

## Verification of the semiempirical mass formulae for photon attenuation coefficients

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**Abstract** : The absorption coefficients of photo-electric, Compton and pair production and hence the total absorption coefficient ( $\mu_a/\rho$ ) have been computed using the semiempirical formulae putforth by Massaro *et al* and the mixture rule for some chemical compounds containing H, C and O atoms in the energy range of 54 to 1333 keV. These values of total absorption coefficients of H, C and O are compared with those of Hubbell and find that there is a good agreement between the two.

**Keywords** : Photon attenuation coefficients, semiempirical mass formula

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### 1. Introduction

The photon attenuation and absorption coefficients in some materials are of interest for industrial, biological, agricultural and medicinal studies. However in certain cases, when the experimental data were insufficient to obtain the total cross sections some theoretical calculations have also been made using empirical relations (1–3). It is well known that the main contribution to mass attenuation values in photon-atom interaction comes from photo-electric, Compton and pair production and the cross sections of these individual effects depend upon the photon energy  $E_r$  and atomic number  $Z$  of the elements. Massaro *et al* [1] have putforth semiempirical formulae for the calculation of  $\Gamma$ -ray absorption coefficients for photoelectric effect ( $\tau/\rho$ ), Compton effect ( $\sigma_c/\rho$ ) and pair production ( $k/\rho$ ). These formulae are given as :

$$(\tau/\rho) = \frac{a_1}{E_r} + \frac{a_2}{E_r^2} + \frac{a_3}{E_r^{3.5}} + \frac{a_4}{E_r^4} \text{ cm}^2 \text{ gm}^{-1}, \quad (1)$$

where  $E_r$  is the photon energy in MeV and  $a_1, a_2, a_3$  and  $a_4$  are constants of elements.

$$(\sigma_a/\rho) = s \frac{c_1 + c_2 E_T + c_3 E_T^2}{1 + d_1 E_T + d_2 E_T^2 + d_3 E_T^3} \text{ cm}^2 \text{ gm}^{-1}, \quad (2)$$

where  $c_1, c_2, c_3$  and  $d_1, d_2, d_3$  are constants whose values are fixed.  $s$  is a constant depending on the element.

$$(k/\rho) = p_1 \log E_T + p_2 + \frac{p_3}{E_T} + \frac{p_4}{(E_T - 0.7)^3} \text{ cm}^2 \text{ gm}^{-1}, \quad (3)$$

in which  $p$ 's are constants. The values of all the constants appearing in eqs. (1–3) are given by Massaro *et al* [1].

Measurements of total mass attenuation coefficient ( $\mu/\rho$ ) for hydrocarbons in the energy range of 33–662 keV were made by Bradley *et al* [4] and have shown that there exists some regularity between total mass attenuation coefficient ( $\mu/\rho$ ) and hydrogen weight fractions  $f_h$ . A similar study has been made by Kateb and Hamid [5] for thirteen compounds containing H, C and O atoms in the energy range of 54–1333 keV.

## 2. Procedure

In the present study we have calculated the absorption coefficients for photo-electric effect, Compton effect and pair production at different energies ranging from 54–1333 keV for ten compounds.

For the calculation of absorption coefficients of the elements H, C and O we have used the semiempirical relations (1–3) [1]. To compare these values with the values obtained using Hubbell [6,7] tables, the total mass attenuation coefficients ( $\mu/\rho$ ), Compton scattering and photo-electric cross sections at different energies for the elements H, C and O are extracted from the tables of Hubbell [6,7] using the log-log interpolation scheme [8],

$$\log \sigma(E) = \frac{\log(E/E_1) \log \sigma(E_2) + \log(E_2/E) \log \sigma(E_1)}{\log(E_2/E_1)}. \quad (4)$$

The Compton scattering attenuation coefficients ( $\sigma_s/\rho$ ) and photo-electric attenuation coefficient ( $\tau/\rho$ ) are calculated using the relation :

$$\sigma_{\text{tot}} = (\mu/\rho) \frac{A}{N_A}; \quad (5)$$

where  $A$  is the atomic weight of the element and  $N_A$  is the Avogadro number.

We get total mass absorption coefficients ( $\mu_a/\rho$ ) for elements H, C and O by using the relation,

$$\frac{\mu_a}{\rho} = \frac{\mu}{\rho} - \frac{\sigma_s}{\rho}. \quad (6)$$

Table 1 . The values of  $\mu_q/\rho$  for H, C, O calculated using Massaro *et al* [1] equations and Hubbell [6,7] tables in the energy range 0.054 – 1.333 MeV.

