the energy range of 4-12 MeV has been installed at our University recently. The salient features of this new facility are presented. Since high-energy electrons are finding increasing applications in medicine and in industry, undertaking of systematic studies using high-energy electrons is very important. R&D programs on radiation dosimetry, bremsstrahlung process and engineering of materials proposed to be undertaken using the microtron facility are outlined and possible applications of the results are discussed.

Keywords: elastic scattering, inelastic scattering, form factors, microtron

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1. Introduction

The interaction of energetic photons with matter, which is the designated theme of the workshop, represents one of the most varied classes of phenomena in the whole of experimental physics in general and radiation physics in particular. The studies in this field are important not only because of the light they throw on many fundamental aspects of physics but also because of important applications in industry, medicine and agriculture. An attempt is made in this paper to present a brief account of the research studies carried out on some of the aspects of this subject with a particular reference to work carried out recently in our laboratory on coherent and incoherent scattering. The R&D programs planned employing energetic photons and electrons using microtron accelerator installed in our university are also outlined.

Although there are as many 12 different processes [1] by which energetic photons interact with matter, in the energy domain of common interest, 0.1 to 10 MeV, the predominant processes are photoelectric effect, photon scattering and pair production. In the intermediate energy range, of about 0.1 to 1 MeV, photon scattering is the predominant process of γ-ray interaction. In this process incident photons are scattered by atomic electrons with or without energy loss and correspondingly we have the inelastic scattering and the elastic scattering processes.

2. Elastic scattering

The elastic scattering of γ rays by atoms occurs mainly through the coherent contribution of the four component processes. Rayleigh scattering, nuclear Thomson scattering, Delbrück scattering and nuclear resonance scattering. However, in the incident energy range up to 1 MeV and scattering angle corresponding to small and intermediate momentum transfer, only Rayleigh scattering makes significant contribution to the elastic coherent scattering.

There are mainly two approaches [2] to obtain theoretically detailed description of the coherent (Rayleigh) scattering process: (i) numerical partial wave calculation of elastic scattering amplitude using second-order S-matrix and (ii) form-factor formalisms. In the numerical calculation method the computer codes are very lengthy and requires many hours of computer time even on fast machines. Also in this method not all subshell contributions are calculated directly and are based on the independent electron model in which full non-local exchange and many electron correlations are not included. The environment of the outer electrons and even the widths of levels are not included. Due to these difficulties and drawbacks the presently available theories on coherent scattering are based on the form-factor formalism. The form factor is defined as the Fourier transformation of electron charge density and gives a measure of the distribution of electron charge density within in the atom. The form-factor predictions are relatively easy to calculate since these involve only the evaluation of the integral over the atomic electron charge distribution. The angular distribution for coherent scattering is expressed as the product of the distribution function of classical Thomson scattering and square of the atomic form factor. Hence differential cross sections for coherent scattering can be obtained theoretically from the knowledge of form factors wherever experimental values are not available and vice versa.

Form-factor derivations assume essentially two things: (i) electrons are loosely bound to atom; and (ii) the photon energy is much greater than the electron binding energy. The form factor is expressed [3,4] as

$$f(\vec{q}, Z) = \int \rho(\vec{r}) \exp(i\vec{q} \cdot \vec{r})\, d\vec{r}. \tag{1}$$

For a spherically symmetric atom it becomes [5,6]

$$f(q, Z) = 4\pi \int_0^\infty \rho(r) \frac{\sin(qr)}{qr} r^2 dr. \tag{2}$$

2.1 Non-relativistic form factors

Using the solution of Schrödinger wave equation for a non-relativistic hydrogen-like atom of charge Ze, the form factor is written as

$$f(x, 1) = (1 + 4\pi^2 a_0^2\, x^2)^{-2} \tag{3}$$

where the momentum transfer variable

$$x = \lambda^{-1} \sin\frac{\theta}{2} \tag{4}$$

and λ is the photon wavelength expressed in units of Å^{-1} ($\lambda[\text{Å}^{-1}] \approx 12.398/E[\text{keV}]$), and $a_0 = \hbar^2/me^2$ is the first Bohr radius. For many electron atoms, exact solution for the atomic form factors are not directly obtainable and therefore a variety of approximations have been used [7-12]. Using the available state of the art theoretical data Hubbell et al. [13] have presented tables of non relativistic atomic form factors extending over the range $0 \leq x \leq 10^9\ \text{Å}^{-1}$ for all elements Z = 1 to 100.

