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Investigation of the influence of nuclear charge on the internal Bremsstrahlung accompanying the β-decay of ^{45}Ca and ^{141}Ce

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Abstract : The internal Bremsstrahlung (IB) spectrum accompanying the allowed β-decay of ^{45}Ca and non-unique first forbidden β-decay of ^{141}Ce was measured using NaI (Tl) scintillation spectrometer The corrected IB distribution was compared with the theories for allowed β-transition of Knipp and Uhlenbeck as well as of Bloch (KUB), Lewis & Ford and Nilsson Further, for ^{141}Ce-isotope the corrected IB distribution was compared with the Ford and Martin theory (for detour β-transition) and modified KUB theory (where the shape correction factor suggested by Konopinski and Uhlenbeck on the Fermi β-decay theory was applied to the calculated IB based on Nilsson theory) This comparison between experiment and theory, and between the measurements of the two isotopes reflects the role played by the nuclear charge and the degree of forbiddenness on the IB spectrum

Keywords : β-decay, internal Bremsstrahlung, influence of nuclear charge

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1. Introduction

Internal Bremsstrahlung (IB) is a weak continuous energy electromagnetic radiation that accompanies β-decay. The basic theory of IB in β-decay was developed for allowed β-transitions by Knipp and Uhlenbeck [1] and independently by Bloch [2] where they neglected the effect of the Coulomb field of the nucleus ($Z = 0$) into IB process. Modifications to the KUB theory have been worked out, taking into account coulomb effects by Lewis and Ford [3] and Nilsson [4], for allowed β-transitions. Further, the KUB theory was extended to forbidden β-transitions by Chang and Falkoff [5]; Ford and Martin [6] and other authors. Ford and Martin considered the contribution to IB from detour transitions in the forbidden decay process and worked out the theory for unique first forbidden transitions only.

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The literature survey in the field of IB reveals [7,8] that there is a lack of agreement between the theory and experiment. This disagreement is more conspicuous in the case of forbidden β-transitions. The divergence between the measurements and the theory is found to increase with increasing energy.

In the present work a new trail of theoretical calculation was carried out for ^{141}Ce-isotope. This is an application of the shape correction factor (C_1), suggested by Konopinski and Uhlenbeck [9,10] for the first-forbidden β-transitions to the theory of Nilsson [4] (where in Nilsson's theory the Coulomb correction factor was considered in a more refined manner than other theories). This is called modified KUB theory. As a result, a good agreement between experimental results of ^{141}Ce and modified KUB calculations was observed in a wide range of energy. Also the experimental results are compared with the theoretical calculations according to KUB, Lewis and Ford [3], Nilsson [4] and Ford and Martin [6] theories.

^{141}Ce was selected with ^{45}Ca for the investigation of IB to obtain information on the Coulomb contribution to IB process. ^{45}Ca was classified as an allowed β-transition, with an end point energy of 0.258 ± 0.002 MeV, log ft value of 5.6 and a half life of 163 days [11]. ^{141}Ce is a β-emitter with two β-energies, the first one with branching ratio of 30% and end point energy of 0.581 MeV, while the second one with branching ratio of 70% and end-point energy of 0.436 MeV. This latter β-group feeds the first excited state of ^{141}Pr nuclei followed by gamma transition of energy 0.145 MeV, while the 0.581 MeV β-group from ^{141}Ce feeds the ground state of ^{141}Pr nuclei. ^{141}Ce-β-decay was classified as non-unique first forbidden β-transitions [12].

The IB from these two isotopes should have about the same probability except for the Coulomb contribution, which would be significant in the case of ^{141}Ce. The comparison of IB probabilities for the two isotopes for the same photon energy would clearly indicate the amount of the Coulomb contribution and the effect of degree of forbiddenness on IB probabilities.

2. Experimental details

^{45}Ca and ^{141}Ce-isotopes were originally prepared in Labo des Produits Biomedicaux C. E. N. Saclay. Each isotope was obtained in a bottle containing 0.4 ml of activity 5 μ Ci. A known quantity of the solution was evaporated drop by drop, on a thin aluminized film mounted on a perspex ring of inner diameter 1 cm. Sufficient care was taken to have uniform spread using drop or two of dilute aqueous solution of insulin and the extent of the source was limited to a circular area of 0.6 cm diameter.