Element	$(\mu_q/\rho)$ cm <sup>2</sup> q <sup>-1</sup>	Energy in MeV											
		0.054 MeV	0.081 MeV	0.125 MeV	0.356 MeV	0.511 MeV	0.662 MeV	1.173 MeV	1.274 MeV	1.333 MeV			
Hydrogen													
	$(\mu_e/\rho)$	0.0211	0.0383	0.0578	0.0579	0.0525	0.0531	0.0565	0.0576	0.0556	0.0515	0.0591	0.0556
	$(\mu_p/\rho)M$	0.0221	0.0389	0.0501	0.0583	0.0511	0.0532	0.0556	0.0515	0.0556	0.0515	0.0591	0.0556
Carbon													
	$(\mu_e/\rho)$	0.0216	0.0228	0.0289	0.0296	0.0262	0.0270	0.0286	0.0282	0.0285	0.0263	0.0251	0.0252
	$(\mu_p/\rho)M$	0.0259	0.0284	0.0223	0.0204	0.0267	0.0275	0.0285	0.0282	0.0285	0.0263	0.0251	0.0252
Oxygen													
	$(\mu_e/\rho)$	0.0427	0.0249	0.0246	0.0296	0.0282	0.0290	0.0287	0.0271	0.0290	0.0276	0.0273	0.0279
	$(\mu_p/\rho)M$	0.0415	0.0223	0.0229	0.0201	0.0265	0.0273	0.0290	0.0276	0.0290	0.0276	0.0273	0.0279

Table 2. The weight fraction of hydrogen vs the total mass absorption coefficient for the energy range 54–1333 keV.

Material	Weight fraction $f_H \times 10^{-2}$	Energy, MeV									
		0.054 ( $\mu_0/\rho$ )	0.081 ( $\mu_0/\rho$ )	0.125 ( $\mu_0/\rho$ )	0.356 ( $\mu_0/\rho$ )	0.511 ( $\mu_0/\rho$ )	0.662 ( $\mu_0/\rho$ )	1.173 ( $\mu_0/\rho$ )	1.274 ( $\mu_0/\rho$ )	1.333 ( $\mu_0/\rho$ )	
Benzaldehyde	5.69	0.1872	0.1676	0.150	0.1059	0.0915	0.0818	0.0619	0.0595	0.0582	
Acetophenone	6.71	0.1885	0.1691	0.152	0.1069	0.0924	0.0826	0.0624	0.06	0.0587	
Benzylalcohol	7.46	0.1899	0.1703	0.153	0.1077	0.0930	0.0831	0.0629	0.0605	0.0591	
Ethylacetate	7.75	0.1938	0.1715	0.153	0.1079	0.0932	0.0833	0.0631	0.0607	0.0593	
Acetylacetone	8.05	0.1935	0.1718	0.154	0.1082	0.0935	0.0836	0.0633	0.0608	0.0595	
Glycerol	8.75	0.1977	0.1736	0.155	0.1088	0.0948	0.0848	0.0643	0.0613	0.0599	
Formalin	8.39	0.1992	0.1751	0.156	0.1098	0.0940	0.084	0.0638	0.0618	0.0604	
Methanol	12.58	0.2034	0.1793	0.156	0.1126	0.0972	0.0869	0.066	0.0634	0.062	
Ethanol	13.12	0.2018	0.1795	0.161	0.1132	0.0977	0.0874	0.0662	0.0636	0.0622	
Ether	17.7	0.2005	0.1798	0.161	0.1137	0.0982	0.0878	0.0664	0.0639	0.0625	

Thus we have calculated the absorption coefficients for H, C and O atoms using Hubbell [6,7] tables.

The total absorption coefficients for different compounds are calculated using the sum rule,

$$(\mu/\rho) = \sum W_i (\mu_i/\rho_i), \tag{7}$$

where  $W_i$  is the proportion by weight of the  $i$ -th element and  $\mu_i$  and  $\rho_i$  are the linear attenuation coefficient and density of the  $i$ -th constituent respectively.

### 3. Results and discussion

In Table 1, we have given the values of total mass absorption coefficients of H, C and O atoms individually. In the first row, the values of total mass absorption coefficients  $(\mu_a/\rho)$  calculated using Hubbell [6,7] tables are given and in the second row, the values  $(\mu_a/\rho)_M$  computed using Massaro *et al* [1] equations are given. We find that there is a good agreement between the two values.

