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Disturbance in a piezo-electric slab sandwiched between an elastic and visco-elastic medium under electrical and mechanical excitations

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Abstract : An attempt has been made to investigate analytically the mechanical disturbance in a piezoelectric slab (quartz) sandwiched between an elastic ($x = 0$) and a visco-elastic ($x = d$) material under electrical and mechanical excitations. The mechanical stress $T_1 = T_0\delta(t)$ is introduced at $x = 0$ The surface $x = 0$ is also excited in the direction of y-axis by an electric field. The problem involves the interaction of two fields *viz.*, electrical and mechanical, and the solution has been obtained with the aid of the operational calculus. The disturbance exhibited by the slab is found to be linear in nature and is of the order of 10^{-5} m, interestingly with an initial disturbance at $t = 0$.

Keywords : Piezoelectric slab, elastic and visco-elastic medium, mechanical disturbance

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The mechanical disturbance of piezoelectric materials have been investigated by many workers [1–4]. Such discussions are of enormous importance in view of various practical applications in different fields of science and technology. Researchers [5–7] considered the problems of the piezoelectric slab bounded by an elastic medium on both sides and the bounding surfaces coated by a thin film of conducting material under different boundary conditions.

The present paper deals with a problem of the mechanical displacement in a piezoelectric slab (quartz), sandwiched between an elastic and a visco-elastic material under electrical and mechanical excitations. The relevant problems are extensively used in different branches of Acoustical Engineering, particularly in piezo-electric microphone [8,9]. The solution involves the interaction of two fields, *i.e.*, electrical and mechanical, and consequently Maxwell's equations and the equations of elasticity have been used simultaneously to obtain the solution, with the aid of the operational calculus.

Let us consider a plane piezo-electric structure consisting of a piezo-electric slab (quartz), bounded by an elastic medium on one of its sides, and visco-elastic medium on the other side. Let $x = 0$ and $x = d$ represent the faces of the slab, so that the regions $x < 0$, $0 < x < d$ and $x > d$ are occupied by elastic, piezo-electric and visco-elastic material. The mechanical stress $T_1 = T_0\delta(t)$ is introduced at $x = 0$. The surface $x = 0$ is also excited in the direction of y-axis by an electric field

$$E_y = E_0 \sin \omega t. \tag{1}$$

A thin film of perfectly non-conducting material is introduced at the face $x = d$ in order to prevent the electromagnetic wave propagation without disturbing the elastic propagation [10,11]. Now we study the disturbances proceeding in the direction of the x-axis.

Following [1–4], the piezo-electric equations of the medium are given by

$$T_{ij} = c^E_{ijkm} S_{km} + e_{lij} E_l, \tag{2}$$

$$D_l = -e_{lij} S_{ij} + \varepsilon^S_{lm} E_m, \tag{3}$$

where T the mechanical stress, D the electric displacement, S the strain, E the electric field intensity, c the elastic compliance at constant electric field, e the piezo-electric coefficient and ε the dielectric permittivity. All indices, i, j,..., run from 1 to 3 and Einstein's summation convention has been employed.

The mechanical displacement u in the direction of the x-axis and the electric field E in the direction of the y-axis satisfy the following equations

$$\partial^2 u / \partial x^2 - \rho_1 / c.\ \partial^2 u / \partial t^2 = -e_1 / c.\ \partial E / \partial x, \tag{4}$$

$$\partial^2 E / \partial x^2 - \mu\varepsilon \partial^2 E / \partial t^2 = -\mu e_1 \partial^2 / \partial t^2.\ (\partial u / \partial x), \tag{5}$$

where ρ_1 is the density of the piezo-electric material.