2.2 Relativistic form factors

The relativistic Hartree-Fock equations which provide accurate generally calculable atomic wave functions, were derived by Swirles [14] and Grant [15]. Solving the equation of Grant numerically, Coulthard [16] obtained the relativistic Hartree-Fock self consistent field for a number of atoms. Using these wave functions, relativistic form factors have been computed by a number of investigators [17-20] yielding extensive tabulations of relativistic atomic form factors.

2.3 Modified form factors

It has been shown [21,2] that the conjecture that the Thomson amplitude represents the high-energy limit is wrong. Consequently the form factor can not be valid in the forward direction at high energies and this has resulted in the modified factor formalism. The modified form factor g(q) is given by [3]

$$g(\bar{q}, Z) = \int \Psi^* \Psi \frac{mc^2}{E - V(r)} \exp(i\bar{q} \cdot \bar{r})\, d\bar{r}, \tag{5}$$

where E is the relativistic total energy of a bound electron and $V(r)$ is the central potential.

Schaupp et al. [22] have presented an extensive tabulation of relativistic form factors from $x = 0\text{-}100\ \text{Å}^{-1}$ for all elements Z = 1-100. These tabulations are based on self consistent wave functions.

2.4 Experimental

The advent of high resolution Ge(Li) and HPGe detectors have led to an enormous increase in the accuracy of experiments on an elastic coherent scattering of γ rays. Quite a number of investigators have reported accurate results on coherent scattering cross sections and form factor [23-30]. Most of these experimental reports were made for 662 keV γ rays and experimental results are not available in the intermediate photon momentum transfer region and for heavy atoms. A systematic study was carried out in our laboratory to bridge this gap and to facilitate a detailed comparison of the experimental results with form-factor predictions. Experiments were carried out using isotopic γ-ray sources and high purity thin absorbers. Experiments were conducted employing high resolution HPGe detector. The sketch of the experimental setup and block diagram of HPGe spectrometer are given elsewhere [31]. A 133 cc HPGe detector (resolution of 2 keV at 1332 keV) was used to detect coherently scattered 84.3, 145.4, 279.2-keV γ rays and a 6 cc HPGe detector (227 eV at 5.9 keV and 500 eV at 59.54 keV) was used to detect 59.54-keV photons.

If $d\Omega_1$ is the solid angle subtended by the scatterer from the source, and $d\Omega_2$ is the solid angle subtended by the detector from the scatterer, the number of γ rays n_c coherently scattered at 90° and reaching the detector is given by

$$n_c = S\left(\frac{d\Omega_1}{4\pi}\right)\frac{d\sigma}{d\Omega}n\varepsilon d\Omega_2, \tag{6}$$

where S is the strength of a point source, n is the number of scattering atoms in the scatterer, ε is the photopeak efficiency of the detector and $d\sigma/d\Omega$ is the differential cross section for coherent scattering at 90° scattering angle. (This expression neglects self absorption in the target.) The differential cross section for coherent scattering is calculated using the above expression. From the measured coherent scattering cross section the form factor is evaluated using the expression

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_T [f(x,Z)]^2. \tag{7}$$

Here $(d\sigma/d\Omega)_T$ is the Thomson cross section, coherent contribution for scattering from a free electron is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_T = \frac{r_0{}^2}{2}[1+\cos^2\theta], \tag{8}$$

where r_0 represents the classical electron radius, θ is the scattering angle and $f(x,Z)$ is the atomic-form factor.

