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Effect of parallel electric fields on the electrostatic ion-cyclotron instability in the presence of ion beam-particle aspect analysis. I

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Abstract : The electrostatic ion cyclotron instability has been studied, investigating the trajectories of the charged particles in the presence of electrostatic ion cyclotron waves The waves propagating at an angle to the geomagnetic field are considered. The effects of parallel electric fields are incorporated through the modification of distribution function which is anisotropic Maxwellian. The stabilising/destabilising effects by the parallel electric fields are studied, pertaining to simultaneous observations of electrostatic ion cyclotron waves and parallel potential drop in the auroral acceleration region. The effects of energetic ion beam on the wave generation processes are investigated, which may be a possible cause for the electrostatic ion cyclotron wave generation due to the upward flowing ion beams.

Keywords : Waves and instabilities, wave-particle interactions, charged particle motion and accelerations

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1. Introduction

Recently, interest has been devoted to the role of the high-latitude ionosphere in influencing magnetospheric dynamics or morphology. In particular, the problem of the high-latitude ionosphere as a source of magnetospheric plasma has come under intense study in the last few years. Several investigators have observed upward flowing accelerated ionospheric ions in the auroral zone space plasma [1–3].

Kintner *et al* [4], using S3–3 satellite data have measured electrostatic ion cyclotron waves near regions of upward flowing ions. At lower altitudes (400–600 Km) Yau *et al* [3] have observed ion cyclotron wave activity near ion transverse acceleration regions. Bering [5] has given evidence of ion cyclotron wave activity at low altitudes (350 Km) in the diffuse aurora. Basu *et al* [6] have observed apparent ion conics at approximately 350 Km in the presence of wave activity in the ion cyclotron range of frequencies and large field aligned currents.

Alfven [7] was first to point out the importance of the parallel electric field on auroral field lines. Axford and Hines [8] attributed the geomagnetic activity during disturbed periods

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to enhanced magnetospheric convection. The connection between magnetospheric convection and parallel electric fields has been studied by Coroniti and Kennel [9], Boström [10], Kan and Akasofu [11], Lennartsson [12], Goertz and Bosewell [13], Chiu *et al* [14], Lyons [15] and Sonnerup [16]. Kan and Lee [17] formulated a theory of steady state imperfect magnetosphere-ionosphere coupling ($E \neq 0$) in which the magnetosphere as well as the ionosphere is allowed to respond to the effects of the parallel electric field.

Therefore, in this paper we have investigated the effect of parallel electric fields on the electrostatic ion cyclotron instabilities, using modified distribution functions [18]. The analysis is based on a physical model used by Dawson [19] in his theory of Landau damping and further extended by Terashima [20], Misra and Tiwari [21] and Tiwari *et al* [22]. The plasma particles are divide into two groups ; non-resonant particles and resonant ones. It is assumed that only resonant particles are responsible for the energy exchange between waves and particles while the main plasma of non-resonant particles supports the oscillatory motions of waves. As compared with the discussion of Landau damping the inclusion of external magnetic field results in some differences in resonant condition and in scheme of wave particle interaction. The change in energy of resonant particles is computed under appropriate initial conditions by following their trajectories. The fluctuations in staying time of interacting particles are expressed as density variations in space. Instability criteria are derived from the rate of energy transfer for the respective cases.

2. Basic assumptions

We consider a homogeneous collisionless plasma in a uniform magnetic field B_0 along the z-direction. The ions are supposed to have the unit charge. It is assumed that an electrostatic wave in the form below starts at the time $t = 0$ when resonant particles are not yet disturbed. We assume a wave of the form :

$$\boldsymbol{k} \parallel E,\ \boldsymbol{k} = (k_\perp,\ 0,\ k_\parallel) \quad \text{and}\ E = (Ex,\ 0,\ Ez) \tag{1}$$

with

$$Ex\ (r,\ t) = E_1\ \cos\ (k_\perp x + k_\parallel z - \omega t)$$
$$Ez\ (r,\ t) = \kappa E_1\ \cos\ (k_\perp x + k_\parallel z - \omega t) \tag{2}$$

and

$$\kappa = k_\parallel / k_\perp \tag{3}$$

The amplitude E_1 is thought to be a slowly varying function of t that is $(1/E_1) \times (dE_1/dt) << \omega$.

3. Perturbed particle velocities

The equation of motion of a particle is given by the equation :

$$m\frac{dv}{dt} = q(E + (1/C)\, v \times B_0). \tag{4}$$

The velocity $\boldsymbol{v}$ can now be expressed in terms of the unperturbed velocity $\boldsymbol{V}$ and the perturbed velocity $\boldsymbol{u}$. The equation for $\boldsymbol{u}$ is:

$$d\boldsymbol{u}/dt - (q/mc)\,\boldsymbol{u} \times \boldsymbol{B}_0 = (q/m)(\boldsymbol{E}\ (\boldsymbol{r}(t),\ t)) \tag{5}$$

where in the argument of $\boldsymbol{E}$, $\boldsymbol{r}(t)$ in the form as below is to be substituted.

