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The thermosolutal instability of a composite rotating plasma with finite Larmor radius

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Abstract : The thermosolutal instability of a composite plasma is studied to include the effects of coriolis force, the finiteness of ion Larmor radius and collisions between ionized and neutral particles in the presence of a uniform vertical magnetic field. It is found in the stationary convection case that the FLR (finite Larmor radius) and stable solute gradient have stabilizing effects. However, the mutual collisions between ionized and neutral particles have no effect on stationary convection.

Keywords : Thermosolutal instability, composite rotating plasma, finite Larmor radius

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1. Introduction

Finite Larmor Radius effect on plasma instabilities has been the subject of many investigations. Rosenbluth *et al* [1] and Roberts and Taylor [2] have studied the effect of finiteness of ion Larmor radius (FLR corrections) on plasma instabilities, showing up in the form of a magnetic viscosity in the fluid equations. Sharma [3] has studied the effect of finite Larmor radius and Hall effects on thermal instability of a rotating plasma. Chandrasekhar [4] has treated the theory of thermal instability of a fluid layer heated from below under varying assumptions. Veronis [5] has studied the thermohaline convection in a layer of fluid, heated from below and subjected to a stable salinity gradient. Nield [6] has considered the same problem, but a horizontal layer of a viscous fluid heated from below and salted from above. In such cases, the buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration.

The conditions under which convective motion is important in stellar atmospheres, require consideration of a fluid acted on by a solute gradient and free boundaries. The problem of the onset of thermal instability in the presence of solute gradient is of great importance because of its applications to astrophysics and atmospheric physics, especially in ionosphere and solar atmosphere [7].

The thermal instability of a composite plasma with finite electrical conductivity, in the absence as well as in the presence of Hall effect separately, has been studied by Sharma and Sharma [8]. The thermal instability in the presence of Hall currents has been studied by Gupta [9], Sharma [10], Sharma and Sharma [11] and Vasiu and Beu [12]. Thermal instability of a compressible FLR, Hall plasma in porous medium has been studied by Sharma and Sunil [13,14]

The thermal instability of plasma subjected to a solute gradient and a uniform magnetic field including finite Larmor radius effect, has been studied by Sharma and Sharma [15], Sharma *et al* [16], Gupta and Singh [17] and Sharma and Rani [18]. In the stellar case, the physics is quite similar to Veronis [5] thermohaline configuration, in that the helium acts like salt in raising the density and in diffusing more slowly than the heat. The finite Larmor radius, rotation and collisional effects are likely to be important in these regions.

The present paper therefore, considers the thermosolutal instability of a composite, rotating plasma including simultaneously the FLR effect and collisions between ionized and neutral particles.

2. The problem formulation

Here we consider the thermosolutal instability of a composite incompressible plasma, in rotation with a uniform angular velocity $\boldsymbol{\Omega}$ (0, 0, Ω), subjected to a vertical magnetic field $\boldsymbol{B}$ (0, 0, B_0) in the presence of FLR We follow Vasiu [19] and neglect Hall effect ($\lambda_e << r_1$, where λ_e, r_1 are the mean free path of electrons and the Larmor radius, respectively).

Here, the plasma is confined in the form of infinite horizontal layer of thickness l_0 and is acted upon by the vertically downward gravitational acceleration $\boldsymbol{g}$ $(0,0,-g)$. This plasma layer has two incompressible components : an ionized one and a neutral one with densities ρ_i and ρ_n respectively. The collisional frequency between ionized and neutral particles is denoted by ν_c and we have neglected the influence of rotational motion and viscosity on neutral plasma component. The effect of FLR on ionized component requires that the pressure must be a tensor quantity depending on ion gyration frequency, because of strong magnetic field action. Futhermore, the effects of viscosity and finite electrical conductivity of ionized component should also be considered. The plasma layer is heated from below and is subjected to a stable solute gradient. We have denoted the uniform temperature and uniform solute gradient by β (= | dT/dZ |) and β' (= | dC/dz |). In stationary state, the plasma layer verifies $T = T_0 - \beta z$ and $C = C_0 - \beta' z$ conditions and $\rho = \rho_0$ $[1 + \alpha (T_0 - T) - \alpha' (C_0 - C)] = \rho_0 (1 + \alpha\beta z - \alpha'\beta' z)$ where T_0, C_0 and T, C are the temperatures and concentrations at the bottom surface $z = 0$ and at any point between $z = 0$

and $z = l_0$, z-axis being taken as the vertical axis, ρ_0 is the density at $z = 0$; α, α' represent the thermal coefficient of expansion and solvent coefficient of expansion, respectively.