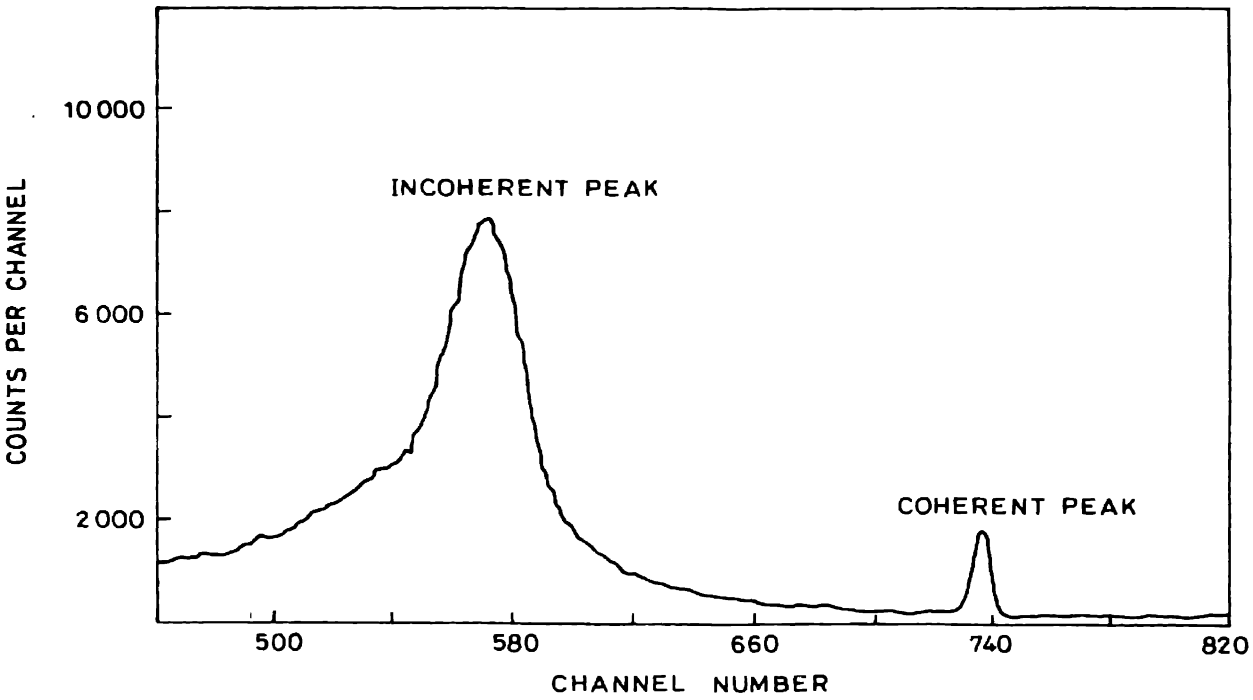

Fig. 1: Spectrum recorded for 145.4 keV γ-rays coherently scattered at 90°

For 90° scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_T = \frac{r_0^2}{2} . \tag{9}$$

Therefore,

$$f(x, Z) = \left[\frac{2\,(d\sigma/d\Omega)}{r_0^2}\right]^{1/2} . \tag{10}$$

A representation spectrum recorded for 145.4-keV γ rays coherently scattered at 90° by Ho is shown in Fig. 1. The differential cross section for coherent scattering of 59.5-, 84.3-, 145.4- and 279.2-keV γ rays at 90° scattering angle were measured for 16 elements in the region $29 \leq Z \leq 92$. In Table 1 is given sample results. The form factors were determined correspondingly at photon momentum transfer 3.395, 4.808, 8.295 and 15.923 Å^{-1}. Some sample results obtained are shown in Table 2 along with the experimental values reported in literature and cross sections computed by numerical calculation methods for available cases. It may be noted that, in all, 49 cross sections measured in our laboratory constitute the first measurement as there are no values reported in literature for these cases. It is also clear from the tables that cross section computed on the basis of numerical calculation method are also available only for a few cases. Graphical comparison of our experimental form factor results with form factor theories are shown in Fig. 2. The comparison clearly shows that the relativistic modified form-factor theory is more appropriate in predicting elastic scattering in the intermediate photon energy range. Further, the results of form factors for Dy-66, Ho-67, Yb-70, Ta-73 at 59.5 keV (x = 3.395Å^{-1}) and for Pb-82 at 84.3 keV (x = 4.808Å^{-1}) confirms the presence of dispersion effect near K edges.

Inelastic scattering

In the inelastic scattering of γ rays the electron absorbs some of the momentum and either remains in excited state or leaves the atom with the result the scattered photon has less energy than the incident photon. In this case there is no phase relation between the radiation scattered by different electrons and hence the process is also known as incoherent scattering. If the incident photon energy is sufficiently high, then the binding energy of the scattering electron can be neglected and the process is referred to as Compton scattering. Klein-Nishina theory [32] is a good attempt to describe Compton

