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Analysis of resonances in heavy ion reactions as barrier region resonances

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Abstract : We numerically study the resonance states generated by the finite range truncated parabolic barrier. In this case, the location of resonances in the vicinity of the barrier are approximately equispaced but deviation for this feature occurs for resonances farther away from the barrier top Using these results as a basis we empirically analyse the resonances in the $^{16}O + {^{16}O}$ and $^{24}Mg + {^{24}Mg}$ systems within the broad framework of barrier region resonance model

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1. Introduction

The prominent intermediate width resonances observed in the energy dependence of cross section of heavy ion reactions is an interesting phenomena generated by heavy ion collision experiments. It is well known that these resonance structures are the characteristic features observed in several nucleus-nucleus collisions around the barrier region. The effective real nucleus-nucleus potential between two heavy ions is characterised by a Coulomb barrier region and a potential pocket which gets shallower and shallower as l increases. Within the framework of such a potential one may look for resonances originating from the pocket region. However, the resonance state can originate not only from the pocket region but also from the barrier top region provided absorption is not large in the barrier region and the barrier is reasonably flat, *i.e.*, in a typical real nucleus-nucleus effective potential, resonances can arise both from the pocket region and the barrier region.

Based on this concept and using the nature of resonance structures observed in the case of a short range repulsive Eckart potential barrier, in an earlier paper [1] we had described the analysis of resonant phenomena in $^{12}C + {^{12}C}$ and $^{12}C + {^{16}O}$ systems using what was referred to as barrier region resonance model [BRRM]. In this paper, we extend BRRM for the description of resonances observed in $^{16}O + ^{16}O$ and $^{24}Mg + ^{24}Mg$ systems.

This paper is organised in the following way. In section 2 we describe the nature of resonances observed in a truncated inverted harmonic oscillator barrier. Based on these results in section 3 the resonance structures in $^{16}O + ^{16}O$ and $^{24}Mg + ^{24}Mg$ systems are analysed using an empirical expression within the framework of BRRM. Section 4 contains discussions and conclusions.

2. Resonances generated by a truncated inverted harmonic oscillator barrier

The analytical formula used in the analysis of $^{12}C + ^{12}C$ and $^{12}C + ^{16}O$ systems [1] was based on the S-matrix for the repulsive Eckart potential which gave $(n+3/4)^2$ dependence on the resonance energies. However, an examination of available experimental data in the $^{16}O + ^{16}O$ system indicate [2–4] that resonance energies for a given partial wave are approximately equispaced. In view of this fact we investigate whether an inverted finite range parabolic barrier can generate in the vicinity of the barrier approximately equispaced resonances. In order to do this we studied the s-wave S-matrix numerically, for the finite range inverted parabolic barrier given by

$$\begin{aligned} V(r) &= V_0 - \beta r^2/2, && r < R \\ &= 0 && r \geq R\,;\; R = (2V_0/\beta)^{1/2} \end{aligned} \tag{1}$$

In Table 1 we list several pole positions of s-wave S matrix in complex k^2 plane in the case of potential given by (1) for different values of β. It is interesting to note that there are

Table 1. Resonance positions in the complex-k^2 plane in the case of cut off parabolic barrier represented by (1)

	k^2 (fm^{-2})	
$\beta = 0.1$ fm^{-4} $V_0 = 10$ fm^{-2}	$\beta = 0.2$ fm^{-4} $V_0 = 10$ fm^{-2}	$\beta = 0.3$ fm^{-4} $V_0 = 10$ fm^{-2}
11.30 – 0.6953i	12.14 – 1.1074i	12.67 – 1.389i
10.56 – 0.5597i	10.94 – 0.8407i	11.11 – 1.051i
10.01 – 0.4472i	10.06 – 0.6299i	10.01 – 0.7762i
9.448 – 0.5524i	9.224 – 0.7794i	8.891 – 1.008i
8.673 – 0.6700i	7.994 – 0.9923i	7.186 – 1.306i

resonances both below and above the barrier and their spacings and widths indicate some approximate symmetry. It is also interesting to note that the two levels in the immediate vicinity of the barrier are approximately equispaced with respect to the resonance closest to the barrier. This linearity is gradually destroyed in the case of levels farther away both above and below the barrier.

We know that in the case of harmonic well $-U_0 + m\omega^2 r^2/2$ of depth U_0, the s-wave energy levels are given by the formula,

$$E = -U_0 + (2n + 3/2)\,\hbar\omega \tag{2}$$

indicating the spacing of $2\hbar\omega$ between the adjacent levels. In this case the ground state energy is $3/2\,\hbar\omega$ above the bottom of the well. On the other hand the corresponding barrier resonances occur very close to the barrier top. We also found the spacing between the barrier top level and the next adjacent level on the either side is approximately $(2)^{1/2}\,\hbar\omega$. However due to the finite range harmonic barrier and the fact that resonant states do not decay exponentially, anharmonic terms significantly affect the levels away from barrier top. The non-linearity or anharmonicity becomes important even for $n = 2$ (assuming the level closest to the barrier top as $n = 0$) as is clear from Table 1. We also observe that sharpest resonance is practically on the top of the barrier.