The IB spectrum of each isotope was measured using a scintillation gamma ray spectrometer utilizing a cylindrical NaI (Tl) crystal (25 mm in diameter and 19 mm in height), mounted on a photomultiplier (type 50 AVP.). The spectrum was then differentiated using a single channel pulse height analyzer and a scaling unit. An Al-sheet of an adequate thickness has been used as a β-stopper to avoid the effect of the escaped β-particles [13,18]. The typical IB spectra of ^{45}Ca and ^{141}Ce are shown in Figure 1a and Figure 1b respectively. As shown in Figure 1b the typical IB spectrum of ^{141}Ce illustrated that the measured photon intensity is a contribution of the photons from the monoenergetic gamma-line energy (0.145 MeV) together with the IB intensity accompanying β-decay. Therefore one must remove the gamma line contribution to obtain the pure IB spectrum.

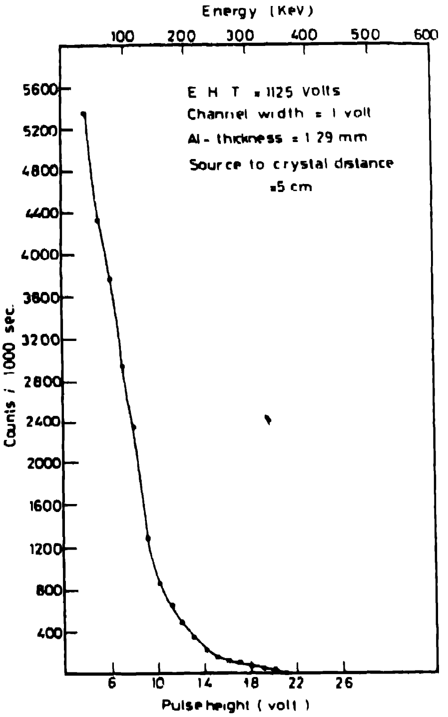

Figure 1a. The typical IB spectrum of ^{45}Ca.

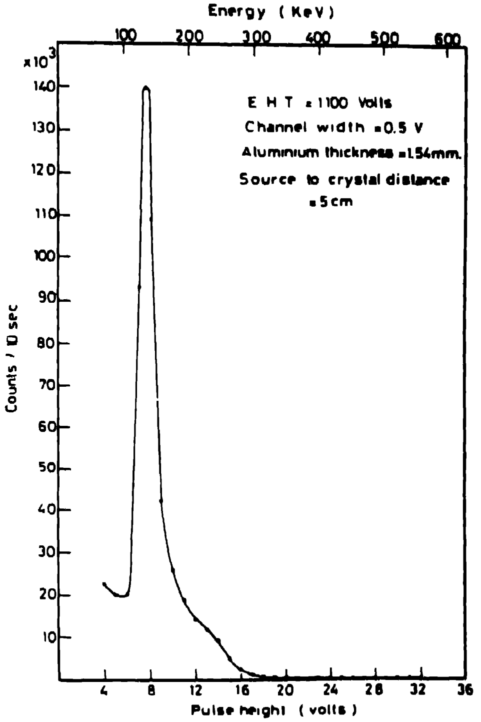

Figure 1b. The typical IB spectrum of ^{141}Ce.

Two approaches have been followed in the present work to remove the gamma line of 0.145 MeV energy. The first one is based on applying the least square fit for energy above that of the gamma-line up to the end-point, after plotting the pulse height distribution multiplied by the corresponding photon energy on a semilogarithmic scale. By extrapolation of the expected straight line (as a result of the exponential character of the IB spectrum [19], the pure IB spectrum was obtained as shown in Figure 2, full curve. The second approach was first suggested by Narasimha Murty and Jnanananda [13], which is based on the choice of a mono-energetic source with gamma-line energy equal to that of the source under investigation and by measuring the spectrum of this monoenergetic source under the same condition and geometry of the IB spectrum. To generalize this approach, the line shape spectrum at any photon energy can be obtained with a considerable accuracy from the study of the line profile of many standard mono-energetic sources covering the energy range of interest. The pure IB spectrum obtained as a result of using the second approach is shown in

Figure 2 (the broken curve). It is clear from this Figure that the two approaches lead approximately to the same result, within the experimental errors.

