NOTE

Indian J. Phys. **68A** (6), 579 – 585 (1994)

I J P A
- an international journal

Study of the behaviour of electron gas

O K Harsh[+], Tapasi Dhar and S P Godia[*]
Department of Post Graduate Studies and Research in Physics, Feroze Gandhi College, Rae Bareli, Uttar Pradesh, India

Received 29 September 1993, accepted 22 July 1994

Abstract : Effect of perturbations on plasma oscillations in degenerate electron gas in metals have been studied. By taking account of perturbed long range forces and the effect of short range interactions, it has been shown that there is a critical wave vector at which the plasma oscillations can be realized The effect of radius parameter r_s, plasma energy and interatomic distance on the dispersion characteristics have been discussed The effect of perturbations decreases the critical wave vector for exciting the plasma oscillations. Our studies reveal that in beryllium both first order perturbed and relative first and second order perturbed dispersion curves are qualitatively similar to that of the unperturbed curve, while for aluminium these contributions are opposite

Keywords : Plasma oscillations in metals, critical wave vector, dispersion relations

PACS No. : 52.35-g

The valence electrons in solids may oscillate collectively under certain conditions with a high eigen-frequency w_p—the plasma frequency [1-11], and is denoted by the relation

$$w_p = \left(\frac{4\pi n e^2}{m} \right)^{1/2},$$

where n is the electron concentration and m is the effective mass of an electron. Such oscillations in electron gas of solids play very important role in the study of various properties *e.g.* determining the frequency range of transparency and reflection coefficient of electromagnetic waves [5], diagnosing various features of bulk metal in the form of thin metallic films *etc.*

[+] Formerly at Lucknow University and Institute of Technology, Banaras Hindu University, Varanasi, India
[*] Physics Department, V. S. S. D. College, Kanpur, India

For metals, if the ratio of phase velocity to the Fermi velocity of electron gas is not very large, then the electromagnetic waves of frequencies much higher than the electron-plasmon frequency are damped out [6]. At the limit of zero temperature, the response function of electron gas in metals is purely real and natural oscillations are sustained, while at non-zero temperatures, the above mentioned velocity ratio decreases and perturbations are initiated. Due to the presence of such perturbations the total energy of the electron gas becomes a negative quantity and the frequency of plasma oscillations decreases. For higher values of the wave vector (k), the phase velocity of electron gas oscillations is less than the net average electron gas velocities and the response function becomes complex. Under these perturbed conditions, for high k, the damping of natural oscillations is enhanced which results into complete cessation of oscillations at $k = k_c$, where k_c is the critical wave vector. Thus the study of perturbation on plasma oscillations for degenerate electron gas plays a vital role in understanding the sharp cut-off of reflection and transmission coefficients and hence in understanding the principle of filters.

In the present work, studies have been made for dispersion characteristics of a degenerate electron gas in the presence of small amplitude perturbations. For such a gas, the unperturbed dispersion equation may be written as [2,7]

$$w_k^2 = w_p^2 + \frac{6}{5}\bar{E}_m \frac{k^2}{m} + \frac{\hbar^2 k^4}{4m^2} + \text{Exchange term}, \tag{1}$$

where $\bar{E}_m$ is the minimum average energy of an electron, which may be approximated to Fermi energy for monovalent metal at a very low temperature.

If we include the bilinear and the exchange interaction between electrons which were not considered by Bohm and Pines [2], in eq. (1), then

$$w_k^2 = w_p^2 + \frac{3}{5}k^2 v_F^2 + \frac{\hbar^2 k^4}{4m^2} + E_{ex}\left(\frac{k^2}{m}\right)\left(-\frac{4}{5}\log\frac{2}{\beta} + \frac{4}{5} - \frac{7\beta}{6} + \frac{113\beta^2}{60} - \frac{\beta^3}{9}\right), \tag{2}$$