Another interesting aspect which we noted is that Im k^2 of two levels adjacent to the barrier top are approximately same indicating resonances of same width. In order to interpret this we calculated the transit time of a classical particle to slide down the truncated harmonic oscillator barrier in the classically allowed region for energy $E = U_0 \pm \Delta;\ \Delta < U_0$. We find that these two times are given by the same formula

$$\tau_\pm = (m/2)^{1/2}\, U_0^{-1/2}\, R\, \ln \left| \frac{U_0^{1/2} + E^{1/2}}{[\pm (E - U_0)]^{1/2}} \right| \tag{3}$$

Further it may be noted that for a given R, τ is proportional to U_0 indicating that for barrier region resonances, width will increase with barrier height, for a given range R. Similar results are obtained in the case of repulsive Eckart potential [1].

3. Barrier region resonance model for $^{16}O + {}^{16}O$ and $^{24}Mg + {}^{24}Mg$ systems

Based on the discussion of Section 2 on the truncated inverted harmonic oscillator barrier and noting that in the case of $^{16}O + {}^{16}O$ system the number of observed resonance states in the Coulomb barrier region is less than or equal to five we seek to analyze these resonances using the empirical expression,

$$E(n,l) = V_B(l) + (C_0 + C_1 l^2)\,(2)^{1/2} n + \varepsilon n^2\ ;\ n = 0,\ +1,\ +2 \tag{4}$$

where $V_B(l)$ is the height of the barrier for the l-th partial wave, C_0, C_1 and ε are parameters. The height of the effective potential $V_B(l)$ for different l's have been calculated using the global nucleus-nucleus potential [5]

$$V(r) = -50\,(R_1 R_2)/(R_1 + R_2)\, \exp\,[(R_1 + R_2 - r)/a] \tag{5}$$

where $R_i = 1.233 \cdot A_i^{1/3} - 0.978\ A_i^{-1/3}$ fm ($i = 1, 2$) and $a = 0.63$ fm. along with centrifugal and Coulomb potentials. $C_1 l^2$ is the term which takes into account the fact that effective barrier varies slowly with l. ε accounts for the deviation from the linear dependence of n in the neighbourhood of the barrier. Using this formula we have fitted the experimental data for $^{16}O + {}^{16}O$ and $^{24}Mg + {}^{24}Mg$ systems. The resonance energies obtained using (4) are

Table 2. The quantities $R_B(l)$, $V_B(l)$ and typical set of $E(n, l)$ corresponding to $^{16}O + {}^{16}O$ and $^{24}Mg + {}^{24}Mg$ systems. The parameters used to compute $E(n, l)$ are listed in corresponding figure captions.

System	l	$R_B(l)$ (fm)	$V_B(l)$ (MeV)	$E(n, l) = V_B(l) + (C_0 + C_1 l^2)$		(MeV)	$(2)^{1/2}$	$n + \varepsilon n^2$
				$n = -2$	-1	0	1	2
	2	8 09	10.70	9 89	10.12	10.70	11.63	12.90
	4	8.03	11.27	10.43	10.68	11.27	12.21	13.51
	6	7.93	12.18	11.28	11.55	12.18	13.15	14.46
	8	7.81	13.44	12.47	12 79	13.44	14.45	15.80
	10	7 67	15.10	14.04	14.40	15.10	16.15	17.55
$^{16}O + {}^{16}O$	12	7.54	17.18	16.00	16.42	17.18	18.29	19 74
	14	7.40	19.71	18.40	18.88	19.71	20.89	22.41
	16	7.27	22.73	21.26	21.82	22.73	23.98	25.57
	18	7.13	26.26	24.62	25.27	26.26	27.61	29.29
	22	6.88	35.02	32.96	33.82	35.02	36.57	38.47
	24	6.76	40.31	38.01	38.99	40.31	41.99	44.00
$^{24}Mg + {}^{24}Mg$	30	8.01	45.16	43.75	44.40	45.16	46.05	47.06
	32	7.94	48.62	47.21	47.85	48.62	49.51	50.52

listed in Table 2. In Figures 1 and 2 we depict the results obtained for even l along with experimental data. It is clear that the present approach gives quite a good fit to the experimental data, for $^{16}O + {}^{16}O$ resonances.

Due to the availability of only limited data, in the analysis of $^{24}Mg + {}^{24}Mg$ we have treated C_0 and C_1 as a single parameter C since l values considered (l = 30, 32) are very close. The reported values of l are 34, 36. However these l values obtained from squared Legendre polynomial fits are larger than the grazing angular momentum by 4-6 $\hbar$ [6].