The energy resolution of the whole gamma spectrometer was determined to be 12%. To determine this value, the complete pulse height spectra of the monoenergetic gamma-ray lines have been measured by a scintillation spectrometer at different high tension and at different sources to crystal distances. Dividing the full width at half maximum (FWHM) by the corresponding energy one can obtain the energy resolution of the spectrometer [17]. The NaI (Tl) crystal-efficiency was calculated for different photon energies and checked experimentally. The measured IB spectrum was analyzed into its constituents by performing a peeling off procedure starting from the high energy tail of the spectrum. In order to accomplish this purpose, the line profiles of monoenergetic gamma lines of energies covering the whole IB range studied were used. The IB intensity was then corrected for the solid angle, absorption, crystal efficiency, back scattering and external bremsstrahlung. The details of the experimental arrangements and all necessary corrections which were applied for the measured IB can be found in the previous investigations [14-18].

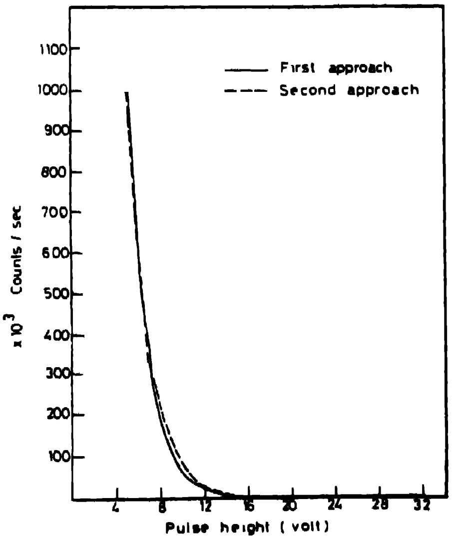

Figure 2. The pure IB spectrum of ^{141}Ce, free from monoenergetic gamma line contributions after using the two approaches

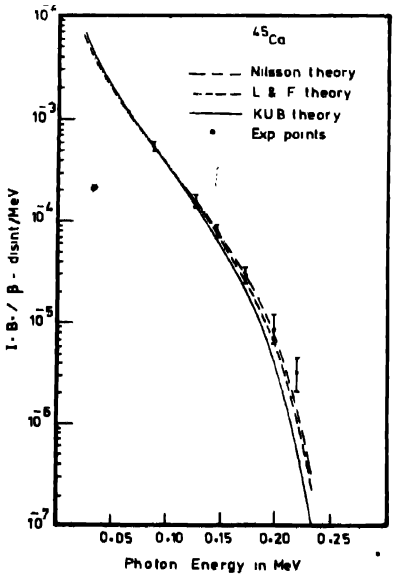

Figure 3. The experimental IB probability of ^{45}Ca and that calculated theoretically according to KUB, Lewis and Ford and Nilsson theories.

3. Result and discussion

^{45}*Ca* :

The experimental IB probability of ^{45}Ca and that of the allowed theoretical calculation according to KUB, [1,2], Lewis and Ford [3] and Nilsson [4] theories, in the energy range from 64–225 keV, are shown in Figure 3. The experimental distribution is normalized to the theoretical value at 88 keV. From Figure 3 one can see that the experimental results agree with the KUB theory up to 150 keV. Above this energy, the experimental results show positive deviation from the KUB theory. A significant improvement is found when the comparison is made with the Coulomb corrected theory by Nilsson [4] over the whole investigated range.

The IB for ^{45}Ca was firstly studied by Kortokov *et al* [20] in the energy range from 40 to 215 keV and the comparison has been carried out with the theoretical predictions KUB [1,2]. An agreement was obtained in the energy range from 60 up 130 keV. Below 60 keV, the experimental results exceeded the theoretical calculations. Above 130 keV, disagreement between experiment and theory occurred. For instance at 180 and 215 keV, the deviation was found to be more than 25% and 35% respectively. Singh and Al-Darguzel [21] reported that the experimental data are lower than the theoretical calculation throughout the studied energy range. This deviation was assumed to be due to improper determination of the IB probability as stated by Powar and Singh [22]. In other studies [23], an agreement is achieved in the low energy, up to 155 keV, with the theoretical calculations of KUB [1,2]. Above 155 keV, the experimental results were in agreement with Lewis and Ford [3] theory up to 210 keV.

In this work, the ratios of the experimental IB probabilities to those calculated according to KUB [1,2] (R_1), Lewis and Ford [3] (R_2) and the Nilsson [4] (R_3) theories for different photon energies are determined and presented in Table 1 along with the values obtained by previous work [22,23]. From this table, the ratios R_3 of the present work are found closer to unity than those previously determined [22,23]. The wider deviation in the results of Powar and Singh [22] may be attributed to the approximation in their method of analysis or in the procedure of estimating the Coulomb correction or both. From the present measurement, a good agreement was obtained between the experimental IB probability and those calculated theoretically, particularly using the theory of Nilsson [4].