The trajectory of free gyration is given by

$$\begin{aligned} x\ (t) &= -(V_\perp/\Omega)\,[\sin(\theta-\Omega t) - \sin\theta] + x_0 \\ y\ (t) &= (V_\perp/\Omega)\,[\cos(\theta-\Omega t) - \cos\theta] + y_0 \\ z\ (t) &= V_\| t + z_0 \end{aligned} \tag{6}$$

where $\Omega = qB_0/(mc)$ is the ion cyclotron frequency and unperturbed velocity is taken as:

$$\begin{aligned} V_x\ (t) &= V_\perp \cos(\theta-\Omega t) \\ V_y\ (t) &= V_\perp \sin(\theta-\Omega t) \\ V_z &= V_\| . \end{aligned} \tag{7}$$

Substituting eq. (6) into eq. (2) we get:

$$\begin{aligned} Ex\ (\boldsymbol{r}\ (t),\ t) &= E_1 \cos\ (k_\perp x\ (t) + k_\| z\ (t) - \omega t) \\ &= E_1 \cos\ [-(k_\perp V_\perp/\Omega)\sin\ (\theta-\Omega t) + (k_\| V_\| - \omega)t \\ &\quad + (k_\perp V_\perp/\Omega)\sin\theta + k_\perp x_0 + k_\| z_0] \\ &= E_1 \sum J_n\ (\mu)\ \cos\psi_n \end{aligned}$$

where $\quad \mu = k_\perp V_\perp/\Omega, \qquad \psi_n = \Lambda_n t + \psi_n^0,$

$$\Lambda_n = k_\| V_\| - \omega + n\Omega, \qquad \psi_n^0 = -n\theta + \mu\sin\ \theta + \boldsymbol{k}.\boldsymbol{r}_0 . \tag{8}$$

Note that $\boldsymbol{r}_0 = (x_0,\ y_0,\ z_0)$ and are parameters at $t = 0$.

Now eq. (5) becomes:

$$du_\|/dt = (q\kappa E_1/m) \sum_{n=-\infty}^{\infty} J_n\ (\mu)\ \cos\left(\Lambda_n t + \psi_n^0\right) \tag{9}$$

$$du_+/dt + i\Omega u_+ = (qE_1/m) \sum_{n=-\infty}^{\infty} J_n\ (\mu)\ \cos\left(\Lambda_n t + \psi_n^0\right)$$

where $\quad u_+ = u_x + iu_y$.

Hence in general, we expect the resonance condition:

$$\Lambda_n\ (V_\| = V_r) = k_\| V_r - \omega + n\Omega = 0, \quad n = \pm 1, \pm 2. \tag{10}$$

We call those particles with $V_\| = V_r$, the resonant particles where V_r is the resonant velocity. To solve eq. (9) we set the initial condition for the resonant particles:

$$\boldsymbol{u}\ (t = 0) = 0. \tag{11}$$

The respective solution of eq. (9) are found to be :

$$u_x(t) = (qE_1/m) \sum_{n=-\infty}^{\infty} J_n(\mu)\Big[\big\{\Lambda_n/\big(\Lambda_n^2 - \Omega^2\big)\big\}\sin\psi_n$$

$$- \delta\sin\big(\psi_n^0 - \Omega t\big)/(2\Lambda_{n+1}) - \delta\sin\big(\psi_n^0 + \Omega t\big)/(2\Lambda_{n-1})\Big]$$

$$u_y(t) = (qE_1/m) \sum_{n=-\infty}^{\infty} J_n(\mu)\Big[\big\{\Omega/\big(\Lambda_n^2 - \Omega^2\big)\big\}\cos\psi_n$$

$$+\delta\cos\big(\psi_n^0 - \Omega t\big)/(2\Lambda_{n+1}) - \delta\cos\big(\psi_n^0 + \Omega t\big)/(2\Lambda_{n-1})\Big]$$

$$u_z(t) = (q\kappa E_1/m) \sum_{n=-\infty}^{\infty} J_n(\mu)\,(1/\Lambda_n)\left[\sin\psi_n - \delta\sin\psi_n^0\right] \quad (12)$$

where $\delta = 0$ for the non-resonant particles and $\delta = 1$ for resonant particles. This $\boldsymbol{u}(t)$ includes the variable t and also the initial parameters and can be written down as a function of $\boldsymbol{r}$ and t by eliminating the initial parameters with the use of eq. (6) that is :

$$u_x(\boldsymbol{r},t) = (qE_1/m) \sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu)J_l(\mu)\Big[\big\{\Lambda_n/\big(\Lambda_n^2 - \Omega^2\big)\big\}\sin\chi_{nl} -$$

$$\delta\sin(\chi_{nl} - \Lambda_{n+1}t)/(2\Lambda_{n+1}) - \delta\sin(\chi_{nl} - \Lambda_{n-1}t)/(2\Lambda_{n-1})\Big]$$

$$u_y(\boldsymbol{r},t) = (qE_1/m) \sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu)J_l(\mu)\Big[\big\{\Omega/\big(\Lambda_n^2 - \Omega^2\big)\big\}\cos\chi_{nl} +$$

$$\delta\cos(\chi_{nl} - \Lambda_{n+1}t)/(2\Lambda_{n+1}) - \delta\cos(\chi_{nl} - \Lambda_{n-1}t)/(2\Lambda_{n-1})\Big]$$

$$u_z(\boldsymbol{r},t) = (q\kappa E_1/m) \sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu)J_l(\mu)\,(1/\Lambda_n)$$

$$\left[\sin\chi_{nl} - \delta\sin(\chi_{nl} - \Lambda_n t)\right] \quad (13)$$

where

$$\chi_{nl} = \boldsymbol{k}.\boldsymbol{r} - \omega t + (n-l)\,(\Omega t - \theta). \quad (14)$$

4. Density perturbation

In order to find out density perturbation associated with the velocity perturbation $\boldsymbol{u}\,(\boldsymbol{r}, t)$, let us consider a group of particles with the same initial condition and let the number density be

$$n\,(\boldsymbol{r}, t; \boldsymbol{V}) = N\,(\boldsymbol{V}) + n_1\,(\boldsymbol{r}, t; \boldsymbol{V}) \quad (15)$$

where $N(\boldsymbol{V}) = N(V_\perp, V_\parallel)$ is unperturbed density and n_1 the perturbed density.