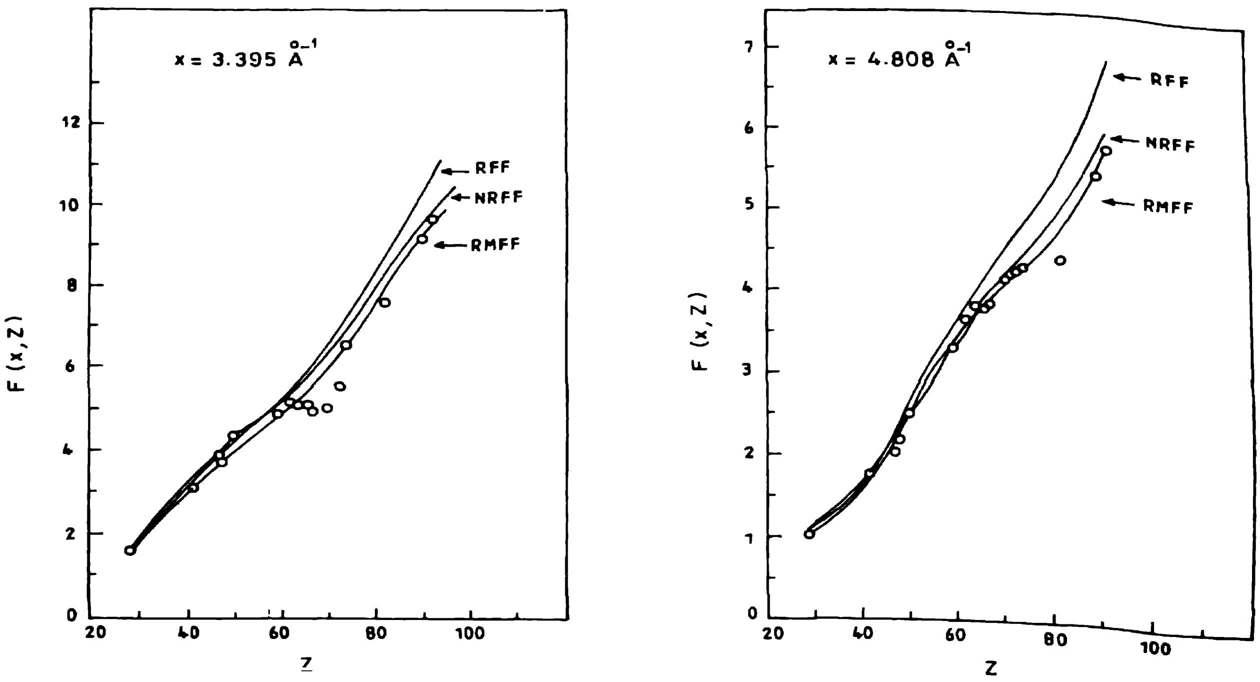

Fig. 2 : Comparison of experimental results with the form factor theories at $x = 3.395\ \AA^{-1}$ and $x = 4.808\ \AA^{-1}$

Table 1. Comparison of differential coherent scattering cross sections.

element	$d\sigma/d\Omega$ (mb/sr) at 59.5 keV			$d\sigma/d\Omega$ (mb/sr) at 145.4 keV		
	present work	other exp.	S-matrix theory (a)	present work	other exp.	S-matrix theory (a)
Cu-29	108±8	111 (b)	118	12±1	14±1 (c)	
Mo-42	380±30	510±20 (d)	500	28±2	45±2 (c)	39
Ag-47	610±40			43±3		
Cd-48	580±30	700b	748	39±3	59±3 (c)	
Sn-50	760±50	840±30 (d)	824	46±3	74±4 (c)	57.2
Pr-59	960±60			71±4		
Sm-62	1060±60			83±5		
Gd-64	1050±60			93±6		
Dy-66	1030±60			108±7		
Ho-67	970±60			113±8		
Yb-70	1020±60			143±10		
Ta-73	1230±70	1100±40 (d)	1070	165±11		219
W-74	1680±80			148±10	219±12 (c)	
Pb-82	2280±90	2240 (b) 2370±80 (d)	2270	304±15	363±14 (e) 290±15 (c)	341
Th-90	3360±120			380±20		
U-92	3720±150		3760	400±20	450±20 (f)	452

(a) Kane et al. (1986), Ref. [41]. (b) Eichler and de Barros (1985), Ref. [42].
(c) B S Gumman (1981), Ref. [43]. (d) Schumacher & Stoffregen (1977), Ref. [24].
(e) . Schumacher (1969), Ref. [23]. (f) Mückenheim & Schumacher (1980), Ref. [44].