Table 1. The ratios between the experimental results and those calculated theoretically according to KUB (R_1), Lewis and Ford (R_2) and Nilsson (R_3) theories for ^{45}Ca.

E in keV	R_1	R_2	R_3	References
180	1.48 ± 0.35	1.07 ± 0.26	1.0 ± 0.23	Present work
	1.21	1.0	0.75	[23]
	3.55	2.13	1.6	[22]
200	1.88 ± 0.51	1.25 ± 0.36	1.09 ± 0.32	Present work
	1.26	0.92	0.68	[23]
220	3.48 ± 1.04	2.0 ± 0.60	1.63 ± 0.49	Present work
	7.7	5.0	3.3	[22]
240			6.0	[22]

^{141}Ce :

The present experimental results of the IB probability per β-disintegration per MeV in comparison with that calculated theoretically on the basis of allowed β-transitions (KUB [1,2] theory and Coulomb corrected theories of Lewis and Ford [3] and of Nilsson [4]) are presented in Figure 4. The investigated energy region was from 133 keV to 495 keV. Normalization was done at 133 keV. This normalization is usually done to avoid the β-activity measurements and in turn eliminate the uncertainties of the measurements. From Figure 4 one observes that there is a disagreement between experimental results and these theories on the basis of allowed β-transitions. This disagreement is attributed to the fact that while the first forbidden radionuclei emit β-spectra analogous to the allowed ones, their IB spectra differ from that of allowed β-transition nuclei.

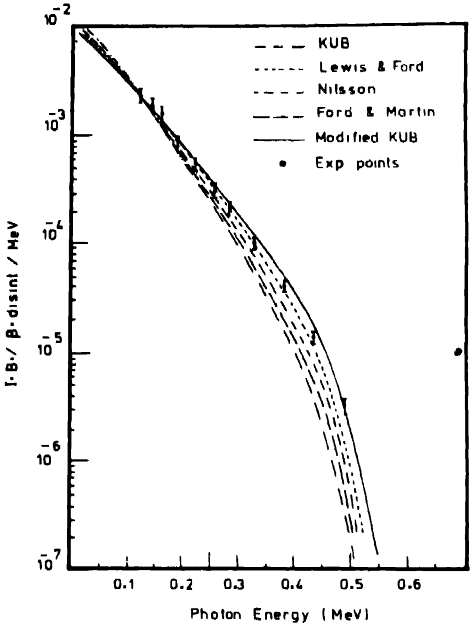

Figure 4. The experimental IB probability of ^{141}Ce and that calculated theoretically according to KUB, Lewis and Ford, Nilsson, Ford and Martin and modified KUB theories.

Therefore, the experimental results were compared with that theoretically calculated according to Ford and Martin [6] theory, (dotted curve in Figure 4), in which the degree of forbiddenness and detour transition was taken into consideration. From Figure 4 one observes that the experimental results are in a better agreement with Ford and Martin [6] calculations up to 438 keV photon energy. Above this energy there is a slight deviation between the experiment and Ford and Martin [6] theory. In the measurement of Gundu Rao and Sanjeeviah [24], they found a total disagreement between their experimental values and theories. For instance, the discrepancy between their results and Lewis and Ford [3] theory was found to be 5%, 21%, 57% and 170% at 200, 250, 300 and 350 keV. The corresponding values with respect to Ford and Martin [6] theory at the same energies were 55%, 50%, 37% and 11%, respectively. The only difference between the present measurement and that of Gundu Rao and Sanjeeviah [24] is their use of magnetic deflection technique. The large discrepancy between their experimental results and theory was not found in the present work.

Furthermore, the present experimental results are compared with those calculated theoretically according to the shape corrected modified KUB theory (solid curve in Figure 4). One observes that the present results are in close agreement with theory over the whole energy range studied. However in the energy range 333 to 438 keV, the experimental results are slightly lower than these modified KUB calculations. This difference may be attributed to the fact that ^{141}Ce β-decay is classified as a non-unique first forbidden while the shape correction factor (C_1) in modified KUB theory was calculated as a first forbidden type. The difference between the two types of β-decay is in the log ft-value, which is the product of a dimensionless function (f), by the half-life of the β-decay [19,25], therefore they must have different IB spectra.

