Indian J. Phys. **67B** (5), 437 – 442 (1993)

Study of propagation characteristics of relativistically moving plasma waveguide

K K Narayan, A B Gautam*, S K Singh** and Dinesh Singh
Department of Electronics, Gaya Post-graduate College, Gaya-823 001, Bihar, India
* S B A N College, Darheta Lari, Jehanabad, India
** Also at Department of Physics, Daudnagar College, Daudnagar, India

Received 9 March 1993, accepted 1 September 1993

Abstract : Propagation characteristics of rectangular waveguide filled with warm lossy plasma is studied Here the plasma is moving relativistically with respect to the guide walls with a constant velocity in the Z direction and a strong static magnetic field is applied in the X direction. It is found that the lossy characteristics of plasma makes propagation possible over all frequency ranges. It is also observed that the more warm is the plasma the higher is the cut off frequency

Keywords : Relativistic plasma, waveguide, propagation constant.

PACS Nos. : 52 40.Db, 52.40.Fd

1. Introduction

The subject of electromagnetic fields and waves in a waveguide filled with relativistically moving media has received considerable interest in recent years [1–8]. The problems of guided waves in moving media was first discussed by Collier and Tai [1] assuming that the medium velocity was much smaller than the velocity of light. Kong and Cheng [2] analysed the guided waves in a cylindrical waveguides of an arbitrary cross section filled with a moving anisotropic medium on the basis of Maxwell-Minkowski theory. For a general anisotropic medium they showed that pure TE or TM modes cannot exist in the waveguide and that the cut off frequencies are lowered by a factor which depends upon the velocity of the medium and is independent of the guide. Jain *et al* [3] studied the propagation of em waves in a rectangular waveguide containing longitudinally and transversely magnetized uniaxial anisotropic plasma moving relativistically by making use of Lorentz transformations and principle of phase invariance. Narayan *et al* [4] derived dispersion relation in case of rectangular waveguide containing cold isotropic lossy plasma. In this paper, the characteristics of propagation through a waveguide containing lossy warm plasma, moving with a high velocity with respect to the guide wall in the presence of strong transverse magnetic field is discussed.

2. Theory

With the help of linearized hydrodynamic equation of motion, the equation of continuity and the equation of state Maxwell's equations can be put into the following form,

$$\nabla' \times \overline{H}' = j\omega'\varepsilon_0\overline{\overline{\varepsilon}}'\overline{E}' + \hat{x}\left[\frac{-je'}{\omega' m\left(1 - \dfrac{jv'}{\omega'}\right)}\right]\frac{\partial p'}{\partial x'}, \tag{1}$$

$$\nabla' \times \overline{E}' = -j\omega'\mu_0\overline{H}', \tag{2}$$

where $\overline{\overline{\varepsilon}}' = \varepsilon_1\hat{x}'\hat{x}' + \hat{y}'\hat{y}' + \hat{z}'\hat{z}'$,

$$\varepsilon_1' = 1 - \frac{\omega_p'^2}{\omega'(\omega' - jv')} \quad \text{and} \quad \omega_p'^2 = \frac{n'e'^2}{m'\varepsilon_0}.$$

Eqs. (1) and (2) are basic equations characterizing the uniaxial and compressible property of plasma inside the guiding structure in the primed system. Making use of eqs. (1) and (2) and assuming Z' dependence of the type $e^{-\gamma' z'}$, one can express transverse fields with respect to the X' direction as

$$L'\begin{pmatrix} E_y' \\ E_z' \end{pmatrix} = \begin{bmatrix} \dfrac{\partial^2}{\partial x'\partial y'} & j\omega'\mu_0\gamma' \\ -\dfrac{\gamma'\delta}{\partial x'} & j\omega'\mu_0\dfrac{\partial}{\partial y'} \end{bmatrix}\begin{pmatrix} E_x' \\ H_x' \end{pmatrix}, \tag{3}$$

$$L'\begin{pmatrix} H_y' \\ H_z' \end{pmatrix} = \begin{bmatrix} j\omega'\varepsilon_0\gamma' & \dfrac{\partial^2}{\partial x'\partial y'} \\ -j\omega'\varepsilon_0\dfrac{\partial}{\partial y'} & -\gamma\dfrac{\partial}{\partial x'} \end{bmatrix}\begin{pmatrix} E_x' \\ H_x' \end{pmatrix}, \tag{4}$$

where $L' = \dfrac{\partial^2}{\partial x'^2} + k_0'^2$, $\quad k_0'^2 = \omega'^2\mu_0\varepsilon_0$.