3. Linearized perturbation equations

We make the assumption that both incompressible viscous ionized fluid and incompressible nonviscous neutral gas behave like continuum fluids and that the neutral gas is not affected by the pressure gradient, gravitational acceleration, temperature gradient and stable solute gradient.

On the basis of the foregoing remarks and following the linearized perturbation theory [4], the linearized hydromagnetic equations of the system are :

$$\frac{\partial}{\partial t}(\delta \boldsymbol{v}_i) = -\frac{1}{\rho_i}\nabla(\delta \boldsymbol{P}) + \nu_i \Delta(\delta \boldsymbol{v}_i) + \nu_c(\delta \boldsymbol{v}_n - \delta \boldsymbol{v}_i) - (\alpha\theta - \alpha'\gamma)\boldsymbol{g}$$

$$+ 2\delta \boldsymbol{v}_i \times \boldsymbol{\Omega} + \frac{1}{\rho_i \mu_0}(\nabla \times \delta \boldsymbol{B}) \times \boldsymbol{B}, \tag{1}$$

$$\frac{\partial}{\partial t}(\delta \boldsymbol{v}_n) = -\frac{1}{\varepsilon}\nu_c(\delta \boldsymbol{v}_n - \delta \boldsymbol{v}_i), \tag{2}$$

$$\nabla \cdot (\delta \boldsymbol{v}_i) = 0, \tag{3}$$

$$\nabla \cdot (\delta \boldsymbol{v}_n) = 0, \tag{4}$$

$$\frac{\partial \theta}{\partial t} - \chi \Delta \theta = \beta w, \tag{5}$$

$$\frac{\partial \gamma}{\partial t} - \chi' \Delta \dot{\gamma} = \beta' w, \tag{6}$$

$$\frac{\partial}{\partial t}(\delta \boldsymbol{B}) = \nabla \times (\boldsymbol{v}_i \times \boldsymbol{B}) + \nu_m \Delta(\delta \boldsymbol{B}), \tag{7}$$

$$\nabla \cdot (\delta \boldsymbol{B}) = 0, \tag{8}$$

where δV_i (u_i, v_i, w_i), $\delta \boldsymbol{v}_n$ (u_n, v_n, w_n), $\delta \boldsymbol{P}$, $\delta \boldsymbol{B}$ $(\delta B_x, \delta B_y, \delta B_z)$, θ, γ denote the perturbations in velocities $\boldsymbol{v}_i$, $\boldsymbol{v}_n$, stress tensor $\boldsymbol{P}$, magnetic field $\boldsymbol{B}$, temperature T, concentration C; whereas χ, χ', ν_m, ν_i, ν_c are the thermal diffusivity, solute diffusivity, electrical resistivity and kinematic viscosity of ionized component and ion-neutral collisional frequency respectively; $\varepsilon = \rho_n/\rho_i$.

The change in the density $\delta\rho$ caused by the perturbation θ in the temperature and γ in the concentration is given by :

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{9}$$

We make the assumption that both ionized fluid and neutral gas behave like continuum fluids and for the neutral gas the influence of magnetic field is negligible.

For the vertical magnetic field $\boldsymbol{B}$ $(0,0,B_0)$, the perturbations $\delta \boldsymbol{P}$ in the stress tensor $\boldsymbol{P}$ have the components written below :

$$
\begin{aligned}
\delta P_{11} &= \delta P_{xx} = \delta P - \rho_i v_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\
\delta P_{12} &= \delta P_{21} = \delta P_{xy} = \delta P_{yx} = \delta P + \rho_i v_0 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right), \\
\delta P_{22} &= \delta P_{yy} = \delta P + \rho_i v_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\
\delta P_{13} &= \delta P_{31} = \delta P_{xz} = \delta P_{zx} = \delta P - 2\rho_i v_0 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\
\delta P_{23} &= \delta P_{32} = \delta P_{yz} = \delta P_{zy} = \delta P + 2\rho_i v_0 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right), \\
\delta P_{33} &= \delta P_{zz} = \delta p
\end{aligned}
\tag{10}
$$

Here, δP is the perturbation in scalar pressure, $\rho_i v_0 = N_i T_i / 4\omega_i$ where ω_i is the ion gyration frequency, v_0 is the ion gyroviscosity, N_i is the number density and T_i is the temperature of the ions.