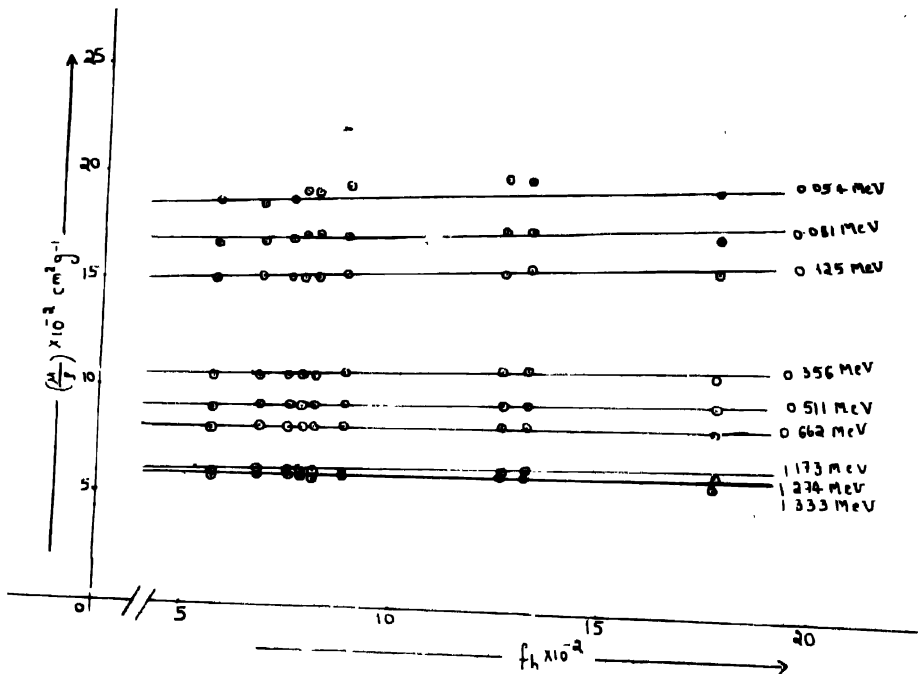


Figure 1. The mass absorption coefficients of HCO-materials calculated using Massaro *et al* [1] equations in the energy range of 54 keV to 1333 keV. Hydrogen weight fraction  $f_h$  ranging from 0.057 to 0.1777.

Further the values of total absorption coefficients for each compound calculated using Massaro *et al* [1] equations are given in Table 2.

In Figure 1, we have plotted the values of total absorption coefficients vs hydrogen weight fractions. The slopes are positive and large for low energetic photons in comparison with high energy photons.

