# *Letters to the Editor*

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## CHERENKOV RADIATION BY LINE CHARGE


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Problems of Cherenkov radiation of various aspects have been investigated in many papers. These are mainly concerned with the motion of a point charge or a beam of electrons. This paper deals with the case in which radiation is produced by a line charge rather than a point charge or a beam of electrons. The line charge consists of a continuous line density $\alpha$ moving with a constant velocity $v$ perpendicular to the line within a dielectric medium. The medium is assumed to be a perfect dielectric and the absorption of radiation, dispersion and scattering are ignored. The medium is unbounded, so the sweeping plane is infinite.

Let the line charge be parallel to $z$ axis and moving along the $y$ axis. Maxwell's equations for field variables $E$ and $H$ after expanding in Fourier series (e.g.)

$\mathbf{E} = \int_{-\infty}^{\infty} E_\omega e^{i\omega t} \, d\omega, \ldots)$ are

$$\left.\begin{aligned} \text{rot}\, H_\omega &= \frac{i\omega n^2}{c} E_w + \frac{4\pi}{c} j_w \\ \text{rot}\, E_\omega &= -\frac{i\omega}{c} H_w \\ \text{div}\, E_\omega &= \frac{4\pi}{n^2}\rho \\ \text{div}\, H_\omega &= 0, \end{aligned}\right\} \quad \ldots (1)$$

where $n$ is the refractive index of the medium at the frequency $w$, $c$ is the velocity of light, $j$ is the current density and $\rho$ is the density of free charges.

Introducing vector and scalar potentials $A$ and $\phi$ we have

$$\nabla^2 A_w + \frac{n^2\omega^2}{c^2} A_w = -\frac{4\pi}{c} j_w \quad \dots (2)$$

with the conditions

$$\left.\begin{aligned} H_w &= \text{rot } A_w \\ E_w &= -\frac{i\omega}{c} A_w - \text{grad } \phi_w \\ \text{div } A_w &+ \frac{i\omega n^2}{c} \phi_w = 0 \end{aligned}\right\} \quad \dots (3)$$

Here $j_x = 0 = j_z$ and $j_y = \alpha v \delta(x)\delta(y-vt)$

$$\therefore \; j_y(\omega) = \frac{\alpha}{2\pi} \delta(x) e^{-iw\frac{y}{v}} \quad \dots (4)$$

From symmetry the motion is independent of $z$.

Taking $A_x = 0 = A_z$ and $A_y = u(x)\, e^{-\frac{iwy}{v}}$ ... (5)

We obtain from (2)

$$\frac{\partial^2 u}{\partial x^2} + s^2 u = -\frac{2\alpha}{c}\delta(x) \quad \dots (6)$$

where $s^2 = \frac{w^2}{v^2}(n^2\beta^2 - 1)$, $\beta = \frac{v}{c}$.

If $s^2 < 0$ the equation (6) does not contribute any wave solution. If $s^2 > 0$, i.e. the velocity of the line charge is greater than the phase velocity of light in the medium, then

$$u = Be^{-isx} \quad \text{when } x > 0$$

and

$$u = B'e^{isx} \quad \text{when } x < 0 \quad \dots (7)$$

where $B$ and $B'$ are constants.

Considering the singularity at $x = 0$,

$$B = -\frac{i2\alpha}{cs} \quad \dots (8)$$

$$\therefore \quad A_y(\omega) = -\frac{i2\alpha}{cs}\, e^{-isx-i\frac{\omega}{v}y+i\omega t} \text{ when } \quad x > 0.$$

In this case

$$\left.\begin{aligned} H_z(w) &= -\frac{2\alpha}{c}\, e^{-isx-i\frac{\omega}{v}y+i\omega t} \\ E_x(w) &= \frac{2\alpha}{vn^2}\, e^{-isx-i\frac{\omega}{v}y+i\omega t} \\ E_y(w) &= -\frac{2\alpha s}{wn^2}\, e^{-isx-i\frac{\omega}{v}y+i\omega t} \end{aligned}\right\}$$

$$\left.\begin{aligned} ReH_z &= -\frac{4\alpha}{c}\int_0^{\infty} \cos\chi\, dw \\ ReE_y &= -4\alpha\int_0^{\infty} \frac{s}{wn^2}\cos\chi\, dw \end{aligned}\right\}$$

where $\chi = w\left\{t-\left(\frac{sx}{w}+\frac{y}{v}\right)\right\}$

when $x < 0,$

$$H_z(\omega) = \frac{2\alpha}{c}\, e^{isx-i\frac{\omega}{v}y+i\omega t}$$

$$E_x(\omega) = -\frac{2\alpha}{vn^2}\, e^{isx-i\frac{\omega}{v}y+i\omega t}$$

$$E_y(\omega) = -\frac{2\alpha s}{\omega n^2}\, e^{isx-i\frac{\omega}{v}y+i\omega t}$$

## CONCLUSION

(i) Equation (9) and (11) reveal that outgoing plane waves are propagating when $s > 0$. At a particular frequency planes are parallel to $sx+\frac{\omega}{v}y=0$ and $sx-\frac{\omega}{v}y=0$.

(ii) Radiation is confined between two planes perpendicular to the above planes similar to cone as in the case of a point charge. The angle between the planes of radiation is $2\cos^{-1}\frac{1}{\beta n}$ which is identical with the Cherenkov relation.

(iii) Cherenkov radiation per unit time is

$$\frac{dw}{dt} = 2\int_{-\infty}^{\infty} \frac{c}{4\pi}[EH]\,dy$$

$$= \frac{c}{2\pi}\int_{-\infty}^{\infty} (ReH_z \,.\, ReE_y)dy$$

$$= 8\alpha^2 \int_0^{\infty} \frac{1}{n^2}\sqrt{n^2\beta^2-j}\,d\omega$$

Thus per unit of frequency interval the amount of radiation is $\frac{8\alpha^2}{n^2}\sqrt{n^2\beta^2-1}$. It is interesting to note that the whole of the frequency dependence is via $n$.

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