The perturbation $\delta\varphi$ in terms of the normal modes has the form :

$$\delta\varphi(x, y, z) = \varphi^*(z) \exp\left[ik_x X + ik_y Y + nt\right], \tag{11}$$

where φ^* is the amplitude, k_x, k_y are the wave numbers along x and y directions, $k^2 = k_x^2 + k_y^2$ and n is the growth rate, which is a complex constant.

Eq. (2) using (11) yields

$$\Omega_n \delta v_n = \frac{1}{\varepsilon} v_c \delta v_i, \tag{12}$$

where $\Omega_n = n + v_c / \varepsilon$.

Eq. (1) using (11–12) has the form :

$$\Omega_{in} \delta v_i = -\frac{1}{\rho_i} \nabla \delta P - (\alpha\theta - \alpha'\gamma) g + 2\delta v_i \times \Omega$$

$$\frac{1}{\mu_0 \rho_i} (\nabla \times \delta B) \times B \tag{13}$$

where : $\Omega_{in} = n^* - v_i \Delta;\ n^* = n(1 + \varepsilon v_c / (n\varepsilon + v_c))$.

Using (10) and applying the 'curl' operator on (13) we can obtain its projection along z-axis :

$$\Omega_{in} \zeta = \left[v_0 (2D^2 + k^2) + 2\Omega \right] Dw + \frac{B_0}{\mu_0 \rho_i} D\zeta, \tag{14}$$

where $D = d/dz$, $D^2 = d^2/dz^2$, $\zeta = [\text{curl}\ \delta v_i]_z$, $\xi = [\text{curl}\ \delta B]_z$.

Using (11) and $\nabla \times (\delta v_i \times B) = B_0 D\delta v_i$, the projection along the z-axis of (7) can be reduced to the following form :

$$\Omega_m \delta B_z = B_0 Dw, \tag{15}$$

where $\Omega_m = n + v_m k^2$.

Applying the 'curl (curl)' operator on (13) and 'curl' on (7) results their projections along z-axis :

$$\Omega_m \Delta\delta w_i = \alpha g\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \alpha' g\left(\frac{\partial^2\gamma}{\partial x^2} + \frac{\partial^2\gamma}{\partial y^2}\right)$$

$$-\left[v_0(2D^2 + k^2) + 2\Omega\right]D\zeta + \frac{B_0}{\mu_0\rho_i} D\Delta\delta B_z, \tag{16}$$

$$\Omega_m \xi = B_0 D\zeta. \tag{17}$$

Analyzing the disturbances in terms of normal modes, we assume that the perturbation quantities are under the form :

$$\{w, \theta, \gamma, \delta B_z, \zeta, \xi\} = \{W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)\}\cdot$$

$$\exp\left[ik_x x + ik_y y + nt\right]. \tag{18}$$

where W, Θ, Γ, K, Z, X are the perturbation amplitudes.

It is convenient to discuss eqs. (5), (6), (14–17) taking into account (18) and in nondimensional variables. Choose the units of length $[L] = l_0$ and of time $[T] = (l_0)^2/v$ and let $z^* = l_0 z$; $a = kl_0$; $\sigma = n(l_0)^2/v$; $\sigma^* = n^*(l_0)^2/v$; $p_1 = v/\chi$; $p_2 = v/v_m$; $p_3 = v/\chi'$. Introducing the following quantities :

$$C_0 = \frac{\beta l_0^2}{\chi};\ C_1 = \frac{\beta' l_0^2}{\chi'};\ C_2 = \frac{\sqrt{T}}{l_0};\ C_3 = \frac{B_0 l_0}{\mu_0 \rho_i v}$$

$$C_4 = \frac{\sqrt{U}}{l_0};\ C_5 = \frac{B_0 l_0}{v_m};\ C_6 = \frac{\alpha g l_0^2}{v}\cdot a^2;\ C_7 = \frac{\alpha' g l_0^2}{v}\cdot a^2 \tag{19}$$