where E_{ex} the exchange interaction between the electrons and $\beta = \frac{k_c}{k_F}$. The results of eq. (2) may further be improved by including the Exchange effect [12] by a proper linearization of the interaction term, known as the generalized random phase approximation (GRPA) [13–15]. For this, one may use the jellium model of the electrons in solids. Using this technique, Glezos [12] obtained the following dispersion relation :

$$\Omega = \Omega_p + \frac{6Q}{5\Omega}(1 - 0.068\, r_s) \tag{3}$$

where $\Omega = w/E_f$, $\Omega_p = w_p/E_f$, $Q = k/k_f$ and $r_s = r_0/a_0$ which is a dimensionless parameter, r_0 being the average interelectronic distance and a_0 is the first Bohr radius.

Using the first-order perturbation correction due to short range Coulomb interactions, the average energy per electron of the oscillating electron gas in metals may be written as [7],

$$\bar{E} = \frac{2.21}{r_s^2} + \frac{\sqrt{3}\beta^3}{2r_s^{3/2}} - \frac{0.916}{r_s}\left(1 + \frac{\beta^2}{2} - \frac{\beta^4}{40}\right) \text{Ry.} \tag{4}$$

The value of $\beta = \frac{k_c}{k_F}$ corresponding to minimum average energy of an electron may be obtained by neglecting the last term in eq. (4) and taking its first derivative with respect to β equal to zero. In this way the value of β obtained is $\beta = 0.353\ r_s^{1/2}$.

The plasma energy for the metal can be computed with the help of formula [8],

$$E_p(0) = \hbar w_p = 28.8\left(\frac{Z\sigma}{W}\right)^{1/2} \tag{5}$$

where Z is the total number of electrons participating in plasma oscillations (valence electrons), σ is the specific gravity and W is the molecular weight. The dimensionless parameter r_s may be expressed as [3]

$$r_s = \left(\frac{47.11}{\hbar w_p}\right)^{2/3} \tag{6}$$

where w_p is the calculated plasma frequency as mentioned in eq. (5).

With the help of eqs.(2), (4) and (6) the effect of first order perturbation on the dispersion characteristics of plasma oscillations in several metals *i.e.* Be, Al, Mg, Li and Na have been studied (Figures 1–5).

The second order perturbation energies can be obtained by subtracting the contribution due to long range Coulomb interaction from the sum of the long and short range second order perturbation contributions of Coulombic interactions as given by Nozieres and Pines [9] and may be written as

$$\Delta\bar{E} = 0.0311 \log r_s - 0.0003\, r_s - 0.096 \text{ Ry.} \tag{7}$$

On taking the sum of eqs. (4) and (7), we draw graphs (Figures 1–5) to show the relative contribution of the first- and second-order perturbation on the dispersion characteristics of plasma oscillations. With the help of these graphs, the effect of various parameters *e.g.* Fermi energy, r_s and plasma energy can be studied (Tables 1 and 2).

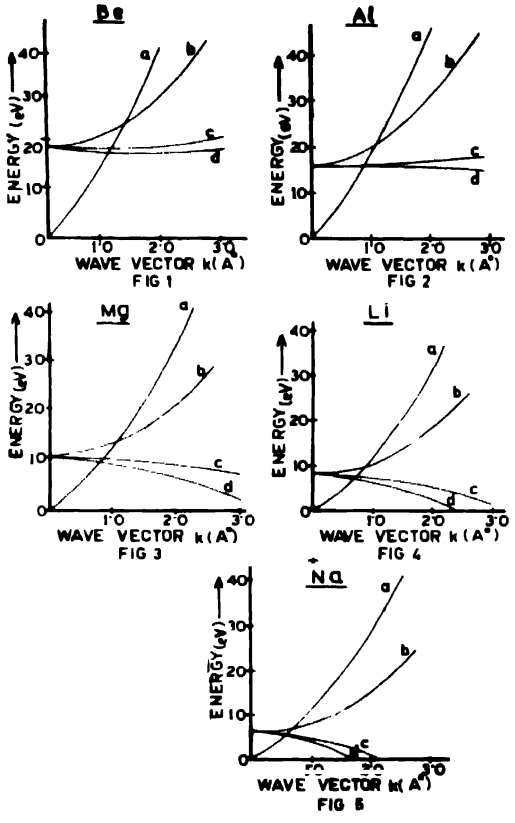
Critical wave vector :