Field components E'_x, H'_x and p are found to satisfy the following equations.

$$\left[\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \gamma'^2 + k_0'^2\varepsilon_1'\right]E_x' = \frac{e'}{\omega' m'\varepsilon_0} \times \left[\frac{k_0'^2}{\left(1 - \dfrac{jv'}{\omega'}\right)} - \frac{\omega'^2}{a'^2}\right]\frac{\partial p'}{\partial x'}, \tag{5}$$

$$\left[\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \gamma'^2 + k_0'^2\right] H'_x = 0\,, \tag{6}$$

$$\left[\frac{\partial^2}{\partial x'^2} + \frac{\omega'^2}{a'^2}\left(1 - \frac{j\nu'}{\omega'}\right)\right] p' = -\frac{n'e'\partial E'_x}{\partial x'} \tag{7}$$

TE modes :

For TE modes set $E'_x = 0$, then p' is found identically zero from eqs. (5) and (7). The only potential function H'_x which must satisfy eq. (6) gives the dispersion relation for TE modes and the characteristics are found to be the same as these of free space waveguide.

TM modes :

Using eqs. (5) and (7), boundary conditions and the transformation relating the parameters in the primed system to those in the unprimed system one obtains the following dispersion relations corresponding to various modes in the unprimed system.

$$B^4 - 2u\Omega B^3 + \left[6u^2\Omega^2 + M\right]B^2 + 2u\Omega\left[2u^2\Omega^2 + M\right]B + \left[u^4\Omega^4 + u^2M\Omega^2 - N\right] = 0\,, \tag{8}$$

where

$$M = \frac{\Omega_c^2}{r^2} - \left[1 - \frac{1}{r^2\left(\Omega'^2 + Z^2\right)}\right]\Omega'^2 - \frac{1}{r^2} \times \left[\frac{\Omega'^2\Omega_1^2\delta^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)}{\Omega'^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)^2 + Z^2\left(2\Omega'^2 - \Omega_1^2\delta^2\right)^2}\right] + \frac{1}{r^4}\left[\frac{\Omega_1^2\left(\Omega_1^2\delta^2 - \Omega'^2\right)}{\left(\Omega_1^2\delta^2 - \Omega'^2\right)^2 + (\Omega' Z)^2}\right],$$

$$N = \frac{\Omega'^2 Z^2}{4}\left[\frac{1}{r^2\left(\Omega'^2 + Z^2\right)} - \frac{\Omega_1^2/r^4}{\left(\Omega_1^2\delta^2 - \Omega'^2\right) + (\Omega' Z)^2} + \frac{\Omega_1\delta\left(2\Omega'^2 - \Omega_1^2\delta^2\right)}{r^4\left[\Omega'^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)^2 + Z^2\left(2\Omega'^2 - \Omega_1^2\delta^2\right)^2\right]}\right]^2$$

$$\Omega' = \Omega - uB\,, \qquad r^2 = \left(1 - u^2\right)^{-1}.$$

By setting $u = 0$ in equation (8) we get

$$B^4 + \left[\Omega_c^2 - \left(1 - \frac{1}{\Omega^2 + Z^2}\right)\Omega^2 - \frac{\Omega^2 \Omega_1^2 \delta^2 \left(\Omega_1^2\delta^2 - \Omega^2 + Z^2\right)}{\Omega^2\left(\Omega_1^2\delta^2 - \Omega^2 + Z^2\right)^2 + Z^2\left(2\Omega^2 - \Omega_1^2\delta^2\right)^2} \right.$$

$$\left. + \frac{\Omega_1^2\left(\Omega_1^2\delta^2 - \Omega^2\right)}{\left(\Omega_1^2\delta^2 - \Omega^2\right)^2 + \left(\Omega^2 Z^2\right)} \right] B^2 - \frac{Z^2\Omega^2}{4}\left[\frac{1}{\Omega^2 + Z^2}\right.$$

$$+ \frac{\Omega_1^2\delta^2\left(2\Omega^2 - \Omega_1^2\delta^2\right)}{\Omega^2\left(\Omega_1^2\delta^2 - \Omega^2 + Z^2\right)^2 + Z^2\left(2\Omega^2 - \Omega_1^2\delta^2\right)^2}$$

$$\left. - \frac{\Omega_1^2}{\left(\Omega_1^2\delta^2 - \Omega^2\right)^2 + \Omega^2 Z^2} \right]^2 = 0 . \qquad (9)$$

The cut-off frequencies for TM modes are found by setting B in eq. (8) equal to zero. Thus,

$$u^4\Omega_{co}^4 + u^2 M'\Omega_{co}^2 - N' = 0, \qquad (10)$$

where M and N' can be obtained by setting $B = 0$ in the expressions of M and N and $\Omega = \Omega_{co}$. Thus, when the plasma is in relativistic movement with respect to guide walls, the cutt-off frequencies for TM mode is changed by terms depending upon plasma parameters.