$$C_8 = \sqrt{U} l_0;\ C_9 = \sqrt{T} l_0;\ \sqrt{\mathrm{T}} = 2\Omega\frac{l_0^2}{v};\ \sqrt{U} = \frac{v_0}{v};$$

and the operators

$$O = D^2 - a^2;\ O_\sigma^* = D^2 - a^2 - \sigma^*;\ O_1 = D^2 - a^2 - \sigma p_1;$$

$$O_2 = D^2 - a^2 - \sigma p_2;\ O_3 = D^2 - a^2 - \sigma p_3;\ O_a = 2D^2 + a^2, \tag{20}$$

the final form is :

$$O_1\Theta = -C_0 W, \tag{21}$$

$$O_3\Gamma = -C_1 W, \tag{22}$$

$$O_\sigma^* Z = -[C_2 + C_4 O_a]DW - C_3 DX, \tag{23}$$

$$O_2K = -C_5DW, \tag{24}$$

$$O_\sigma^* OW = C_6\Theta - C_7\Gamma + (C_8O_a + C_9)DZ - C_3ODK, \tag{25}$$

$$O_2X = -C_5DZ. \tag{26}$$

4. Dispersion equation

Eliminating $\Theta(z)$, $\Gamma(z)$, $K(z)$, $X(z)$ and $Z(z)$ from eqs. (21–26) and introducing the Chandrasekhar number $Q = C_3C_5 = \dfrac{B_0 l_0}{\mu_0 \rho_i \nu \nu_m}$, the differential equation is :

$$\begin{aligned}
&[(D^2-a^2-\sigma^*)(D^2-a^2-\sigma p_2) - QD^2]\{(D^2-a^2)\cdot(D^2-a^2-\sigma p_1)\\
&\cdot(D^2-a^2-\sigma p_3)\cdot[(D^2-a^2-\sigma^*)\cdot(D^2-a^2-\sigma p_2) - QD^2]\\
&+(D^2-a^2-\sigma p_2)\cdot a^2\cdot[\mathrm{R}(D^2-a^2-\sigma p_3) - S(D^2-a^2-\sigma p_1)]\}\\
&W(z) = -U[V+(2D^2+a^2)]^2(D^2-a^2-\sigma p_1)(D^2-a^2-\sigma p_3)\\
&\qquad (D^2-a^2-\sigma p_2)^2 D^2 W(z)
\end{aligned} \tag{27}$$

where $R = \dfrac{g\alpha\beta l_0^4}{\chi\nu}$ is the Rayleigh number,

$$S = \frac{\alpha'\beta' g l_0^4}{\chi'\nu} \text{ is the solute Rayleigh number,} \tag{28}$$

$V = \dfrac{2\Omega l_0^2}{\nu_0}$. is a non-dimensional number.

Eq. (27) is identical to that of Gupta and Singh [17] where the $D^2 - a^2 - \sigma$ operator has been replaced by $D^2 - a^2 - \sigma^*$ operator and

$$\sigma^* = \frac{n^* l_0^2}{\nu}; \quad n^* = n\cdot\left[1 + \frac{\varepsilon\nu_c}{\varepsilon n + \nu_c}\right] \tag{29}$$

5. Particular cases

(i) In the absence of FLR ($\nu_0 = 0$) and rotational motion ($V = 0$), eq. (27) reduces to :

$$\begin{aligned}
&[(D^2-a^2-\sigma^*).(D^2-a^2-\sigma p_2) - QD^2]\{(D^2-a^2)\cdot(D^2-a^2-\sigma p_1)\cdot\\
&(D^2-a^2-\sigma p_3)\cdot[(D^2-a^2-\sigma^*)\cdot(D^2-a^2-\sigma p_2) - QD^2]\\
&+(D^2-a^2-\sigma p_2)\cdot a^2[.R\cdot(D^2-a^2-\sigma p_3) - S\cdot(D^2-a^2-\sigma p_1)]\}\\
&W(z) = 0,
\end{aligned} \tag{30}$$

because $UV^2 = T$. Eq. (30) is identical to Sharma and Sharma [8] taking into account the same operator modification.