Table 2. Comparison of differential incoherent scattering cross sections.

element	$d\sigma/d\Omega$ (barns/atom-sr) at		
	59.5 keV	145.4 keV	
	present work	present work	literature value
Cu-29	0.87±0.04	0.72±0.04	
Mo-42	1.23±0.06	1.03±0.05	
Ag-47	1.38±0.07	1.15±0.06	
Cd-48	1.39±0.09	1.16±0.06	
Sn-50	1.40±0.07	1.21±0.06	
Pr-59	1.61±0.08	1.42±0.07	
Sm-62	1.67±0.08	1.49±0.07	
Gd-64	1.72±0.09	1.53±0.08	1.58±0.05 (a)
Dy-66	1.78±0.09	1.57±0.08	1.65±0.05 (a)
Ho-67		1.60±0.08	
Yb-70		1.67±0.08	
Ta-73	1.87±0.09	1.73±0.09	
W-74	1.87±0.09	1.76±0.09	1.82±0.05 (a)
Pb-82	2.06±0.10	1.93±0.10	2.01±0.06 (a)
Th-90		2.11±0.11	

(a) Rao & Rao (1983), Ref. [40].

scattering process. Departure from Klein-Nishina theory are found to occur at low energies because of the binding effect of the electron and at the high energies because of the possible emission of an additional photon and radiation correction associated with the emission and absorption of the virtual photon [33]. The binding effects of atomic electrons become important at low photon energy, high atomic number and at small scattering angle.

The electron binding effect has been taken into account [34-36] in the impulse approximation by including a factor as a multiplicative correction factor to the Klein Nishina cross section to give the incoherent scattering cross section for the bound electron. The multiplicative cross section factor thus introduced is known as the incoherent scattering function. Thus the differential cross section for incoherent scattering is written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{KN}} S(x, Z) \,. \tag{11}$$

There are two methods to estimate incoherent scattering functions. In the method based on Thomas-Fermi model [9,10] the atomic electrons are treated as a degenerate gas obeying Fermi-Dirac statistics and the Pauli principle, with the ground state energy of the atom equal to the zero-point energy of this gas. In the other method, based on the Hartree-Fock model, the accurate computation of wave functions of many electrons atoms are made by the self consistent field method [37]. This is an independent-particle model in which each electron is assumed to be in the field of nucleus and in average field due to the other electrons. Another approach to describe the bound-electron Compton scattering of γ rays is based on S-matrix theory [38,39]. This method involves evaluation of second-order S matrix of quantum electrodynamics in the bound interaction picture where the relativistically bound electron propagator is used to describe intermediate electron-positron states and relativistic external field eigenfunctions are used for the initial and final atomic states. The results of direct numerical integration calculation based on S-matrix formalism and non-relativistic Hartree-Fock formalism tabulated by Hubbell et al. [13] have clearly established an excellent agreement between the predictions of two methods.

Employing the experimental setup outlined in the previous section, the whole atom differential incoherent scattering cross section of 84.3-, 145.3-, 279.2- and 59.6-keV γ rays were measured for 16 elements in the region $29 \leq Z \leq 92$. Some sample results are shown in Table 2. It can be seen that most of the results constitute the first measurement. There is a lone report [40] of the experimental values for Gd, Dy, W and PB and our results agree within experimental uncertainties with these literature values.

From the measured cross sections incoherent scattering functions were extracted. The results of he incoherent scattering functions are compared with the theoretical formalism in Fig. 3. The disagreement of the experimental results with the Thomas-Fermi model is slightly more than that for the non-relativistic Hartree-Fock formalism; the disagreement of experiment with theory is greater at larger momentum transfer values. The incoherent scattering formalism based on non-relativistic Hartree-Fock theory is better than Thomson-Fermi model in predicting the incoherent scattering functions.

3. R&D using Microtron

Electron and other charged particle accelerators are widely used in developed countries like the USA, Canada, France, Germany, the UK, Sweden and Russia for R&D activities relevant to technological and industrial applications. As far as electron accelerators are concerned, linear accelerators are extensively used in the USA, Canada, the UK and other European coun-

