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Analysis of bent microstrip resonator using finite element method

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Abstract : The microstrip resonator is analysed with fixed height and breadth for different length using finite element method (FEM). The effect of length variation on 110, 210, 111, 211 mode frequencies is observed. Further, by considering the straight patch resonator to be divided into two equal parts along its length, and by bending the second part through an angle θ, with respect to the first part, the effect of such bending on the above mentioned mode frequencies is observed. The effect of bending on the equivalent length of the resonator is also studied for these modes.

Keywords : Finite element method, microstrip resonator

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1. Introduction

The resonant frequencies of open microstrip ring resonators are determined by [1,2]. The microstrip resonator with equilateral triangular patch is studied by Wolf and Knoppik [3], Helszain and David [4] and Kuester and Chang [5]. Helszain and David [4] have obtained transverse magnetic (TM) mode solutions from duality with transverse electric (TE) mode solutions with electric boundaries, whereas Kuester and Chang [5] have obtained the required solutions by geometrical theory. The triangular and rectangular patch microstrip resonator is analysed by Kalamse and Patil [6,7] using finite element method.

In this paper, we have analysed the rectangular patch microstrip resonator with different length and the effect of bending of the half part of the patch, on different mode frequency and on equivalent length of the resonator, using FEM.

2. Statement of the problem

Consider the rectangular microstrip patch resonator bounded by six faces B1, B2, B3, B4, B5, B6, the cross section of which is shown in Figure 1(a). The two side surfaces B_1 and B_2

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are magnetic walls. Similarly, front and back *i.e.* B_3 and B_4 are magnetic walls. The top and the bottom surfaces are electric walls.

The electric field within the resonator will satisfy Maxwell's equations

$$\text{Curl Curl } \overline{E} - K^2\overline{E} = 0, \quad (1)$$

$$\text{grad div } \overline{E} - \nabla^2\overline{E} - K^2\overline{E} = 0. \quad (2)$$

Since the medium is charge free, div $\overline{E} = 0$.

$$\nabla^2\overline{E} + K^2\overline{E} = 0. \quad (3)$$

The electric field within the substrate has only Z component and magnetic field has X and Y components. The tangential component of magnetic field at the edge is negligible.

$$\text{div grad } E_z + K^2E_z = 0. \quad (4)$$

The fields within the resonator corresponding to TM modes will be generated by the equation

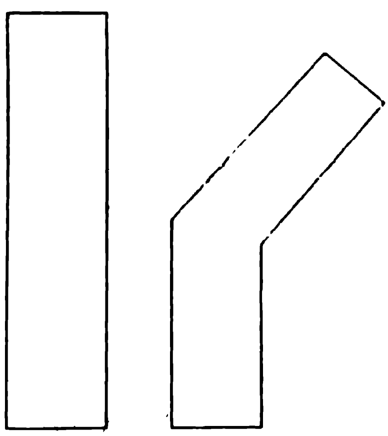
$$\nabla^2E_z + K^2E_z = 0, \quad (5)$$

subjected to the boundary condition

$$\left.\frac{\partial E_z}{\partial n}\right|_{B_1, B_2, B_3, B_4} = 0, \quad (6)$$

$$\left.E_z\right|_{B_5, B_6} = 0, \quad (7)$$

where E_z is z component of $\overline{E}$, $\partial/\partial n$ represents normal derivative



(a) (b)

Figure 1. (a) Cross section of straight patch resonator. (b) Cross section of bent patch resonator

3. Variational formulation

To get the expression for the functional Π in variational formulation, multiply eq. (4) by some weight function V^* and integrate it over the domain of the resonator. Then

$$\Pi = \iiint_{\Omega} V^* \operatorname{div} \operatorname{grad} E_z \, d\Omega + K^2 \iiint_{\Omega} V^* E_z \, d\Omega. \tag{8}$$

Using the vector identity $S \operatorname{div} \bar{A} = \operatorname{div} (S\vec{A}) - (\operatorname{grad} S). \bar{A}$ for the first term, eq. (8) becomes

$$\Pi = \iiint_{\Omega} \operatorname{div} (V^* \operatorname{grad} E_z) \, d\Omega - \iiint_{\Omega} (\operatorname{grad} V^*). (\operatorname{grad} E_z) \, d\Omega$$

$$+ K^2 \iiint V^* E_z \, d\Omega. \tag{9}$$

Applying Gauss Divergence theorem to the first term in eq. (9), we get

$$\Pi = - \iiint_{\Omega} (\operatorname{grad} V^*). (\operatorname{grad} E_z) \, d\Omega + k^2 \iiint_{\Omega} V^* E_z \, d\Omega$$

$$+ \iint_{S} V^* \operatorname{grad} E_z . \bar{n} \, ds. \tag{10}$$

Using eq. (6), the last term in eq. (10) will vanish.