(ii) In the absence of FLR ($\nu_0 = 0$), and solute gradient ($S = 0$), eq. (27) becomes :

$$[D^2 - a^2 - \sigma p_1]\Big\{(D^2 - a^2)\cdot\left[(D^2 - a^2 - \sigma^*)\cdot(D^2 - a^2 - \sigma p_2) - QD^2\right]^2$$

$$+ T(D^2 - a^2 - \sigma p_2)^2 D^2\Big\} W(z) = -R\cdot a^2\cdot\left[(D^2 - a^2 - \sigma^*)\cdot\right.$$

$$\left.(D^2 - a^2 - \sigma p_2) - QD^2\right]\cdot(D^2 - a^2 - \sigma p_2)\ W(z). \qquad (31)$$

(iii) For a single plasma component (pure plasma) in the absence of ion-neutral collisional frequency ($\nu_c = 0$; $\sigma^* = \sigma$), eq. (31) has the form :

$$(D^2 - a^2 - \sigma p_1)\Big\{(D^2 - a^2)\left[(D^2 - a^2 - \sigma)(D^2 - a^2 - \sigma p_2) - QD^2\right]^2$$

$$+ T\cdot(D^2 - a^2 - \sigma p_2)D^2\Big\} W(z) = -R\cdot a^2 \Big\{\big[(D^2 - a^2 - \sigma)$$

$$.(D^2 - a^2 - \sigma p_2) - QD^2\big]\cdot\left[D^2 - a^2 - \sigma p_2\right]\Big\} W(z), \qquad (32)$$

which is identical with Chandrasekhar's result (1961).

6. Discussion

The boundary conditions in the case in which both boundaries are free as well as perfect conductors are :

$$W(z) = D^2W(z) = DZ(z) = 0;\ \Theta(z) = \Gamma(z) = X(z) = 0 \qquad (33)$$

at $z = 0$ and $z = 1$ ($z^* = 0, z^* = l_0$) and δB_x, δB_y, δB_z are continuous. The proper solution of eq (27) characterizing the lowest mode has the form :

$$W(z) = W_0 \sin(\pi z), \qquad (34)$$

where W_0 is a constant.

Substituting (34) in (27) we obtain the characteristic equation :

$$R_1 = \left\{\frac{(1+x)\left[(1+x+\tilde{\sigma})(1+x+b_1)(1+x+b_2)(1+x+b_3)\right]}{\left[x(1+x+b_2)(1+x+b_3)\right]}\right.$$

$$\left. + \frac{S_1\cdot x(1+x+b_1)(1+x+b_2) + Q_1(1+x)(1+x+b_1)(1+x+b_3)}{\left[x(1+x+b_2)(1+x+b_3)\right]}\right\}$$

$$+ \frac{U[V_1 - (2-x)]^2\,(1+x+b_1)(1+x+b_2)]}{x[(1+x+\tilde{\sigma})(1+x+b_2) + Q_1]}, \qquad (35)$$

where $x = \dfrac{a^2}{\pi^2}$; $R_1 = \dfrac{R}{\pi^4}$; $S_1 = \dfrac{S}{\pi^4}$; $Q_1 = \dfrac{Q}{\pi^2}$; $V_1 = \dfrac{V}{\pi^2}$;

$\tilde{\sigma} = \dfrac{\sigma^*}{\pi^2}$; $b_1 = \dfrac{\sigma p_1}{\pi^2}$; $b_2 = \dfrac{\sigma p_2}{\pi^2}$; $b_3 = \dfrac{\sigma p_3}{\pi^2}$.

In the case of stationary convection ($n = \sigma = n^* = \sigma^* = \tilde{\sigma} = 0$; $b_1 = b_2 = b_3 = 0$), eq. (35) has the form :

$$R_1 = \frac{1+x}{x}\left[(1+x)^2 + Q_1\right] + U\,\frac{[V_1 - (2-x)]^2(1+x)^2}{x[(1+x)^2 + Q_1]} + S_1, \tag{36}$$

identical with Gupta and Singh's [17] results.

