EFFECT OF HIGHER ORDER PARTIAL WAVE PHASE-SHIFTS IN ELASTIC ELECTRON SCATTERING BY HELIUM ATOM

S. N. BANERJEE AND N. C. Sil

DEPARTMENT OF THEORETICAL PHYSICS,

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE,

JADAVPUR, CALCUTTA-32.

(*Received September* 29, 1966)

ABSTRACT. In elastic e^-He scattering in the energy range of 13.6 ev to 54.4 ev we have computed in the Born approximation the higher order phase-shifts ($l \geqslant 3$) ; making use of lower order phase-shifts (*i.e.* for $l=0, 1, 2$) from the calculation of LaBahn and Callaway (1966), we find that the inclusion of the higher order phase-shifts considerably improves the value of the total and differential cross-sections in bringing them to better agreement with the experimental results.

INTRODUCTION

Recently LaBahn and Callaway (1966) have investigated the elastic scattering of electrons by helium atom in the energy range of 0 to 50 ev. Taking into account both the exchange and polarisation effects, they have calculated the S, P and D-wave phase-shifts in three different approximations viz. i) adiabatic—exchange (ii) Dynamic-exchange with all components and (iii) dynamic-exchange with only the dipole component. They have not, however, considered the phase-shifts for $l \geqslant 3$, which are expected to be of considerable importance specially at higher energies.

For scattering in the energy of 13.6 ev to 54.4 ev, we have computed in the Born approximation those higher order phase-shifts which contribute appreciably to the scattering cross-sections. For such energies and for such higher order phase-shifts, it will be enough to take only the asymptotic part of the potential which in effect will be a term of the form $-\alpha/r^4$ coming from the polarisation potential, the screened coulomb potential and other short-range potentials will have no appreciable contribution to it. As the exchange effect decreases with increase in energy and increase in order of phase-shifts, it will be consistent to neglect exchange terms. The fact that the higher order phase-shifts have small values gives us the reasonable justification for using the Born's approximation in calculating the phase-shifts.

THEORY

The Born approximation for the phase-shift is given by

$$\eta_l = -\frac{2mk_0}{\hbar^2} \int_0^\infty V(r)[j_l(r)]^2 r^2 dr$$

where
$$j_l(r) = \left(\frac{\pi}{2k_0 r}\right)^{\frac{1}{2}} . J_{l+\frac{1}{2}}(k_0 r),$$

and the symbols have their usual significances, $V(r)$ is the total potential.

With potential $V(r) = -(\alpha/r^4)$, we may write (cf. Mott and Massey, 1965)

$$\eta_l = -\frac{4\pi^3 m\alpha}{h^2} \int_0^\infty \{J_{l+\frac{1}{2}}(kr)\}^2 . r^{-3} dr$$

$$= -\frac{4\pi^3 m\alpha}{h^2} . \left(\frac{k_0}{2}\right)^{4-1} . \frac{\Gamma(3).\Gamma(l-2+3/2)}{[\Gamma(2)]^2.\Gamma(l+2+\frac{1}{2})}$$

the constant α is the polarisability of the He atom and its value is 1.376 a_0^3. The differential cross section $I(\theta)$ is given by

$$I(\theta) = \frac{1}{k_0^2} |\Sigma(2l+1)e^{i\eta_l}. \sin \eta_l P_l(\cos\theta)|^2$$

and total cross section $Q = \frac{4\pi}{k_0^2} \Sigma(2l+1)) \sin^2\eta_l$.

RESULTS AND DISCUSSIONS

The phase-shifts in radions for $l = 3, 4, 5$ and 6 waves are given in Table I for five different energies.

TABLE II

Energy in ev	$l=3$	$l=4$	$l=5$	$l=6$
13.6	.0374	.0124	.0067	—
21.25	.04285	.019493	.010496	.00629
30.60	.066834	.030379	.016358	.00981
41.65	.08439	.0383	.02065	.01239
54.40	.109786	.049903	.02687	.01622

LaBahn and Callaway (1966) have calculated S, P, D wave phase shifts in three different types of approximations, of which the dynamic exchange approximation with only the dipole component gives the best agreement with experiment. Hence we have calculated the total and differential cross sections by using the S, P, D wave phawe-shifts of Labahn and Callaway (1966) dynamic exchange approximatin with dipole component only and the higher order phase shifts computed here in the Born approximation.

