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A new quantum number for qqq baryons

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Abstract . A complex harmonic oscillator basis is proposed for a three-body system obeying S_3–symmetry, so as to generate an extraquantum number (N_a), over and above the (standard) total quantum number (N), for a more quantitative S_3–classification of the various qqq states than is possible in the usual (real) representation Further, certain bilinears in their complex forms with definite S_3–symmetry properties that can be constructed out of the linear h.o. operators (a, $a^\dagger$, a^*, $a^{*\dagger}$), satisfy several distinct SO (2, 1) algebras with spectra bounded from below. The 3–body wave function is written down in the complex basis

Keywords Three-body system, S_3–symmetry

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Introduction

It is very well known that the quantum state of a three-body system in the real basis is specified by | NJLS ; λ >. Here λ characterizes the S_3–symmetry status of the system in terms of 'symmetric' (s), 'antisymmetric' (a) and 'mixed–symmetric' (m), all of which are expressible in terms of two independent (Jacobi) variables (ξ_i, η_i) forming a [2, 1] representation of S_3–symmetry. However, a disadvanntage of this real representation is that the above S_3–symmetry charactrization (λ) is <u>not</u> quantitatively expressed in terms of some additional quantum numbers. This shortcoming is more acute in the case of relativistic three-body problem wherein a large number of S_3–symmetry breaking terms are present in the interaction hamiltonian than for the corresponding non-relativistic system. This can be taken care of by the use of a more promising approach, viz., the complex basis defined through

$$\sqrt{2}\, Z_i = \xi_i + i\, \eta_i \ ; \ \sqrt{2}\, Z_i^* = \xi_i - i\, \eta_i, \tag{1}$$

A new quantum number can be generated in a simple way, without going into any particular representation, through the quantity $\tan \nu = 2\xi\cdot\eta/(\xi^2 - \eta^2)$ and the related operator $- i\, \nu$ whose eigennfunctions are $\exp(i\nu\lambda)$ with eigenvalues $\lambda/2$ where $\lambda = 3n-1$, $3n$, $3n + 1$ (n=o, 1, 2. .) [1].

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The Harmonic representation, on the other hand, offers a more specific information since the dynamics is controlled by the two number operatiors

$$N_\xi = a_{\xi_i}^{\dagger} a_{\xi_i} \quad ; \quad N_\eta = a_{\eta_i}^{\dagger} a_{\eta_i} \tag{2}$$

where a, $a^\dagger$ are normalized ladder operators in ξ, η indices deffined in the standard manner. In this $(\vec{\xi}, \vec{\eta})$basis, however, only the sum $N = N_\xi + N_\eta$ is diagonal while the difference $N_\xi - N_\eta$ is not. This shows that the real $(\vec{\xi}, \vec{\eta})$basis does not provide an h.o. representation with good S_3-symmetry properties. A more promising structure, i.e. one more in conformity with S_3-symmetry would be obtained in terms of the following complex combinations,

$$\sqrt{2}\ (a_i,\ a_i^{*}) = a_{\xi_i} \pm i\ a_{\eta_i} \quad ; \quad \sqrt{2}\ (a_i^{\dagger},\ a_i^{*\dagger} = a_{\xi_i}^{\dagger} \mp i\ a_{\eta_i}^{\dagger} \tag{3}$$

satisfying the commutation relations

$$[a_i, a_j^{\dagger}] = [a_i^{*}, a_j^{*\dagger}] = \delta_{ij} \quad ; \quad [a_i, a_j^{*}] = 0 \tag{4}$$

In the new complex basis, the number operators

$$N_c = a_i^{\dagger} a_i \ ; \ N_c^{*} = a_i^{*\dagger} a_i \ ; \ N_m = a_i\ a_i^{*\dagger} \ ; \ N_m^{*} = N_m^{\dagger} = a_i^{*}\ a_i^{\dagger} \tag{5}$$

exhibit better symmetry properties manifested by the separate S_3-singlets

$$N = N_c + N_c^{*} = N_\xi + N_\eta \ ; \ N_a = N_c - N_c^{*} \neq N_\xi - N_\eta \tag{6}$$

N_a is thus a new quantum number having no counterpart in the $(\vec{\xi}, \vec{\eta})$ basis. Its eigenvalues, modulo 3, are a measure of departure of a given state with total quantum number N from a fully symmetric (antisymmetric) state. [The situation is analogous to the diagonality of the charge operator for a scalar field when expressed in complex basis (ϕ, ϕ^{*}), but not in the real basis, while the energy remains diagonal in both.] Thus the charge for the complex scalar field seems to play a role analogous to N_a in the three-body problem when viewed in the complex h.o. basis, a feature which clearly brings out the superiority of the complex basis over the real one.