If the FLR corrections are not taken into account ($v_0 = 0$), eq. (36) can be reduced to the form :

$$R_1 = \frac{1+x}{x}\left[(1+x)^2 + Q_1\right] + T_1\,\frac{(1+x)^2}{x[(1+x)^2 + Q_1]} + S_1, \tag{37}$$

where $$T_1 = U[V_1 - (2-x)]^2 = UV_1^2 - 2UV_1(2-x) + U(2-x)^2$$

$$= \frac{4\Omega^2 l_0^4}{\pi^4 v^2} - \frac{4\Omega l_0^2}{\pi^2 v^2}\cdot v_0\cdot(2-x) + \frac{v_0^2}{v^2}(2-x)^2;$$

$$T_1 = \frac{T}{\pi^4};\ (\text{when } v_0 = 0;\ T = \frac{4\Omega^2 l_0^4}{v^2} \text{ is the Taylor number}).$$

If furthermore, we do not consider the solute gradient ($S = 0$), we obtain the Chandrasekhar result [4].

Generally, the investigation of FLR, rotational motion and solute gradient effects is facilitated by the analytical study of dR_1/dU, dR_1/dV_1, dR_1/dS_1. It follows from (35) that

$$\frac{dR_1}{dU} = \frac{[V_1 - (2-x)]^2\,(1+x+b_1)(1+x+b_2)}{x[(1+x+\tilde{\sigma})(1+x+b_2) + Q_1]},$$

$$\frac{dR_1}{dV_1} = U\,\frac{2[V_1 - (2-x)]\,(1+x+b_1)(1+x+b_2)}{x[(1+x+\tilde{\sigma})(1+x+b_2) + Q_1]},$$

$$\frac{dR_1}{dS_1} = \frac{1+x+b_1}{1+x+b_3}. \tag{38}$$

Our discussion is limited only for stationary convection (see eq. 36) where the modified Rayleigh number R_1 attains the minimum when $dR_1/dx = 0$. We obtain :

$$x^7 + a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 + a_5x^2 + a_6x + a_7 = 0, \tag{39}$$

where $$a_1 = 5.5 + 0.5,$$

$$a_2 = 12 + 2(Q_1 + U),$$

$$a_3 = 12.5 + 6.5Q_1 - 0.5T_1 + U(1 + 1.5Q_1) + \sqrt{UT_1},$$

$$a_4 = 5 + 6Q_1 + Q_1^2 - 2T_1 + 0.5U(-12 - 4Q_1) + 2\sqrt{UT_1}(4 + Q_1),$$

$$a_5 = -1.5 + 0.5Q_1(Q_1 - 2) - 3T_1 + 0.5Q_1T_1 + 0.5U(-23 - 3Q_1) + 12\sqrt{UT_1}\cdot$$

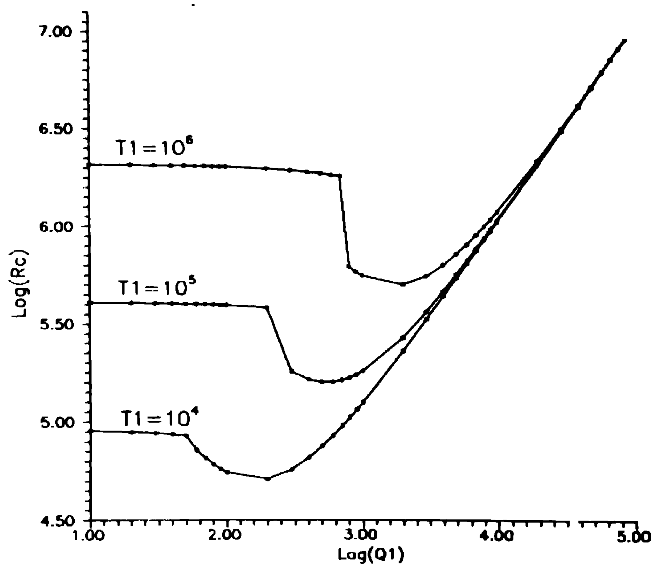

Figure 1. The critical Rayleigh number R_c for the onset of ordinary convection as a function of Q_1 for various assigned values of T_1 in Chandrasekhar model The curves are labelled by values of T_1 to which they refer and in the absence of FLR ($U = 0$) and thermosolutal effect ($S_1 = 0$).