In the elastic medium, the mechanical displacement u satisfies the equation

$$\rho\, \partial^2 u / \partial t^2 = (\lambda + 2\mu)\, \partial^2 u / \partial x^2, \tag{6}$$

where ρ is the density of the elastic material and λ, μ are Lame's constants of the material. In the visco-elastic medium, the displacement u satisfies the relation

$$\rho_2\, \partial^2 u / \partial t^2 = \{(\lambda_0 + 2\mu_0) + (\lambda_1 + 2\mu_1)\, \partial / \partial t\}\, \partial^2 u / \partial x^2 \tag{7}$$

where ρ_2 is the density of the visco-elastic material and $\lambda_0, \mu_0, \lambda_1, \mu_1$ are constants.

Now we state the boundary conditions as follows :

$$E_1 = E_0 \sin \omega t \qquad \text{at } x = 0 \tag{8}$$

$$T_1 = T_0 \delta(t) \qquad \text{at } x = 0 \tag{9}$$

$$T_1 = T_2 \qquad \text{at } x = d \tag{10}$$

$$u_1 = u_2 \qquad \text{at } x = d \tag{11}$$

$$E_1 = 0 \qquad \text{at } x = d \tag{12}$$

where the suffixes 1 and 2 stand for the entities of the piezo-electric and visco-elastic materials and $\delta(t)$ is the Dirac delta function.

We use the Laplace transform method of parameter p [Re $p > 0$]. Taking Laplace transform of eqs. (4), (5) and (7), we get

$$(D^2 - \rho_1 p^2 / c)\,\bar{u}_1 + e_1 D\bar{E}_1 / c = 0, \tag{13}$$

$$\mu e_1 p^2 D\bar{u}_1 + (D^2 - \mu\varepsilon p^2)\,\bar{E}_1 = 0, \tag{14}$$

$$\partial^2\bar{u}_2 / \partial x^2 - \rho_2 p^2 \bar{u}_2 / \{(\lambda_0 + 2\mu_0) + (\lambda_1 + 2\mu_1)p\} = 0, \tag{15}$$

where, $\quad D = \partial / \partial x$ and $D^2 = \partial^2 / \partial x^2$

Solving (13) and (14) we get

$$\bar{u}_1 = C_1 \exp(-m_1 x) + C_2 \exp(-m_2 x) + C_3 \exp(m_1 x) + C_4 \exp(m_2 x), \tag{16}$$

where C_1, C_2, C_3 and C_4 are constants, and m_1^2, m_2^2 are roots of

$$m^4 - m^2 p^2 (\rho_1 / c + \mu\varepsilon + \mu e_1^2/c) + \mu\rho_1 \varepsilon p^4 / e_1 = 0 \tag{17}$$

The solution of eq. (15) subjected to the condition $\bar{u}_2 \to 0$ as $x \to \infty$, is given by

$$\bar{u}_2 = C_5 \exp(-m'_1 x), \tag{18}$$

where m'_1 is a root of the equation

$$m^2 - \rho_2 p^2 / \{(\lambda_0 + 2\mu_0) + (\lambda_1 + 2\mu_1)p\} = 0. \tag{19}$$

From eq. (16), we get

$$\bar{u}_1|_{x=0} = C_1 + C_2 + C_3 + C_4. \tag{20}$$

The values of the constants C_1, C_2, C_3 and C_4 are obtained by solving the eqs. (16) and (18), with the aid of eqs. (8)–(12) and substituting these values in eq. (20) we get $\bar{u}_1$. The inversion of $\bar{u}_1$ is a complicated task, to get an approximate value, we shall consider small values of time [11,12].

$$m_1 = \theta_1 p; \quad m_2 = \theta_2 p; \quad m'_1 = \gamma p,$$

where θ_1, θ_2 and γ are positive constants involving material parameters of the problem. Adopting the procedure of approximation [3,6], we get

$$u = -dtT_0A + d\exp(-A/B.t)/A + [t/2A - Bt/A^2T_0$$

$$+ \exp(-A/B.t)B/A^2]/(\theta_1 + \theta_2) + e_1E_0(t - \cos\omega t)/\omega c(\theta_1 + \theta_2) \quad (21)$$

where, $A = 2(\lambda_0\gamma + 2\mu_0\gamma) + \rho(\theta_2 - \theta_1)/\theta_1\theta_2;$

$$B = 2(\lambda_1\gamma + 2\mu_1\gamma) + \rho d(\theta_1 + \theta_2)/\theta_1.$$

Eq. (21) shows the displacement of the piezo-electric slab sandwiched between an elastic and a visco-elastic material under electrical and mechanical excitations. The displacement is found to be partly linear, partly periodic, and partly transient in nature.