The critical wave vector k_c which limits the plasma excitations, is determined by the intersection of the dispersion function of eq. (2) and the following express on for the maximum energy to excite an individual electron

$$E(k) = \frac{(\hbar k_F + \hbar k)^2}{2m} - \frac{\hbar^2 k_F^2}{2m}, \tag{8}$$

where $\hbar k_F$ is the momentum of an electron at the Fermi surface.

The plot in Figures 1 to 5 for metals Be, Al, Mg, Li and Na show the effect of first and second order perturbations over the unperturbed cases. One can see that the value of k_c decreases as we go towards the first and the second order perturbations respectively. Also,



Figures (1–5). Excitation Energy (E) *vs* wave vector (k) graphs for Be, Al, Mg, Li and Na respectively, representing

(a) Maximum energy to excite an individual electron (eq 8)

(b) Plasmon dispersion (unperturbed)

(c) Plasmon dispersion with first order perturbation

(d) Plasmon dispersion with second order perturbation.

the value of k_c is higher for metals with lower r_s. The rate of decrease of critical wave vector for both the first and the second order perturbation increases as the plasma energy decreases or as the value of r_s decreases (Tables 1 and 2).

At the higher value of k, the dispersion characteristics of Li, Na and Mg are divergent downwards in both the first- and second-order perturbations, while in case of Be, they

diverge upwards. For Al, these two perturbation graphs diverge in opposite direction (see the graphs). This abnormal behaviour has only been found in case of Al. This may be due to the fact that aluminium may behave as a superconductor under certain condition.

Table 1. Effect of various parameters on plasmon energy.

Sl No.	Element	Total no of electrons participating in plasma oscillation	Nearest neighbour distance (in Å)	Radius parameter	Plasmon Energy (eV)	
					Calculated (eq 5)	Experimental Ref [10,11]
1	Be	2	2.22	1.88	18.45	19 1
2.	Al	3	2.86	2 07	15 8	15.3
3	Mg	2	3.20	2.65	10 89	10 6
4	Li	1	3 023	3.25	7.99	7 12
5	Na	1	3 659	3 93	5 92	5 71

Table 2. Relative contribution of the first & second order perturbations

Sl no.	Element	Unperturbed (k_c) Å^{-1}	Perturbed k_c		Relative decrease of k_c (percent)	
			1st order	2nd order	1st order	2nd order
1	Be	1 223	1 0154	1 002	16 97	1.32
2.	Al	1.14	0 94	0.929	17.54	1.17
3	Mg	0 9784	0.8015	0 7803	18.08	2.64
4	Li	0 8570	0 6949	0.6753	18 91	2 82
5	Na	0 747	0 5905	0.577	20.96	2 26

Graphs have also been drawn in Figure 6 showing the variation of excitation energy *vs* wave vector, by using the eqs. (3) and (8) for Be for which experimental results exist [12]

From these graphs (unperturbed, first- and second-order), it can be concluded that the inclusion of effect of exchange interaction in the Be reduces the value of critical wave vector (k_c) for exciting the plasmon oscillations. The value of the critical wave vector for Be have been found as 1.0096 Å^{-1}, 0.9947 Å^{-1} and 0.9796 Å^{-1} for the unperturbed, first- and second-order perturbations respectively. These values are in excellent agreement with the work of other authors [16,17].