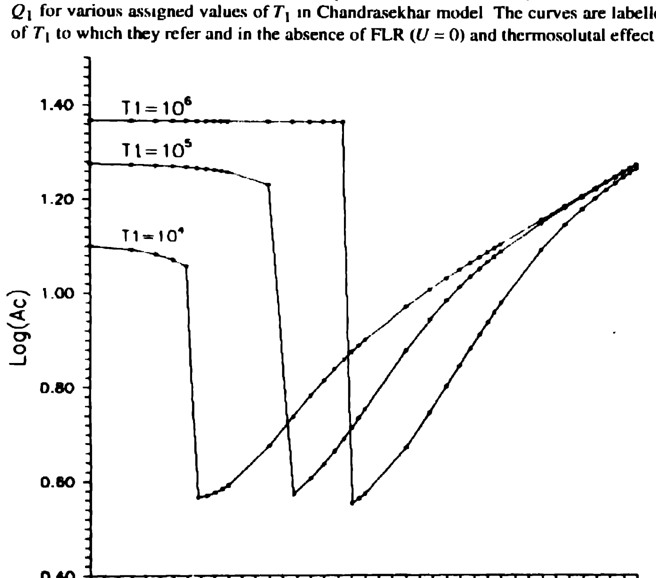

Figure 2. The dependence on Q_1 for various values of T_1 of the critical wave number A_c (in Çhandrasekhar model) of disturbance at which instability first sets in as convection. It will be observed that a discontinuous change in A_c occurs when (for increasing Q_1) the maner of instability changes from overstability to ordinary convection.

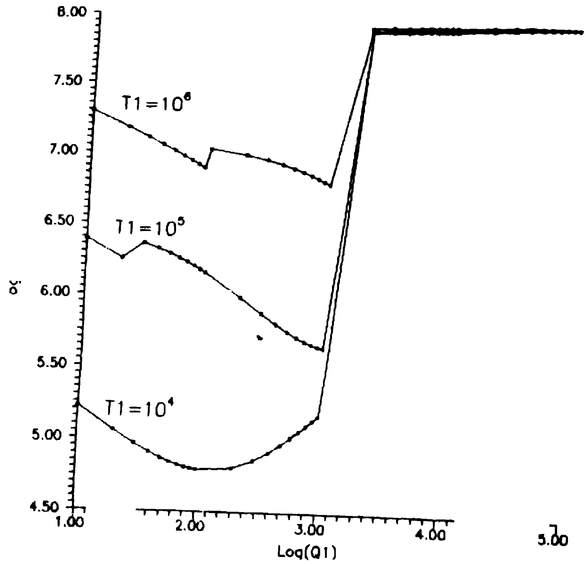

Figure 3. The critical Rayleigh number R_c as a function of Q_1 for various assigned values of T_1 in the Gupta and Singh model, in the presence of FLR ($U = 1000$) effect but in the absence of thermosolutal influence ($S_1 = 0$)

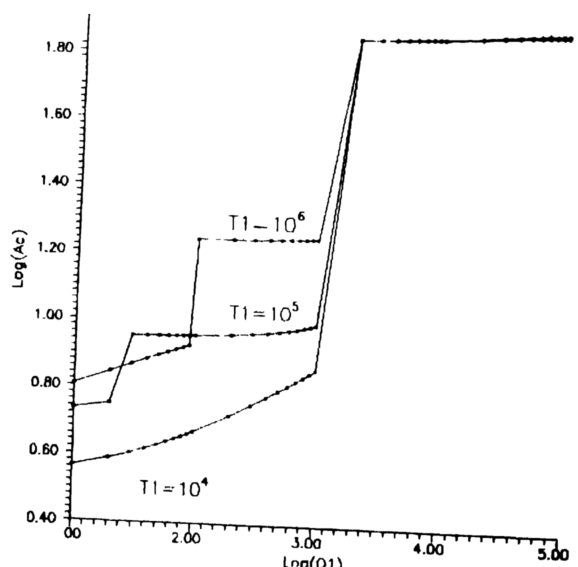

Figure 4. The dependence on Q_1 (for various values of T_1) of the critical wave number A_c in Gupta and Singh model, of the disturbance at which instability first sets in as ordinary convection in the presence of FLR effect ($U = 1000$) and in the absence of thermosolutal influence ($S_1 = 0$) It will be observed a stabilizing influence on discontinuous change in A_c induced by FLR.

$$a_6 = -2[(1+Q_1)^2 + T_1] - 8U + 8\sqrt{UT_1},$$

$$a_7 = -0.5(1+Q_1)[(1+Q_1)^2 + T_1 + 4U - 4\sqrt{UT_1}], \quad (40)$$

with X_i (i = 1,7) determined as a solution of eq. (39), the relation (36) will give the required critical Rayleigh number R_c (if $R < R_c$ the system is stable and for $R > R_c$ the system is unstable).