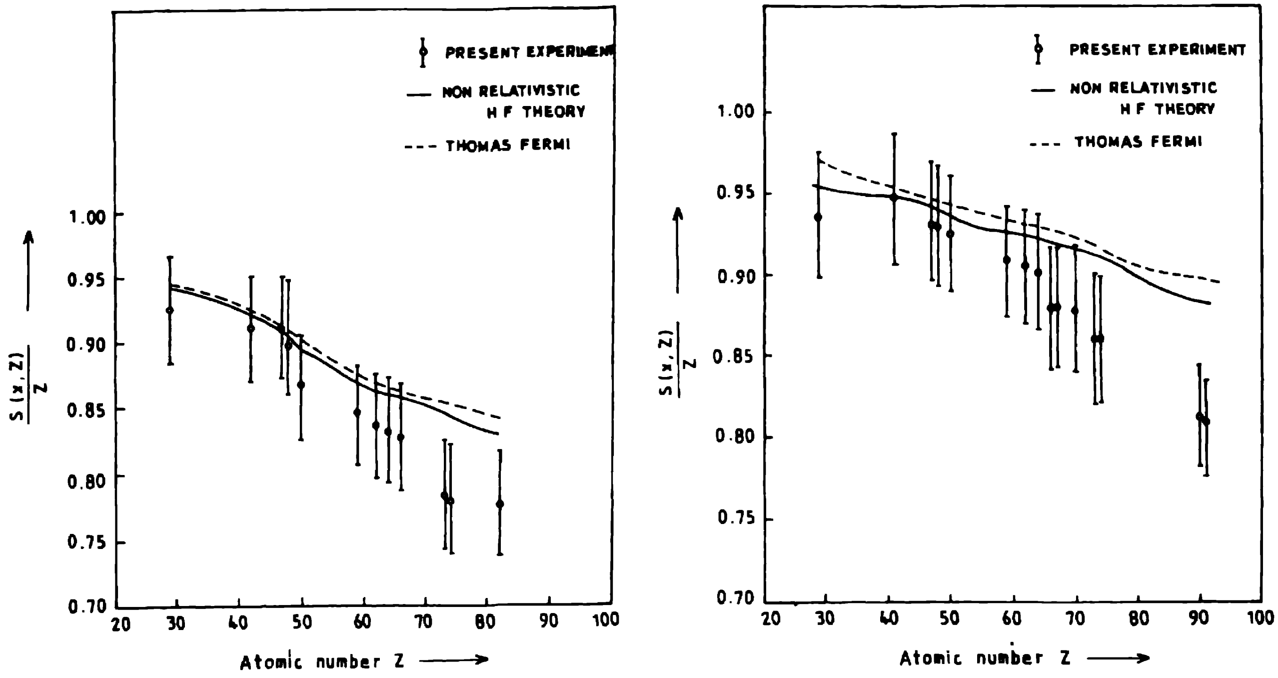

Fig. 3 : Comparison of experimental results of incoherent scattering functions with theory at $x = 3.395\ \mathring{A}^{-1}$ and $x = 4.808\ \mathring{A}^{-1}$

tries while microtron accelerators are more extensively used in Russia.

A variable energy microtron has been setup at our university in collaboration with CAT (Indore) and BARC (Bombay). The facility would provide electrons of 4-12 MeV energy and bremsstrahlung radiation of 3-10 MeV energy. It is also possible to obtain neutrons through photo neutron reaction. The important characteristics features of the microtron are given below:

Beam energy: 8/12 MeV
Pulse current: 50 mA (max)
No. of electron orbits: 14
Beam size: 3 mm × 5 mm
Pulse duration: 2.5 μs
Average beam power: 250/375 W
Electromagnet diameter: 740 mm
Electromagnet weight: 550 kg
Magnetic field strength: 1285/1927.5 G
Magnetron power: 2 MW
Operation frequency: 2998 MHz
Voltage in RF cavity: 682/1023 kV
Dose rate at 1 m: 5 Gy/min

The machine has been successfully commissioned in September, 1995, and the facility has opened new vistas in teaching and research programs of radiation physics and in R&D work relevant to industry. In our country studies on radiation shielding, radiation dosimetry, radiation damage in materials and in several basic aspects of atomic and nuclear physics are being carried out using accelerators at VECC (Calcutta), NSC (New Delhi), TIFR (Bombay), BARC (Bombay), Institute of Physics (Bhuvaneshwar) and also in a few universities with smaller accelerators. Research at the national centers is conducted mainly using accelerated heavy charged particles like proton, alpha particle and other heavy ions. Investigations employing accelerated electrons are relatively sparse even at the international level in spite of the fact that linear accelerators and microtrons are readily available to the scientific community. Precise experimental data on several basic aspects such as radiation dosimetry, bremsstrahlung and other processes are not available above 2 to 4 MeV energy. Therefore, it is proposed to undertake R&D programs using the newly commissioned variable energy microtron in the following areas:

- bremsstrahlung and other basic interaction process in the electron energy range 4-12 MeV;
- radiation damage and radiation processing in semiconductor

materials and devices;

- irradiation effects on electro-optic and nonlinear optical properties in inorganic and organic crystals;
- irradiation effects on the physical and mechanical properties in polymers and other materials;
- industrial radiation processing for developing electronic components with tailored characteristics;
- geochemical analysis of mineral, soil, marine sediment and other samples.

The facility will also be utilized to train graduate students in all these important areas needed for Science and Technology programs.

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