For numerical calculations, the standard values of the material constants have been taken [2,13–15], while values like E_0, d, ω and T_0 have been chosen suitably to facilitate the numerical calculations as follows :

$$E_0 = 300\ \text{V}, \quad d = 0.05\ \text{m}, \quad \omega = 1.5\ \text{rad/s}, \quad T_0 = 1\ N/m^2.$$

The values of the amplitude of free electric field (E_0 = 300 v), the mechanical stress (T_0 = 1 N/m^2) and the thickness of the piezoelectric slab (d = 0.05 m) as considered here are quite in conformity to those of other researchers [2,7]. Again the arbitrary value for frequency of the applied periodic excitations (w = 1.5 rad/s) is conveniently chosen as conceived by [8,15]. The variation of the mechanical disturbances in a piezo-electric slab sandwiched between an elastic and a visco-elastic medium with time is shown in Table 1.

Table 1. The mechanical disturbance (u) exhibited by the piezo-electric slab sandwiched between an elastic and a visco-elastic medium against time (t).

t (sec).	$u \times 10^{-5}$ (m)
0.0	0.560
0.1	0.615
0.2	0.671
0.3	0.727
0.4	0.783
0.5	0.839
0.6	0.895
0.7	0.951
0.8	1.007
0.9	1.063
1.0	1.119

From eq. (21), we find that the displacement is partly linear, partly periodic, and partly transient in nature. The contribution of the transient part of eq. (21) is insignificant compared to the linear and periodic parts and as a result, the disturbance exhibits linear relationship with time (Figure 1). The electric field $E_y = E_0 \sin \omega t$ is applied at the surface $x = 0$ in the direction of the y-axis and this periodic electric field is reckoned from the moment $t = 0$ but from investigation it is revealed that at $t = 0$ some disturbance persists.

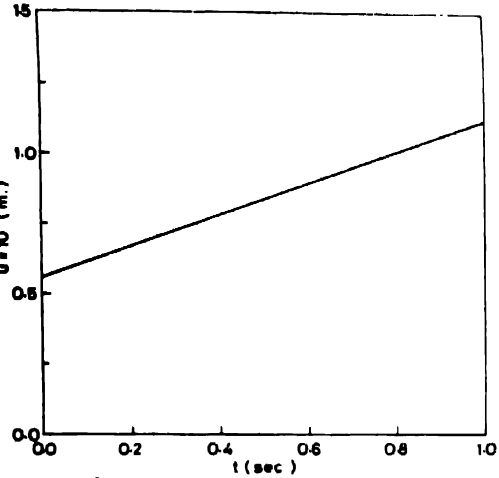

Figure 1. The variation of the mechanical disturbance of the piezo-electric slab with time.

This finite response at $t = 0$ is coming from the time dependent boundary condition $T_1 = T_0 \delta(t)$ introduced at the face $x = 0$ of the piezo-elctric slab. Researchers [5–7] obtained similar result in a problem in which piezo-quartz slab is bounded by an elastic medium on both the sides. The order of magnitude of the disturbances obtained therein being 10^{-7} m and a finite deformation also exists at $t = 0$. It is very interesting to note that the order of disturbance (10^{-5} m.) when the piezo-quartz sandwiched between elastic and visco-elastic medium, rises hundred times than the piezo-quartz sandwiched between elastic medium and coated with a perfectly conducting material.

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