It is easy to show that for stationary convection ($\sigma = 0$; $\sigma^* = 0$; $\bar{\sigma} = 0$; $b_1 = b_2 = b_3 = 0$), the results are identical with the relations obtained by Gupta and Singh [17] (eqs. 27–31).

The numerical result and comparatively graphic analysis for our model can be seen in Figures 1–5. It may be observed in Figure 5, by comparision with Figure 3, a special behaviour of a critical Rayleigh number R_c as a function of Q_1 for the same assigned values of T_1.

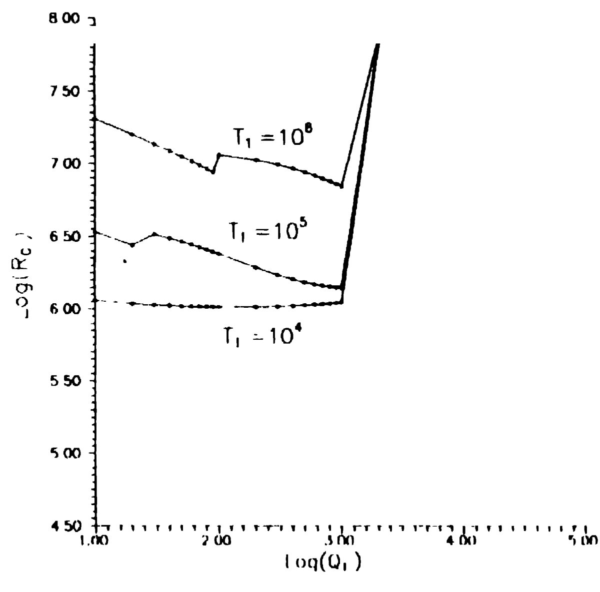

Figure 5. The critical Rayleigh number R_c as a function of Q_1 for various assigned values of T_1 in our present model for stationary convection in the presence of FLR (U = 1000) and thermosolutal (S_1 = 10000) effects. It will be observed a stabilizing influence of FLR and stable solute gradient on thermosolutal instability.

The positive value for dR_1/dV_1, where $x = a^2/\pi^2 > 2$ for all values of V_1 (or $V_1 > 2$ for all values of x) permits to conclude that for $x > 2$, the effect of rotation is always stabilizing (see Figure 6). Because dR_1/dU is always positive and $dR_1/dS_1 = 1$ according to Gupta and Singh [17], the FLR and stable solute gradient have stabilizing effects on the thermosolutal instability (see Figure 7).

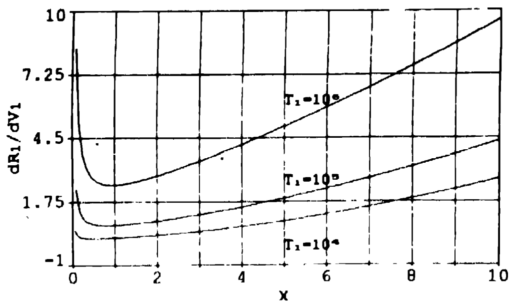

Figure 6. Variation of dR_1 / dV_1 for different x and several values of T_1 (10^4, 10^5, 10^6) and $Q_1 = 10^5$

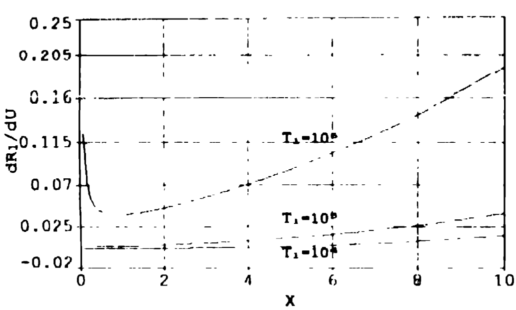

Figure 7. Variation of dR_1 / dU for different x and several values of T_1 (10^4, 10^5, 10^6) and $Q_1 = 10^5$

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