4. Discussions and conclusions

The present approach for the analysis of heavy ion resonance data comes within the general framework of potential scattering, that if the potential pocket is highly absorptive resonances can be attributed to the barrier region. It may be noted that in the case of $^{16}O + {}^{16}O$ system study of resonances has been carried out using different approaches like orbiting cluster model [7] and using shallow potentials [8–11], deep potentials [12] and potentials like Morse and anharmonic well [4]. The orbiting cluster model essentially incorporates implicitly the spirit of barrier region resonances in a sense that resonances for a given l are in effect rotating clusters with separation approximately corresponding to the barrier. The calculations based on the effective shallow potential is somewhat akin to the present approach. In the case of potentials used in [4], the minima of the effective potential is almost at the Coulomb barrier position. Hence, by construction these potentials used are such that they generate pocket resonance states in the vicinity of the barrier.

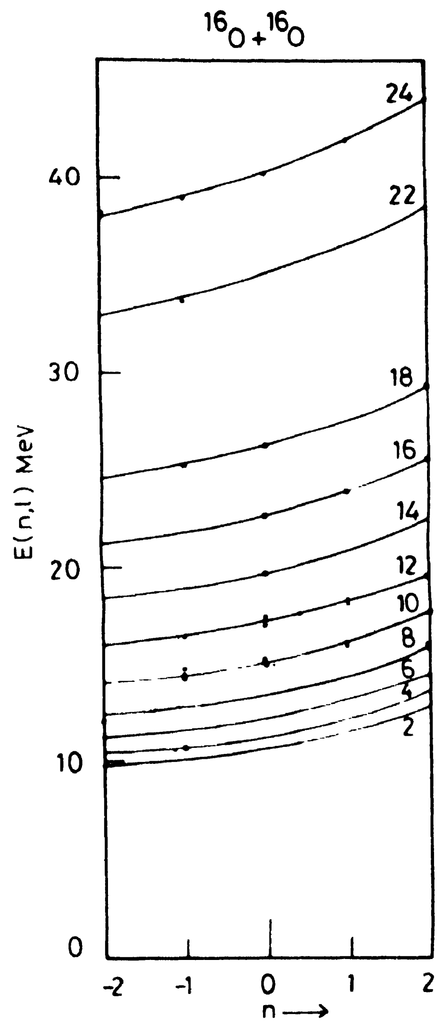

Figure 1. Plots of $E(n, l)$ against the assumed values of n for the resonance data of the ^{16}O + ^{16}O system. The parameters used to compute $E(n, l)$ are C_0 = 0.5288 MeV, C_1 = 0.0009 MeV, and ε = 0 1724 MeV.

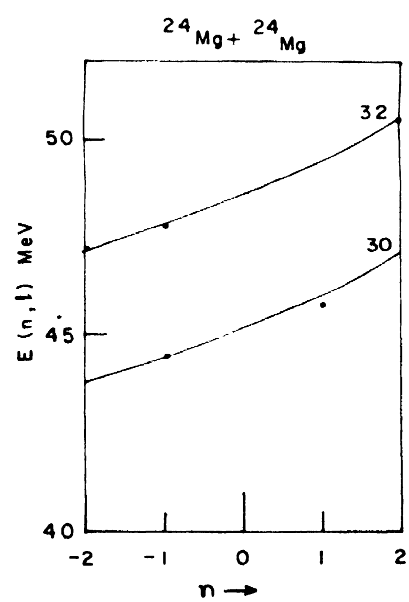

Figure 2. Plots of $E(n, l)$ against the assumed values of n for the resonance data of the ^{24}Mg + ^{24}Mg system. The parameters used to compute $E(n, l)$ are C = 0.5835 MeV and ε = 0.0618 MeV

The deep potential approach [12] is quite interesting. This potential also gives barrier position approximately equal to that obtained by using global nucleus-nucleus potential. However it gives a deeper pocket because of the large values of strength of the attractive part of the nuclear potential. In [12] the strength of the imaginary part in the pocket is 2.5 MeV. In the absense of imaginary part such deep potential can be expected to generate very sharp resonances but their width will be drastically enhanced and will be of the order of the imaginary part if present. Even for such potentials if the barrier region is surface transparent, one can expect resonances close to the barrier to be narrower because imaginary part of the potential will affect them comparatively to a lesser extent. This point is illustrated using model calculations in [13]. These demonstrate the fact that in the case of surface transparent potential (small imaginary part in the barrier region and large imaginary part in the pocket region) the barrier region resonances are more important than the pocket resonances.

In the case of $^{16}O + {}^{16}O$ system the spacing between the levels is of the order of 0.6 to ~ 1 MeV [2–4] and hence one expects their widths to be in the KeV region. Hence even in the fits obtained using the deep potential the resonances are perhaps dominated by the barrier only.

The calculations on the repulsive Eckart potential [1] and the truncated harmonic barrier indicate that different resonance level spacing can be expected if one uses different types of potential barriers. This can be fruitfully exploited in constructing appropriate models for the description of resonances in heavy ion reactions.

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