The conservation of particle number is :

$$n\,[\boldsymbol{r}\,(t'),\, t' = t + \Delta t;\, \boldsymbol{V}\,(t')]\, d\boldsymbol{r}' = n\,[\boldsymbol{r}\,(t),\, t;\, \boldsymbol{V}\,(t)]\, d\boldsymbol{r} \tag{16}$$

where $\boldsymbol{r}\,(t)$ is the particle trajectory, that is :

$$d\boldsymbol{r}(t)/dt = \boldsymbol{V}\,(t) + \boldsymbol{u}\,(t).$$

Using Taylor expansion of eq. (16) we get,

$$dn_1/dt = -N(V)\ (\nabla\cdot \boldsymbol{u}). \tag{17}$$

Changing the variables of the right hand side of eq. (17) in terms of t and the initial parameters with the help of eqs. (13) and (6) we get :

$$\begin{aligned} dn_1/dt = {} & qE_1k_\perp N(V)/m \sum_{n=-\infty}^{\infty} J_n(\mu)\Big[\Lambda_n\big/\left(\Lambda_n^2 - \Omega^2\right)\cos\psi_n - \\ & \delta\cos\left(\psi_n^0 - \Omega t\right)\big/(2\Lambda_{n+1}) - \delta\cos\left(\psi_n^0 + \Omega t\right)\big/(2\Lambda_{n-1})\Big] - \\ & qE_1\kappa k_\parallel N(V)/m \sum_{n=-\infty}^{\infty} J_n(\mu)\ (1/\Lambda_n)\ \left(\cos\psi_n - \delta\cos\psi_n^0\right). \end{aligned} \tag{18}$$

Here, the second bracket on the right hand side denotes the density variation due to velocities along $\boldsymbol{B}_0$ while the first term denotes the plane normal to $\boldsymbol{B}_0$. By integration we find $n_1\,(t)$ the perturbed density for the non-resonant particles :

$$\begin{aligned} n_1(t) = {} & -\ qE_1N\,(V)/m \sum_{n=-\infty}^{\infty} J_n\,(\mu)\left[k_\perp\big/\left(\Lambda_n^2 - \Omega^2\right) + \right. \\ & \left.\left(k_\perp\kappa^2/\Lambda_n^2\right)\right]\ \sin\psi_n. \end{aligned} \tag{19}$$

The transformation of $n_1\,(t)$ into $n_1\,(\boldsymbol{r},\,t)$ is :

$$\begin{aligned} n_1(\boldsymbol{r},t) = {} & -\ qE_1N(V)/m \sum_{n=-\infty}^{\infty} \sum_{l=}^{\infty} J_n(\mu)\ J_l\,(\mu) \\ & \left[k_\perp\big/\left(\Lambda_n^2 - \Omega^2\right) + k_\perp\kappa^2/\Lambda_n^2\right]\sin\chi_{nl} \end{aligned} \tag{20}$$

The perturbed density for the resonant particles under the assumption that $n_1\ (t = 0) = 0$, we get :

$$\begin{aligned} n_1(t) = {} & -\ (qE_1N(V)/m)\ \kappa^2k_\perp\ J_l(\mu)\big/\Lambda_l^2 \\ & [\sin\psi_l - \sin(\psi_l - \Lambda_l t) - \Lambda_l t\cos(\psi_l - \Lambda_l t)] \end{aligned} \tag{21}$$

$$\begin{aligned} n_1(\boldsymbol{r},t) = {} & -\ (qE_1N(V)/m)\ \kappa^2k_\perp \sum_{l'=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} J_{l'}(\mu)J_l(\mu)\ \left(1/\Lambda_l^2\right) \\ & [\sin\chi_{ll'} - \sin(\chi_{ll'} - \Lambda_l t) - \Lambda_l t\cos(\chi_{ll'} - \Lambda_l t)] \end{aligned} \tag{22}$$

provided that

$$\omega \sim l\Omega \quad \text{and} \quad \kappa^2 = k_{\parallel}^2/k_{\perp}^2 > \left|\Lambda_l^2/\left(\Lambda_l^2 - \Omega^2\right)\right| = \Lambda_l^2/\Omega^2. \tag{23}$$

where

$$\chi_{ll'} = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t + (l - l')\,(\Omega t - \theta).$$

5. Distribution function

A detailed treatment of the inclusion of parallel electric field through the zeroth order distribution function has been incorporated in various papers [18,23,24] performing mathematical analysis, through the kinetic approach, Mishra *et al* [18] have mentioned that by the inclusion of parallel electric field, the kinetic equations remain unchanged, in that case the zeroth order distribution function is converted from,

$$f_0 = \frac{n_0}{(2\pi)^{3/2}\alpha_{\perp}^2\alpha_{\parallel}} \exp\left[-\frac{V_{\perp}^2}{2\alpha_{\perp}^2} - \frac{(V_{\parallel} - V_D)^2}{2\alpha_{\parallel}^2}\right] \tag{24}$$

to the another distribution function defined by

$$g_0 = \frac{n_0}{(2\pi)^{3/2}\alpha_{\perp}^2\alpha_{\parallel C}} \exp\left[-\frac{V_{\perp}^2}{2\alpha_{\perp}^2} - \frac{(V_{\parallel} - V_D)^2}{2\alpha_{\parallel C}^2}\right] \tag{25}$$

where

$$\alpha_{\parallel C} = \alpha_{\parallel}\left(1 - \frac{JeE_{\parallel}}{m\alpha_{\parallel}^2 k}\right)^{1/2}, \quad \alpha_{\parallel} = \left(\frac{KT_{\parallel}}{m}\right)^{1/2}, \quad \alpha_{\perp} = \left(\frac{KT_{\perp}}{m}\right)^{1/2} \tag{26}$$