Influence of nuclear charge on IB process :

As mentioned by Nilsson [4] the influence of the Coulomb field on the IB probability is introduced by an approximation factor as

$$F(Z,W_e) = \frac{2\pi\alpha\ ZW_e/P_e}{1-\exp\left(-2\pi\alpha\ ZW_e/P_e\right)},$$

which includes the dependence of the Coulomb field on the nuclear charge Z of the source, and the energy of the electron W_e. To illustrate the influence of the electrostatic Coulomb field on the IB spectrum, the ratio [26] of the IB probability $S(K)$ dk calculated according to Nilsson to that calculated according to KUB ($Z = 0$) is shown as a function of the energy ratio $K/(W_0 - m_0C^2)$ for ^{45}Ca and ^{141}Ce-nuclei in Figure 5a. It is clear from this figure that the effect of the Coulomb field is high at the high photon energies and it decreases as the photon energy decreases in both cases of ^{45}Ca and ^{141}Ce isotopes. This may reflect the importance of the Coulomb field on IB process, specially at high photon energies. Also Figure 5a reflects the importance of the Coulomb correction for high Z, which is the case of ^{141}Ce-isotope. It is known that the Coulomb effect is higher in case of higher values of Z, and lower values of end point W_0. Then on comparing the Coulomb effect for the two isotopes ^{45}Ca ($Z = 20$, $E\beta^{max} = 257$ keV), and ^{141}Ce ($Z = 58$, $E\beta^{max} = 581$ keV), one may expect that the IB process will be affected with approximately the same degree of Coulomb effect in both nuclei. This was not observed as can be seen from Figure 5a, where the Coulomb effect for ^{141}Ce is larger than that for ^{45}Ca at all values of energies. Therefore, one may suggest, that the influence of Z on the Coulomb effect is more pronounced than that due to the end point.

Also the IB measurements for ^{45}Ca and ^{141}Ce nuclei enable one to clarify the influence of the degree of forbiddenness on the IB probability. For this purpose to be accomplished, the ratios of the experimental IB probability to the corresponding values of Nilsson calculations, as a function of the fraction energy are drawn in Figure 5b. From this figure, one finds that the deviation from the Nilsson [4] calculations is higher for ^{141}Ce than that for

^{45}Ca. This deviation is due to the forbidden character of the β-transition in case of ^{141}Ce (which is non-unique first forbidden one).

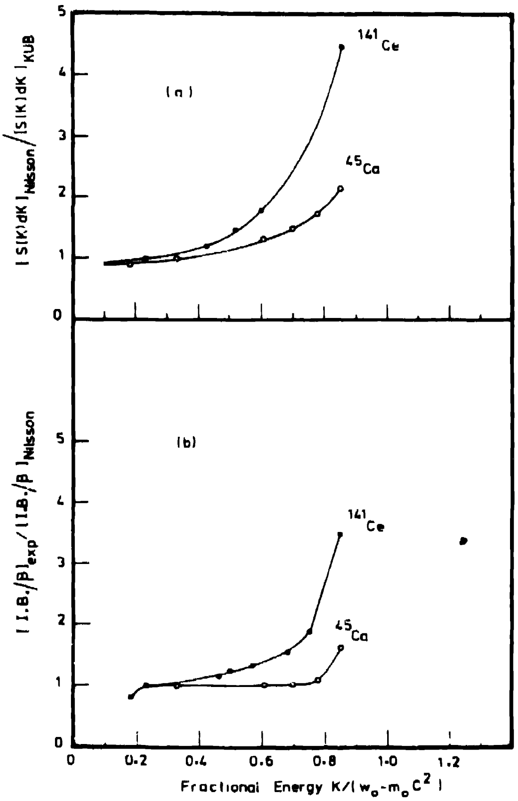

Figure 5. (a) The ratio of the IB probability of ^{45}Ca and ^{141}Ce-isotopes calculated according to Nilsson's theory to that of the KUB theory *versus* the energy ratio $K/(W_0 - M_0C^2)$. (b) The ratio of the experimental IB probability of ^{45}Ca and ^{141}Ce-isotopes to that calculated according to Nilsson's theory *versus* the energy ratio $K/(W_0 - M_0C^2)$

Therefore, the above study may lead to the previously reported conclusion [15,16] that, if the effects of Coulomb correction, degree of forbiddenness and detour contribution are considered in a more refined manner than those of the aforementioned theories, a satisfactory agreement between experiment and theory particularly at the higher energy values can be expected.

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