The values of the critical wave vectors obtained for Be in latter case may be applied to explain the minimum plasmon scattering angle, as experimentally observed by Suzuki and Tanokura [11] and Papademetriou *et al* [18] using incident CrK β_1 and CuK α_{w2} radiation for studying the inelastic scattering phenomenon in Be. In our present results the values for the critical wave vectors (using GRPA) are 1.0096 Å^{-1}, 0.9947 Å^{-1} and 0.9796 Å^{-1} for

unperturbed, first- and second-order perturbations respectively. Using the slow-fast interaction effects of Langreth [19] as $\beta' = \beta - 0.1$, (where $\beta = \frac{k_c}{k_F}$, k_F being the Fermi wave

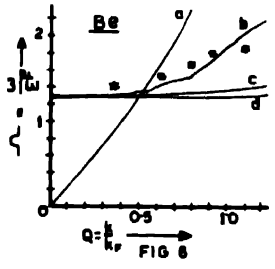


Figure 6. Excitation energy $\Omega = \frac{w}{E_F}$ *vs* reduced wave vector $Q = \frac{\kappa}{k_F}$ graphs for Be, representing

(a) Limit of particle-hole spectrum [12]

(b) Plasmon dispersion [12] (unperturbed)

(c) Plasmon dispersion with first-order perturbation

(d) Plasmon dispersion with second order perturbation, while asterisks stand for experimental values [12]

vector), the values of minimum critical angles have been calculated as 15.57°, 15.28° and 14.9° for unperturbed, first- and second-order perturbations respectively which are close to 10° and 16° as observed by Suzuki and Tanokura [11] and Papademetriou *et al* [18].

The propagation of electromagnetic waves and their optical properties are determined by the above mentioned features of electron gas oscillations. It appears that the precise measurements of optical properties at various temperature and different fractions of impurities would help in diagnosing the electron gas in metals. The role played by the mode of preparation of thin films for various electron densities can be analysed by having such features of electron gas oscillations.

References

[1] D Pines in *Solid State Physics* ed. F Seitz and D Turnbull (New York : Academic) **Vol. 1** p 368 (1955)

[2] D Bohm and D Pines *Phys. Rev.* **92** 609 (1953)

[3] D Pines in *Elementary Excitations in Solids* (New York · Benjamin) (1964)

[4] O K Harsh in *DSc Thesis* (Kanpur University : Kanpur, India) (1987)

[5] R H Ritchie *Prog. Theo. Phys.* **29** 603 (1963)

[6] A A Maradudin and D L Mills *Phys. Rev* **B7** 2787 (1973)

[7] S Raimes *Many Electron Theory* (Amsterdam · North Holland) (1972)

[8] L Marton, L B Leder and H Mendlowitz *Advances in Electronics and Electron Physics* edited by L Marton **Vol. 7** (New York . Academic) p 225 (1955)

[9] P Nozieres and D Pines *Phys. Rev.* **111** 442 (1958)

[10] C Kittel *Introduction to Solid State Physics* 4th edn. p 277

[11] T Suzuki and A Tanokura *J. Phys. Soc. Jpn.* **29** 972 (1970)

[12] N M Glezos *Phys. Rev.* **B43** 7538 (1991)

[13] D Pines and P Noziers *The Theory of Quantum Liquids* (New York : Benjamin) (1966)

[14] P M Platzmanf and X Wolff *Solid State Physics Supplement* **13** (New York : Academic) (1966)

[15] S Ichimaru *Rev. Mod. Phys.* **34** 1017 (1982)

[16] F Brosens, J T Devreese and L F Lemmens *Phys. Rev* **B21** 1363 (1980), **21** 1349 (1980); *Phys. Stat. Solidi* **B74** 45 (1976)

[17] H Kanazawa and Sho-ichiro Tani *Prog. Theo. Phys* **19** 153 (1958)

[18] D K Papademetriou, D Katsanos and A G Doukas *Phys Stat Sol.* (b) **133** 223 (1986)

[19] D C Langreth *Phys. Rev.* **B1** 471 (1970)