K is the Boltzmann constant and V_D is the drift velocity of the electrons due to the parallel electric field. They have also mentioned that the velocity space integration of the perturbed distributions $f\_$ and $g\_$ are equivalent. That is

$$\int g_- d^3v = \int f_- d^3v\,.$$

Adopting the same model we have selected $N(V)$ as the equilibrium distribution function of the ions which is of the form,

$$N(V) = \left(n_0/(2\pi)^{3/2}\alpha_{\perp}^2\alpha_{\parallel C}\right) \exp\left[-\frac{V_{\perp}^2}{2\alpha_{\perp}^2} - \frac{(V_{\parallel} - V_D)^2}{2\alpha_{\parallel C}^2}\right] \tag{27}$$

The applied electric field parallel to B_0 modifies the electron thermal velocity in that direction, and the temperature $T_{\parallel}$ in the direction of magnetic field modifies to the complex temperature $T_{\parallel C}$ as:

$$T_{\parallel C} = T_{\parallel}\left[1 + jeE_{\parallel}/(k_{\parallel}KT_{\parallel})\right] \tag{28}$$

Here we follow the techniques of Pines and Schrieffer [23] and Bers and Brueck [25] where, the change has been introduced in the zero order distribution function by a change in the temperature parallel to $E_{\parallel}$. This method has been further considered by Misra *et al* [18] for the investigation of whistler mode instability and by Tiwari and Varma [24] for the study of drift wave instability.

There are several observational evidences where the parallel electric fields as large as 100 mV/m are reported along the auroral field lines [26,27]. Various theories have been proposed for the existence of parallel electric fields on the auroral field lines [28,29].

6. Dispersion relation

We consider the electrostatic ion cyclotron instability in the system of hot electrons and hot ions under the conditions,

$$\omega - l\Omega_i << k_{\parallel}v_{\parallel e}, k_{\parallel}v_{\parallel i} < |\omega - l\Omega_i|$$

$$1 > \kappa^2 = k_{\parallel}^2/k_{\perp}^2 > (\omega - l\Omega_i)^2/\Omega_i^2 \quad \text{and} \quad \rho_e << k_{\perp}^{-1} \sim \rho_i \tag{29}$$

where ρ_e and ρ_i are the mean gyroradii of the electrons and the ions respectively.

The resonant particles in this case also are the ions with parallel velocities near to $V_r = (\omega - l\Omega_i)/k_{\parallel}$, except for them, the other particles are non-resonant. The integrated perturbed densities for the electrons and the ions are estimated respectively as :

$$\tilde{n}_e = - 1/\left(k_{\perp}d_{\parallel e}^2\right). \quad E_{\parallel}/(4\pi e). \quad \sin(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)$$

$$\tilde{n}_i = -k_{\perp}\kappa^2\omega_{pi}^2/(\omega - l\Omega_i)^2 \left\langle J_l^2\right\rangle\left[1 + \left(2V_D/V_\phi\right)/\{1 - (l\Omega_i/\omega)\}\right]$$
$$[E_{\parallel}/(4\pi e)]\sin(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t) \tag{30}$$

where

$$\omega_{pi,e}^2 = 4\pi n_0 e^2/m_{i,e}$$

and

$$\left\langle J_l^2\right\rangle = \int_0^\infty 2\pi V_{\perp}dV_{\perp}J_l^2(k_{\perp}V_{\perp}/\Omega)f_{\perp i}(V_{\perp}) \tag{31}$$

$$\exp\left(-\frac{1}{2}k_{\perp}^2\rho_i^2\right) I_l\left(\frac{1}{2}k_{\perp}^2\rho_{\perp}^2\right) \qquad \text{for the Maxwellian } f_{\perp i}.$$

$d_{\parallel e}$ is the Debye length defined as :

$$d_{\parallel e}^2 = T_{\parallel ce}/\left(m_e\omega_{pe}^2\right) \tag{32}$$

The Poisson's equation can be written in the form

$$\nabla\cdot\boldsymbol{E} = -k_{\perp}\left(1 + \kappa^2\right)E_{\parallel}\sin\ (\boldsymbol{k}.\boldsymbol{r} - \omega t)$$
$$= 4\pi e\ (\tilde{n}_i - \tilde{n}_e) \tag{33}$$

Substitution of eqs. (9 – 12) in eq. (13) results the dispersion relation in the form :

$$1 + \left(1/\left(1 + \kappa^2\right)\right)\left(1/\left(k_{\perp}^2 d_{\parallel e}^2\right)\right) - \left(\kappa^2/\left(1 + \kappa^2\right)\right)\left(\omega_{pi}^2/(\omega - l\Omega_i)^2\right)$$

$$\left[1 + \{(2V_D)/V_\phi/(1-(l\Omega_i/\omega))\}\right]\left\langle J_l^2\right\rangle = 0 . \quad (34)$$

In this method, we consider the principal part of plasma dispersion function, therefore; the dispersion relation is real. We follow the energy exchange procedures to evaluate the growth rate and, therefore, complex analysis of the dispersion relation is not required. The involvement of Bessel functions $\left\langle J_l^2\right\rangle$ in the dispersion relation shows that the procedure is equivalent to kinetic approach. In case $V_D = E_0 = 0$ our result is same as derived by Terashima [20] using the standard Maxwellian forms.