By substituting $V^* = E_z$ and changing the sign, eq. (10) becomes

$$\Pi = \frac{1}{2} - \iiint_{\Omega} (\operatorname{grad} E_z) \cdot (\operatorname{grad} E_z) d\Omega - k^2 \frac{1}{2} - \iiint_{\Omega} E_z E_z d\Omega, \tag{11}$$

1/2 is introduced since Π is bilinear functional.

The first variation $\delta\Pi$ is given by

$$\delta\Pi = \delta - \frac{1}{2} - \iiint_{\Omega} (\nabla E_z) \cdot (\nabla E_z) d\Omega - k^2 \iiint E_z E_z d\Omega \tag{12}$$

For Π to be stationary, $\delta\Pi$ should be minimum.

4. Discretization

According to standard finite element method [8,9], the volume of resonator is divided into hexahedral elements with 20 nodes. The mapping functions assumed for these elements are quadratic in nature. The functional over an element is given by

$$\Pi^e = \sum_{ele} - \frac{1}{2} - \{E_z^e\}^T [S^e] \{E_z^e\} - \frac{1}{2} - K^2 \sum_{ele} \{E_z^e\}^T [T^e] \{E_z^e\}, \tag{13}$$

where $$S_{ij} = \int \left[\frac{\partial F_i}{\partial x} \frac{\partial F_j}{\partial x} + \frac{\partial F_i}{\partial y} \frac{\partial F_j}{\partial y} + \frac{\partial F_i}{\partial z} \frac{\partial F_j}{\partial z} \right] dx dy dz \tag{14}$$

and $$T_{ij} = \int F_i F_j \, dx \, dy \, dz. \tag{15}$$

Here, F_i is the mapping function due to i-th node and integrations are over mesh element surface.

The functional for the whole region Ω is given by

$$\Pi = -\frac{1}{2} - \{E_z\}^T [S] \{E_z\} - \frac{1}{2} - K^2 \{E_z\}^T [T] \{E_z\}. \tag{16}$$

The condition that variation of Π must be minimum *i.e.* zero, gives

$$[S]\{E_z\} - K_0^2 [T]\{E_z\} = 0. \tag{17}$$

Eq. (17) is the matrix equation to be solved to get eigenvalues and eigenvectors.

5. Numerical calculations for bent m.w. resonator

A rectangular microstrip patch of breadth 0.4 mm and height 0.318 mm is considered. The length of the resonator is changed from 3 mm to 5.0 mm in the steps of 0.1 mm. For each length, the eigenvalues and eigenvectors are calculated. These eigen values are square of

Table 1. Variation of frequency with length of the resonator for different modes.

Length in mm	f in GHz			
	110 Mode	210 Mode	111 Mode	211 Mode
3.0	47 465	48.249	51.044	51 774
3 1	47 448	48 183	51 029	51 712
3 2	47 433	48.123	51 015	51 657
3.3	47 419	48.068	51 002	51 606
3.4	47 407	48 019	50 990	51.559
3.5	47.395	47 973	50 979	51 517
3.6	47 384	47.931	50.969	51 478
3.7	47.375	47 892	50 960	51 442
3 8	47.366	47 857	50.952	51 409
3 9	47 357	47.824	50.952	51 392
4 0	47 350	47.793	50.937	51 394
4 1	47.343	47.765	50.930	51 323
4.2	47.336	47.738	50.924	51.299
4.3	47.330	47.714	50.919	51.277
4 4	47.324	47.691	50.913	51.254

Table 1. *(Cont'd.).*

Length in mm	*f* in GHz			
	110 Mode	210 Mode	111 Mode	211 Mode
4.5	47.319	47.670	50.914	51.245
4.6	47.313	47 649	50.904	51.217
4.7	47.309	47.631	50.905	51.203
4.8	47.304	47.618	51.034	51.482
4.9	47.300	47.597	50.891	51.167
5.0	47.296	47.581	50.888	51.152

the ratio of angular frequency and velocity in vacuum. The different modes of propagation are identified using the field plots. The variation of frequencies with length of resonator is given in Table 1 and shown graphically in Figure 2.

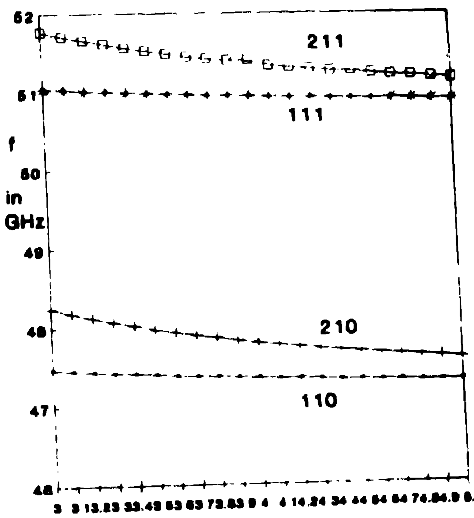


Figure 2. Variation of frequency for different modes with length of straight patch resonator.