SO (2, 1) Algebras of Bilinear Operators

To consider the SO(2, 1) algebras of two-step bilinear operators made out of the set (3), it is convenient to distinguish them in accordence with their S3-symmetry properties as follows

symmetric : $A = 2\, a_i a_i^{*} \ ; \ A^{\dagger} = 2\, a_i^{\dagger}\ a_i^{*\dagger}$ (7)

mixed-symmetric :

$$C = a_i\ a_i \ ; \ C^{*} = a_i^{*}\ a_i^{*} \ ; \ C^{\dagger} = a_i^{\dagger}\ a_i^{\dagger} \ ; \ C^{*\dagger} = a_i^{*\dagger}\ a_i^{*\dagger} \tag{8}$$

We can identify three distinct sets of commuting SO (2, 1) algebras whose spectra are bounded from below. Of these, the S_3-symmetric set (A, $A^{\dagger}$, N+3) satisfy the commutation relations

$$[A, N] = 2A\ ,\ [A^{\dagger}, N] = -\ 2A^{\dagger}\ ,\ [A, A^{\dagger}] = 4\ (N+3) \tag{9}$$

so that the three normalized components are

$$Q_+ = A^{\dagger}/2 \ ; \ Q_- = -\ A/2 \ ; \ Q_3 = (N+3)/2$$

and the corresponding Casimir is

$$u(u+1) = -(AA^\dagger + A^\dagger A)/8 + (N+3)^2/4. \qquad (10)$$

Since this spectrum is bounded from below [2], the eigenvalues of Q_3 are $-u+k$, $k = 0, 1, 2..$ on the one hand, and $(N+3)/2$ on the other. Thus

$$u(u+1) = 3/4 \text{ (even N) } ; + 2 \text{ (odd N)}. \qquad (11)$$

Similarly, for the mxed symmetric set $C, C^\dagger, N_c$ satisfying

$$[C, N_c] = 2C\ ,\ [C^\dagger, N_c] = -2C^\dagger\ ,\ [C, C^\dagger] = 4\,(N_c + 3/2)\ , \qquad (12)$$

the corresponding Casimir is

$$u_c\,(u_c+1) = -(CC^\dagger + C^\dagger C)\,/\,8 + (N_c+3/2)^2/4 \qquad (13)$$

where

$$u_c\,(u_c+1) = -3/16 \text{ (even } N_c) \ ;\ 5/16 \text{ (odd } N_c). \qquad (14)$$

Identical results hold for starred counterpart $(C^*, C^{*\dagger}, N_c^*)$.

Finally, the antisymmetric set $(N_m, N_m^\dagger, N_a)$ satisfies a normal SO(3) algebra

$$[N_m, Na] = 2\,N_m\ ,\ [N_m^\dagger, N_a] = -2\,N_m^\dagger\ ,\ [N_m, N_a^\dagger] = -N_a \qquad (15)$$

with the Casimir

$$s(s+1) = (N_m\,N_m^\dagger + N_m^\dagger\,N_m)\,/\,2 + N_a^2\,/\,4 \qquad (16)$$

so that spectra is bounded ffrom both below and above, just like (spin) angular momentum.

$$-s \le Na \le s\ ;\ s = N\,/\,2 \qquad (17)$$

Three-Quark Wave Function

As to the signatures of the new quantum number, N_a, (Sec. 1) consider a quark-level three-body problem, viz., the baryon which, after taking out the antisymmetric color part $(\varepsilon_{\alpha\beta\gamma}\,/\,\sqrt{6})$, has a symmetric wave unction in orbital (ψ), spin (χ), isospin (ϕ) degrees of freedom. In the same phase and normalization as eq.(1) for the complex quantity Z_i, we denote $(\psi_c\ ;\ \psi_c^*)$ and $(\psi_s\ ;\ \psi_a)$ as the respective doublet and singlet representations of S_3 w.r.t. the permutation symmetry, and the same notation applies for the χ, ϕ. In this notation, the various SU(6) states are expressed as

$$|56\rangle^q = \psi^s\chi^s\phi^s \quad ; \quad |56\rangle^d = \psi^s\,(\chi_c\phi_c^* + \chi_c^*\phi_c)/\sqrt{2} \qquad (18)$$

$$|70\rangle^q = \chi^s(\psi_c\phi_c^* + \psi_c^*\phi_c)/\sqrt{2}\ ;\ |70\rangle^d = (\psi_c^*\chi_c^*\phi_c^* + \psi_c^*\chi_c^*\phi_c^*)/\sqrt{2} \qquad (19)$$

$$|20\rangle^q = \psi^a\chi^s\phi^a \quad ; \quad |20\rangle^d = \psi^a\,(\chi_c\phi_c^* - \chi_c^*\phi_c)/\sqrt{2} \qquad (20)$$

Applications to baryonic spectra are given elsewhere [3].

The authors felicitate Prof. Haridas Banerjee on his 60th Birthday

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