7. Energy balance and growth rate

Wave energy density per unit wave-length W_ω is defined as a sum of pure field energy and the change in the energy of non-resonant particles i.e.,

$$W_\omega = \left(\lambda E_1^2/8\pi\right) + W_e + W_i \quad (35)$$

where

$$W_{e,i} = \int_0^\lambda ds \int dV \left(m_{e,i}/2\right)\left[(N+n_1)\ (V+u)^2 - NV^2\right]_{i,e} \quad (36)$$

After substituting values from eq. (13) with $\delta = 0$, and eqs. (20), (26) and (27), we get the energy associated with the ion and electron components of the non-resonant particles as :

$$W_e = \left(\lambda E_1^2/16\pi\right)\left(1/k_\perp^2 d_{\parallel e}^2\right), \quad (37)$$

$$W_i = \left(\lambda E_1^2/16\pi\right)\omega_{pi}^2\Big/(\omega - l\Omega_i)^2 \frac{1}{2}\left\langle J_{l-1}^2 + J_{l+1}^2\right\rangle$$
$$\left[1+\{(2V_D/V_\phi)\ /\ 1-(l\Omega_i/\omega)\} - R\ k_\parallel^2 V_\parallel^2/\omega(l\Omega_i - \omega)\right] \quad (38)$$

Hence the expression for the wave energy density per unit wave length can be written by substituting eqs. (37) and (38) into eq. (35) as :

$$W_\omega = \lambda E_1^2/8\pi + \lambda E_1^2/16\pi\ \ 1/k_\perp^2 d_{\parallel e}^2 + \left(\lambda E_1^2/16\pi\right)\left(\omega_{pi}^2/(\omega - l\Omega_i)^2\right)$$
$$\frac{1}{2}\left\langle J_{l-1}^2 + J_{l+1}^2\right\rangle\left[1+\ \{(2V_D/V_\phi)\ /\ 1-(l\Omega_i/\omega)\}\right.$$
$$\left. - R k_\parallel^2 V_\parallel^2/\omega(l\Omega_i - \omega)\right] \quad (39)$$

where

$$R = \left\langle (J_{l-1} + J_{l+1})^2\right\rangle \Big/ \left\langle\left(J_{l-1}^2 + J_{l+1}^2\right)\right\rangle \quad (40)$$

Here in the above, the ions' contribution is dominant unless $k_\perp^2 d_{\parallel e}^2 < 1$.

The changes in energy of the resonant particles are :

$$W_{r\perp} = \int_0^{\lambda} ds \int_0^{\infty} V_\perp dV_\perp \int_D^{2\pi} d\theta \int_{V_r\,\Delta V}^{V_r+\Delta V} dV_\parallel \,(m/2)\left[(N+n_1)\,(V_\perp+u_\perp)^2 - NV_\perp^2\right] \quad (41)$$

$$W_{r\parallel} = \int_0^{\lambda} ds \int_0^{\infty} V_\perp dV_\perp \int_0^{2\pi} d\theta \int_{V_r\,\Delta V}^{V_r+\Delta V} dV_\parallel \,(m/2)\left[(N+n_1)\,(V_\parallel+u_\parallel)^2 - NV_\parallel^2\right] \quad (42)$$

where eq. (13) with $\delta = 1$ and eqs. (22), (26) and (27) are to be substituted to evaluate the change in energy W_r.

The change in energy for resonant particles due to the presence of EIC wave, perpendicular to the ambient magnetic field for $l = 1$ is given as :

$$W_{r\perp} = \left(\lambda E_1^2/8\right)\left(\omega_{pi}^2/\Omega_i^2\right)\frac{1}{2}\left\langle J_0^2 + J_2^2\right\rangle(\Omega_i/\omega)\,(\omega/k_\parallel V_{\text{thi}})\;\Omega_i t/(2\pi)^{1/2}$$

$$\frac{\exp-\left[\omega^2\left\{1-(\Omega_i/\omega)-\left(V_D/V_\phi\right)\right\}^2 \Big/ 2k_\parallel^2 V_{\text{thi}}^2\left\{1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right\}\right]}{\left[1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right]^{1/2}}$$

$$\left[1-R\left((\Omega_i/\omega)+\left(V_D/V_\phi\right)-1\right)\Big/(\Omega_i/\omega)\frac{T_{\perp i}}{T_{\parallel i}\left\{1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right\}}\right]. \quad (43)$$

Similarly, the change in energy for resonant particles due to presence of wave, along the ambient magnetic field for $l = 1$:

$$W_{r\parallel} = \left(\lambda E_1^2/8\right)\left(\omega_{pi}^2/\Omega_i^2\right)(\Omega_i t)\,(\omega/k_\parallel V_{\text{thi}})\,\frac{1}{2}\left\langle (J_0+J_2)^2\right\rangle\left(T_{\perp i}/T_{\parallel i}\right)$$

$$\left[1-(\Omega_i/\omega)-\left(V_D/V_\phi\right)\right]^2 \Big/ \left((\Omega_i/\omega)(2\pi)^{1/2}\right)$$

$$\frac{\exp-\left[\omega^2\left\{1-(\Omega_i/\omega)-\left(V_D/V_\phi\right)\right\}^2 \Big/ 2k_\parallel^2V_{\text{thi}}^2\left\{1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right\}\right]}{\left[1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right]^{3/2}} \quad (44)$$

where use is made of :

$$\left\langle J_l^2(\mu)\right\rangle = \{J_{l-1}(\mu) + J_{l+1}(\mu)\}^2\left(\mu^2/4l^2\right)$$

The growth rate of the wave is found from the energy conservation equation and defined as :

$$Y = (1/E_1)\,(dE_1/dt) = -(dW_r/dt)\,/\,2W_\omega . \quad (45)$$

Hence, the growth rate in eq. (45) is now :

$$Y/\omega = (\pi/2)^{1/2}\,\frac{\omega\{1-(l\Omega_i/\omega)\}^2\left\{1+\left((2V_D/V_\phi)\,/\,1-(l\Omega_i/\omega)\right)\right\}}{k_\parallel V_{\text{thi}}\left\{1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right\}^{1/2}}$$

$$\exp -\left[\omega^2 \left\{1-(l\Omega_i/\omega) - (V_D/V_\phi)\right\}^2 / 2k_\parallel^2 V_{thi}^2\left\{1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right\}\right]$$

$$\left[R\left((l\Omega_1/\omega)+\left(V_D/V_\phi-1\right)/(1\Omega_1/\omega)\right) \frac{T_{\perp i}}{T_{\parallel i}\left[1+\left(e^2E_\parallel^2/k_\parallel^2K^2T_\parallel^2\right)\right]}-1\right] \quad (46)$$

where $l = 1, 2, 3$ is to be substituted and we have used the eqs. (25–27), (39) and (43) to evaluate the growth rate. If the applied parallel electric field $E_\parallel = 0$ and $V_D = 0$, the result is same as derived by Terashima [20].