The straight patch resonator of size 4 mm × 0.4 mm × 0.318 mm is then divided in two parts along its length, with equal size. The second part is bent through an angle θ with respect to the first part, as shown in Figure 1(b). The variation of frequency with angle of bend for these different modes is given in Tables 2 and 3 and shown graphically in Figure 3.

The frequencies for different modes in the bent position are compared with the corresponding mode frequencies for straight resonator and equivalent length of straight

Table 2. Variation of frequency and equivalent length with angle of bend for 110 and 210 modes.

Bending angle in degree	Mode 110		Mode 210	
	f in GHz	Equi. Length in mm	f in GHz	Equi. Length in mm
7 5	47.350	3.98	47.793	4.00
15.0	47.350	3.98	47.793	4.00
22 5	47 351	3.98	47.792	4.01
30.0	47.352	3.96	47.792	4.01
37 5	47.353	3.95	47.791	4.01
45 0	47.355	3 92	47.789	4.02
52 5	47.357	3.90	47.788	4.03
60.0	47.360	3.86	47.785	4.04
67.5	47.368	3.83	47.779	4.04
75.0	47 374	3.77	47 773	4.05
82.5	47.381	3.70	47 766	4.06
90.0	47 390	3.65	47.757	4 09
97.5	47.402	3.55	47 743	4.14
105.0	47.419	3.45	47 726	4 18
112.5	47 443	3 32	47 699	4 26
120.0	47.482	3.31	47 664	4 37

Table 3. Variation of frequency and equivalent length with angle of bend for 111 and 211 modes

Bending angle in degree	Mode 111		Mode 211	
	f in GHz	Equi Length in mm	f in GHz	Equi Length in mm
7 5	50 938	4 00	51 347	4 00
15.0	50 941	3.96	51.338	4 05
22 5	50 946	3 88	51 324	4 10
30 0	50.957	3 73	51.312	4 15
37 5	50 963	3 67	51 282	4.27
45 0	50.975	3 53	51 254	4.40
52 5	50 990	3 40	51 222	4 55
60 0	51 009	3 33	51.188	4 72
67 5	51 029	3.10	51.150	5 02
75 0	51 047	2.99	51 108	5.47
82 5	51.066	2.92	51 070	5.93
90 0	51 020	2.87	51 092	6.45

patch resonator is obtained. The variation of such equivalent length with angle of bend is given in Tables 2 and 3 and is shown graphically in Figure 4.

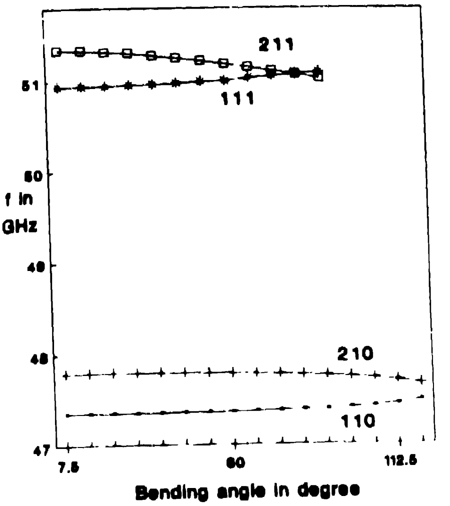


Figure 3. Variation of frequency for different modes with bending angle

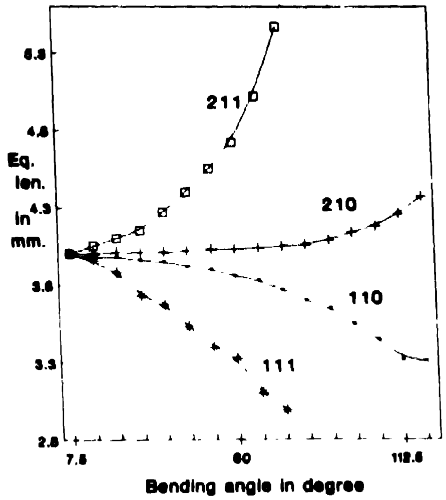


Figure 4. Variation of equivalent length for different modes with bending angle

6. Conclusions

The bending of half part of the patch of microstrip resonator affects the values of the frequency for different mode which is discussed below.

For 110 and 111 mode, the value of the frequency increases whereas that for 210 and 211 modes decreases slightly than the normal straight patch microstrip resonator.

The effective length of the patch resonator appearing because of bending, undergoes different changes for different modes. For modes 110 and 111, the effective length decreases than the actual length, with increase in bending angle θ. The rate of decrease is more for 111 mode than for 110 mode.

For modes 210 and 211, the effective length increases than the actual length, with increase in bending angle θ. The rate of increase in effective length, with θ, is more for 211 mode than for 210 mode.

These conclusions are useful in designing the microstrip resonator of particular frequency. Instead of using the resonator of more length, the same effect can be obtained by bending a small length resonator at its centre.

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