3. Results and discussion

Eq. (8) has been used to compute B for laboratory plasma parameters *i.e.*, $\Omega_c = 0.5$, $\Omega_1 = 0.5$, $\delta = 0.02$, $u = 0.5$ and $Z = 0.5$ and the result is shown in Figure (1). For comparison, eq. (9) has been used to compute normalized phase constant for stationary plasma medium and is shown in Figure (2). It is found that for lossy compressible plasma propagation is possible over all frequency ranges with cut-off frequency dependent on plasma parameters. The cut-off frequency for the lossy warm plasma shifted towards the high frequency side as compared to the cut off frequency obtained by Tuan [5] and Allis *et al* [6]. It is also observed that in the low frequency range, the propagation constant increases showing the forward wave and then decreases in non propagating region obtained in the study of Allis *et al* [6] showing the existence of a backward wave. Finally at higher microwave frequencies, the value of B approaches that of free space waveguide showing that there is no interaction between electrons and electromagnetic waves. It is concluded that lossy characteristics of plasma makes propagation possible over all frequency ranges. The cut-off frequency corresponding to the perturbed waveguide mode is shifted towards the higher frequency side due to the relativistic motion. The more warm is the plasma, the higher is the cut-off frequency.

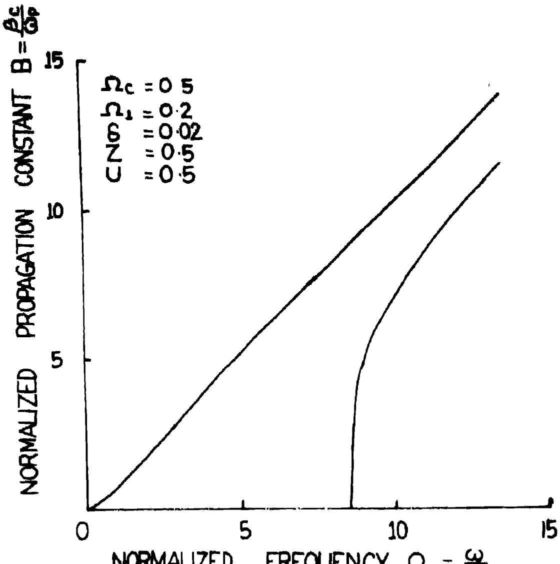

Figure 1. (*Ω*-*B*) diagram for TM modes i a rectangular waveguide filled with relativistically moving lossy warm plasma in presence of strong transverse magnetic field

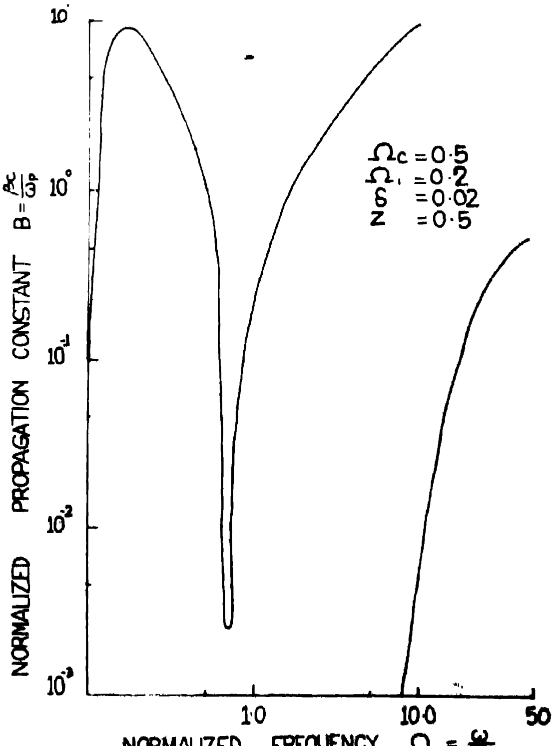

Figure 2. (*Ω*-*B*) diagram for TM modes in a rectangular waveguide filled with relativistically stationary lossy warm plasma in the presence of

References

[1] J R Collier and C T Tai 1965 *IEEE Trans Microwave Theory and Technique* **MTT-13** p 441

[2] J A Kong and D K Cheng 1968 *IEEE Trans Microwave Theory and Technique* **MTT-16** p 99

[3] P K Jain, D Singh and S K Tolpadi 1973 *IEEE Ant. Prop* A P 21

[4] K K Narayan, R K P Singh and D Singh 1992 *J Natl Acad. Sci Lett.* **Vol 15** 339

[5] H S Tuan 1969 *IEEE Trans Microwave Theory and Technique* **MTT-173** p 164

[6] W P Allis, S J Buchsbaum and A Bers 1963 *Waves in Anisotropic Plasma* (Cambridge, Massachusetts MIT Press)

[7] S Biswas and J Basu 1987 *IV Proc. Nat Symp Plasma Science and Technology* (Gandhinagar, Gujarat, Dec 28 - 30, 1987) p 16

[8] R K Singh, K K Narayan and D Singh 1993 *Indian J Pure Appl Phys.* **Vol 31** p 148