Here, the growth rate is valid for electrostatic ion cyclotron waves in the presence of ion beam and parallel electric field. The contribution of electron beam and interaction of resonant electrons with the wave has not been taken into account. The results with electron drifts and energy exchange with the electrons have been considered by Drummound and Rosenbluth [30], Kindel and Kennel [31]. The constant parameters used in the calculations are : $\lambda = 300$ m, $E_1 = 50$ mV/m $KT_\parallel = 5$ eV, $k_\parallel = 0.002$ m^{-1}, $bi = 0.5$ $\omega/k_\parallel V_{th} = 2$.

8. Results and discussion

In this paper, we have concentrated upon the study of electrostatic ion-cyclotron instability which has been observed in the inverted – V structure of the auroral acceleration region. We have emphasized the study to investigate the possibilities of EIC wave emissions in the presence of EIC wave and unflowing ion beam relevant to the auroral acceleration processes. The wave electric field choosen for numerical evaluation of the results are pertaining to the wave fields observed in the acceleration region [32]. Thus the ionospheric physicist may use the results as the ion beam may be the cause of EIC wave emissions, which have been observed in the auroral acceleration region. The study on saturation spectra and the saturation level has not been considered in the present analysis which may require the further investigation.

The expressions for the change in energy of the resonant particles due to presence of wave perpendicular and along the ambient magnetic field are derived in eqs. (43) and (44). Hence the growth rate is evaluated as eq. (46).

Figures 1 – 3 show the variation of the normalised growth rate versus Ω_i/ω for different values of parallel electric field at the harmonics of the ion cyclotron frequency. The variation of Y/ω versus Ω_i/ω for $l = 1$ is shown in Figure 1. The growth rate reaches the maximum value for the particular value of Ω_i/ω and electric field enhances the growth rate, when $\omega \sim \Omega_i$ the growth rate vanishes, when $\Omega_i > \omega$ the growth rate goes on increasing and after a certain value of Ω_i/ω growth rate starts decreasing. This is due to the term $1 - (\Omega_i/\omega)$ appeared in eq. (46) of the growth rate. Higher parallel electric fields tend to enhance the growth rate of wave and the band of Ω_i/ω is expanded. At the particular band of Ω_i/ω the generation of wave is possible which may correspond to the wave emissions around $\omega \approx \Omega_i$ above the wave frequency ω. The shifting of maximum of the growth rate at the higher electric fields indicate that the wave generation may be possible away from the wave frequency ω and increasing growth rate with higher electric fields indicate the possibility of generation at the higher fields.

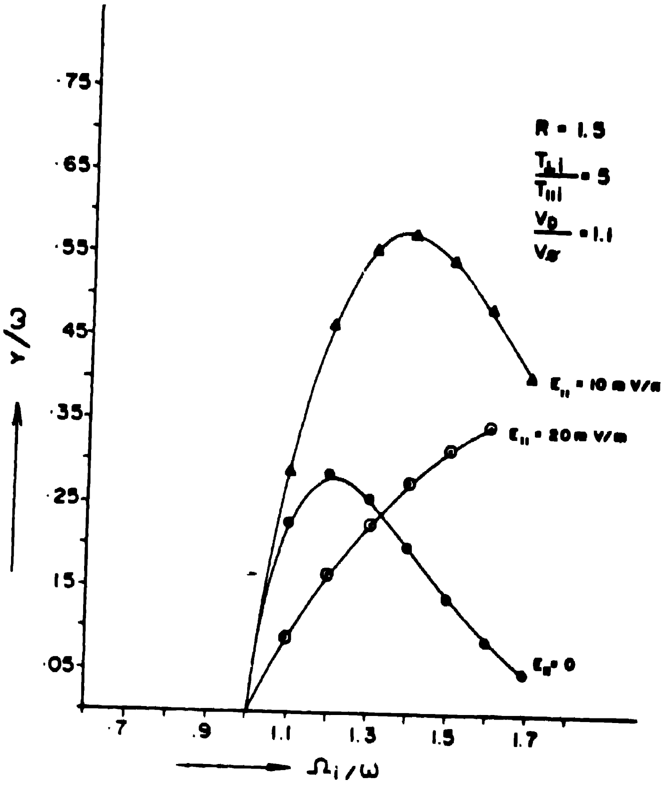

Figure 1. Variation of Y/ω with Ω_i/ω for different values of $E_\parallel$ for $l = 1$.

Figure 2 shows the variation of Y/ω Versus Ω_i/ω for $l = 0$ for different parallel electric fields ($E_\parallel$). Figure predicts that $E_\parallel$ expands the bandwidth of Ω_i/ω. At a particular value of $\Omega_i/\omega \approx 0.5$, growth rate becomes maximum and afterwards, it approaches to zero. This predicts that the most possible mode of the excitation is second harmonic, that is $\omega \approx 2\Omega_i$. Beyond the first harmonic that is $\omega \approx \Omega_i$, the growth rate reduces to minimum values. Thus the possible band of wave generation may be in between Ω_i to $2\Omega_i$.