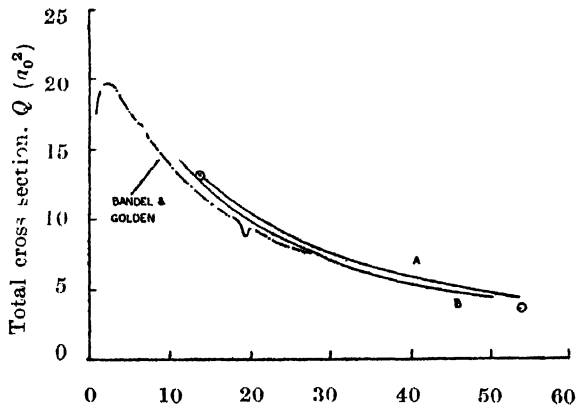

Fig. 1. Energy (ev). The total cross section Q is plotted against energy. Curve A—Present Calculation; Curve B—Calculation by LaBahn and Callaway (1966); .—experimental results of Normand (1930); dashed and dotted curve is that of Bandel and Golden (1965).

In fig. 1, we have plotted our calculated values of the total cross section Q against energy for the dynamic exchange case with the dipole component only.

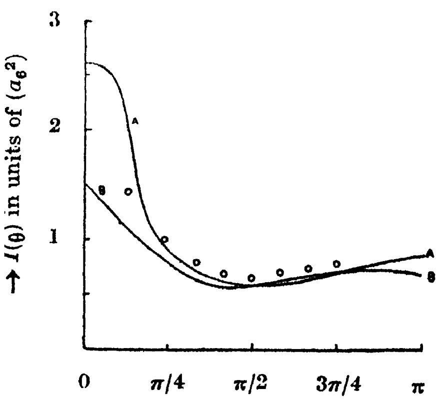

Fig. 2. → θ.

Differential cross section $I(\theta)$ is plotted against θ for 21.25 ev energy. Curve A—Present Calculation; Curve B—Calculation using only S, P. D wave phase-shifts of LaBahn andCallaway; ⊙— experimental results at 20 ev.

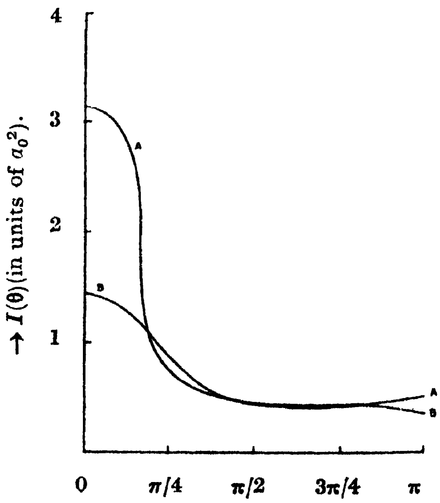

Fig. 3. → θ.

Differential cross section $I(\theta)$ is plotted against θ for 30.6 ev energy. Curve A—Present Calculation; Curve B—Calculation using the S, P, D wave phase-shifts of LaBahn and Callaway.

The theoretical results of LaBahn and Callaway (1966) in the dynamic exchange approximation with dipole component only and the experimental values of Bandel and Golden (1965) and Normand (1930) are shown for comparison. To get accurately Normand's experimental data of elastic scattering excitation cross section data of Gabriel and Heddle (1960) are added to those of ionisation cross section of Smith (1930) and the sum is subtracted from the total cross section data of Normand (1930), after increasing its value by 25% according to the suggestion of Gabriel and Heddle (1960). The discrepancy still left may be partially due to over simplification of the dynamics of the scattering electron in describing the distortion interaction and due to the influence of inelastic processes on the elastic scattering. In figs. 2 and 3, we have shown calculated differential cross section $I(\theta)$ for 21.25 ev and 30.6 ev electron energies respectively against scattering angles θ together with the available experimental results. These calculations have been done only for the dynamic exchange case in dipole approximation.

From the above figures, we see that the tendency of higher order phase-shifts is to give a sharp increase in the differential cross section for forward scattering angles. It should be mentioned that we have neglected exchange in the energy range of 13.6 ev to 54.4 ev in our computation of the phase shifts. This is justified because in the energy range concerned, influence of exchange decreases particularly for higher order partial waves.

ACKNOWLEDGMENT

The authors are thankful to Prof. D. Basu for his kind interest and valuable discussions throughout the progress of the work.

REFERENCES

Bandel, H. W., and Golden, D. E., 1965, *Phys. Rev.* **138**, A14.

Gabriel, A. H. and Heddle, D.W.O., 1960, *Proc. Roy. Soc.*, *A***258**, 124.

LaBahn, R. W. and Callaway, J., 1966, *Phys. Rev.*, **147**, 28.

Mott, N. F. and Massey, H. S. W., 1965, *The Theory of Atomic Collisions*, Oxford at the Clarendon Press, Third Edition.

Normand, C. E., 1930, *Phys. Rev.* **35**, 1217.

Smith, P. T., 1930, *Phys. Rev.*, **36**, 1293.