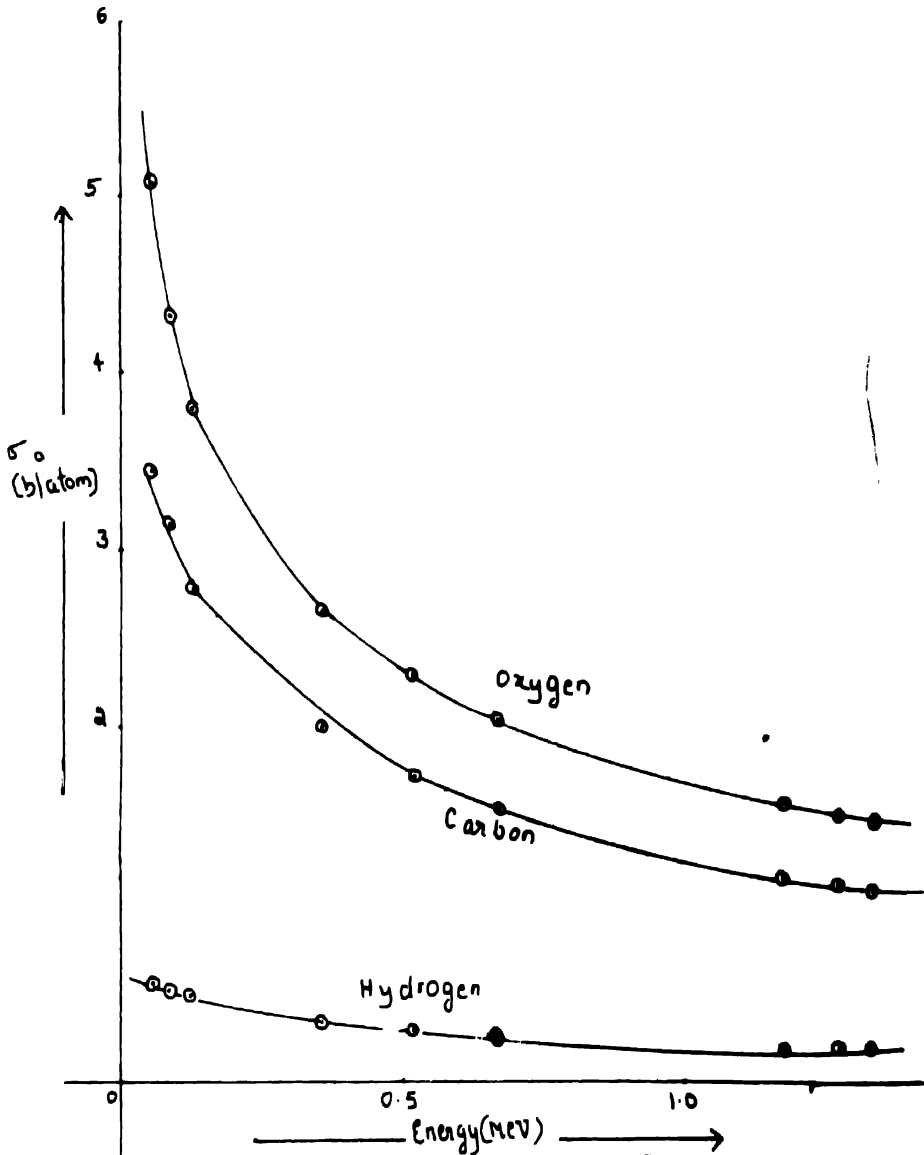


Figure 2. The values of absorption cross sections of hydrogen, carbon and oxygen at energy  $E_\gamma$  in the energy range of 54 keV – 1333 keV

Further, we have determined absorption cross sections for H, C and O using the calculated values of total absorption coefficients for different compounds using the relation,

$$\sigma_a = (\mu_a/\rho) \frac{A_r}{N_A} \quad (8)$$

where  $A_r$  is the relative molecular weight and  $N_A$  is the Avogadro number. The values are given in Table 3.

**Table 3.** The values of absorption cross section  $\sigma_a$  of H, C and O at  $54 \text{ keV} < E_\gamma < 1333 \text{ keV}$ .

$E_\gamma$ in MeV	$\sigma$ (in barns)		
	Hydrogen	Carbon	Oxygen
0.054	0.5560	3.5081	5.0880
0.081	0.5171	3.1591	4.3121
0.125	0.4689	2.8380	3.7967
0.356	0.3320	2.0024	2.6596
0.511	0.2864	1.7291	2.2982
0.662	0.2564	1.5457	2.0538
1.173	0.1935	1.1667	1.5676
1.274	0.1867	1.1228	1.5038
1.333	0.1826	1.0989	1.4693

In Figure 2, we give the plot of absorption cross section vs photon energy. From the figure we see that absorption cross section increases as the energy decreases and the decrease is more pronounced in the case of oxygen and carbon than hydrogen. This may be attributed to the atomic number of the elements.

#### 4. Conclusion

From the above results, we may conclude that in the absence of experimental data, the semiempirical formulae of Massaro *et al* [1] can be used to compute  $(\mu_a/\rho)$  of any element.

Further, the results support the mixture rule.

#### References

- [1] E Massaro, E Costa and M Salvati 1982 *Nucl. Instrum. Meth.* **192** 423
- [2] G C Chikkur and S M Hanagodimath 1989 *Indian J. Pure Appl. Phys.* **27** 132
- [3] G C Chikkur and S M Hanagodimath 1992 *Asian J. Phys.* **1** 71
- [4] D A Bradlev, C S Chang and A M Ghose 1986 *Appl. Radiat. Isot.* **37** 1195

- [5] A H El-Kateb and A S Abdul Hamid 1991 *Appl. Radiat. Isot.* **42** 303
- [6] J H Hubbell 1969 *Nat. Stand. Ref. Data Ser.* (National Bureau of Standards, Centre for Radiation Research, Washington D C.) **202341** 29
- [7] J H Hubbell 1982 *Int. J. Appl. Radiat. Isot.* **33** 1269
- [8] D E Cullen, M H Chen, J H Hubbell, S T Perkins, E F Plechaty, J A Rathkopf and J H Scofield October 1989 *UCRL-50400* 6