The similar pattern is seen in Figure 3, where, growth rate is plotted with Ω_i/ω for $l = 3$ and different values of parallel electric field. The electric field controls the growth rate and increases the emission band.

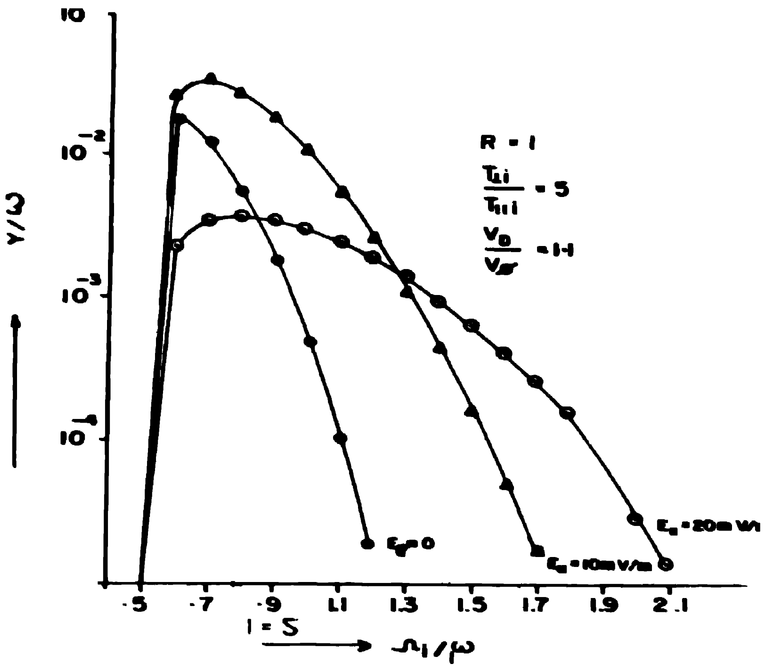

Figure 2. Variation of Y/ω with Ω_i/ω for different values of $E_{\parallel}$ for $l = 2$.

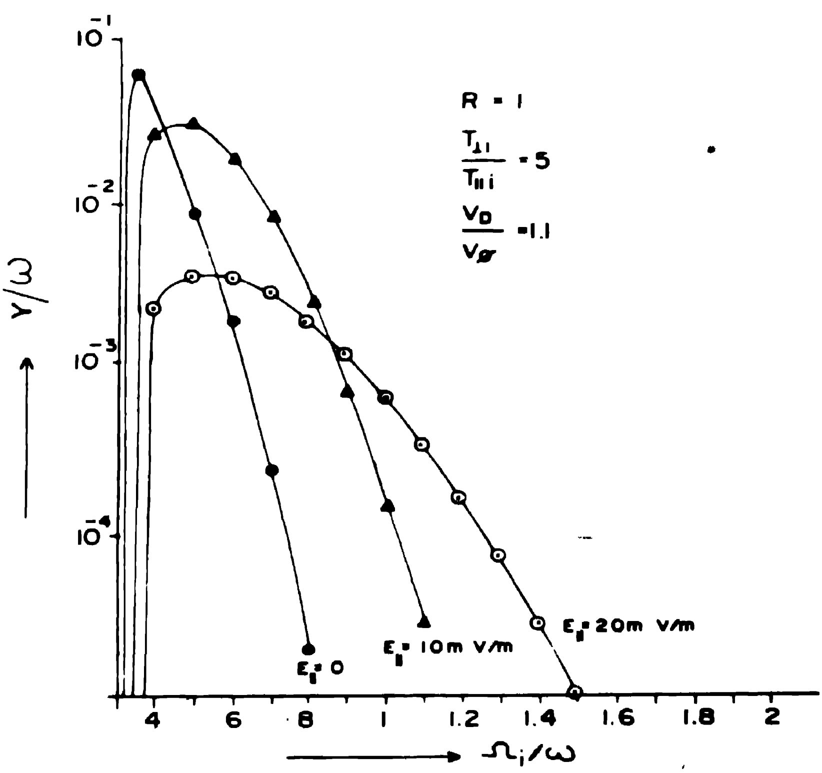

Figure 3. Variation of Y/ω with Ω_i/ω for different values of $E_{\parallel}$ for $l = 3$.

Comparing the above mentioned figures, it can be noticed that the possible mode of excitation is the lowest harmonics. The resonant particles corresponding to the first harmonics of the wave feed maximum energy to the wave around $\omega \approx \Omega_i$ and less effective for the higher harmonics. Thus, the electric field is less effective to provide energy to the wave through the modification of thermal velocity parallel to the magnetic field. The particles resonating with the first harmonics may be less effective for the higher harmonics. Thus the electric field may reduce the growth rate.

Figures 4 – 6 show the variation of Y/ω with V_D/V_ϕ for different parallel electric fields at the harmonics of the ion cyclotron frequency. In Figure 1, the parallel electric fields expand the band width of V_D/V_ϕ . $E_\parallel$ may alter the beam velocity required for growth rates at

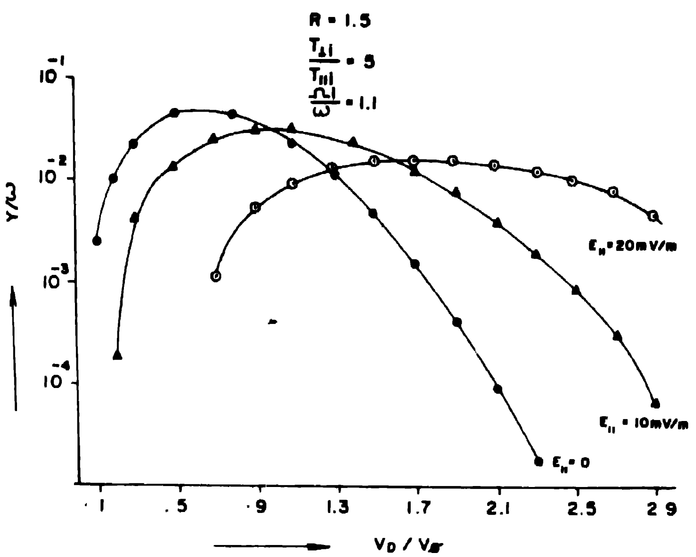

Figure 4. Variation of Y/ω with V_D/V_ϕ for different values of $E_\parallel$ for $l = 1$.

smaller electric fields. When parallel electric field is zero, the growth rate increases rapidly with the increase of V_D/V_ϕ and after a certain value of V_D/V_ϕ starts decreasing. It is seen in figure, that whenever, the parallel electric field is above 10 mV/m, the growth rate increases rapidly but after a certain value of V_D/V_ϕ it decreases slowly and approaches to zero. In Figures 5 and 6 between Y/ω and V_D/V_ϕ for $l = 2$ and for $l = 3$, the similar nature may be visualised. In Figure 4, it may be noticed that when phase velocity V_ϕ, approaches beam velocity V_D, the effect of electric field is to reduce the growth rate. However, beyond this limit the effect of electric field is to enhance the growth rate. In the case of the higher harmonics of the ion cyclotron frequency, the conditions for V_D/V_ϕ are modified as seen in Figures 5 and 6. The higher electric fields require higher values of V_D/V_ϕ, for the excitation of the wave. This may be due to the reason that the electric field may be additional energy source for the generation of EIC waves in the presence of beam velocity V_D. The condition on V_D/V_ϕ may thus be modified. The presence of electric field may change the beam velocity

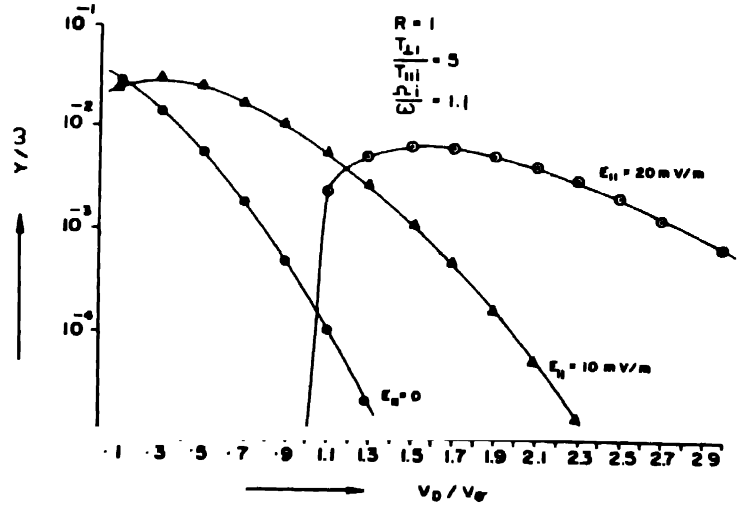

Figure 5. Variation of γ/ω with V_D/V_ϕ for different values of $E_\parallel$ for $l = 2$.

required for the generation of the wave. The condition for the instability may be predicted by the Figure 4 as the growth is possible as V_D is around V_ϕ. These results may be consistent

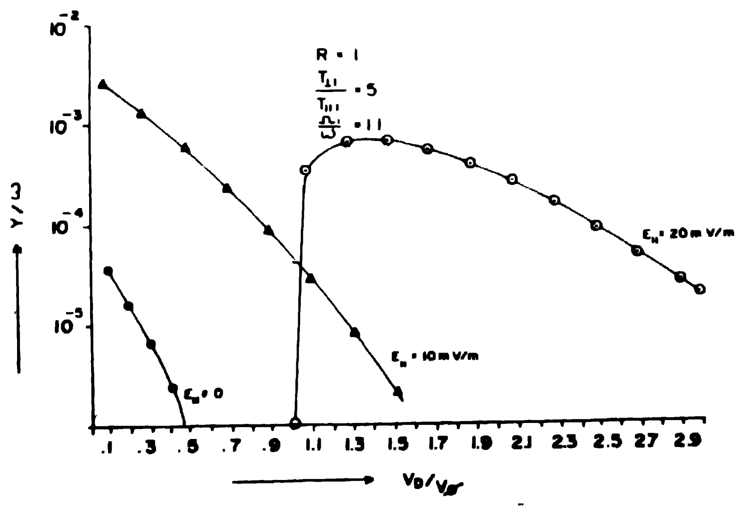

Figure 6. Variation of γ/ω with V_D/V_ϕ for different values of $E_\parallel$ for $l = 3$.

with the results reported by Mozer *et al* [1], Lundin and Hultqvist [33], Singh *et al* [34] and the references therein; about the up flowing ion beams and the parallel acceleration in the auroral region.

The high altitude observations by S3–3 indicated that the altitude range upto 10,000 Km was a prime acceleration heating region for the topside ionosphere. Upward flowing ion beams were believed to be predominantly accelerated by parallel electric fields. Several theoretical studies on the topic of ionospheric ion acceleration have also been made during the past year [35,36]. Since the beam velocity is the critical parameter that determines whether beam driven instabilities, can be excited. The plasma in the upward current regions may be more easily destabilised due to the upflowing ion beams reported by various satellites. Over the altitude range of 5000 to 8000 Km beam the velocity in the acceleration region also maximises and therefore, plasma at those altitudes will be most easily distablished by ion beams driven instabilities supported by the parallel electric fields.

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