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Growth of breakdown susceptibility in random composites and BTW model : Prediction of dielectric breakdown and other catastrophes

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Abstract : The responses to short duration pulses (of electric field, of additional 'particles', *etc*) have been studied numerically for metal-insulator composites before dielectric breakdown and the BTW (sand pile) model before the critical avalanches. The study of the breakdown susceptibility (defined in the text) indicates universal behaviour near the catastrophic breakdown or the self-organised critical points, and its study can locate these breakdown points very accurately. We make a numerical study of the Laplace's equation of a dielectric with random bond conductors below its percolation threshold, and of the BTW model. We show that, if one applies weak pulses of appropriate external field (*e.g.*, electrical pulse in the case of dielectric breakdown) and studies the breakdown susceptibility, one can locate accurately the breakdown or disaster point, much before its occurrence, by extrapolating the inverse breakdown susceptibility to its vanishing point. The growth of the susceptibility, coming from stress correlations, in the Burridge-Knopoff model of earthquake is again shown to be exponential in time, as for pulse susceptibility in this model (*J Phys France*) **I**, 5 153 (1995). Prediction of the earthquake point in time is also possible in the model from the study of its inverse logarithm with straightline extrapolation to its vanishing point.

Keywords : Dielectric breakdown, metal-insulator composites, BTW model, earthquake

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Considerable progress has recently been made in the statistical study of the breakdown strength of disordered solids [1]. These solids may be porous media, random composites, granular packings or layers in earth mantle; the relevant phenomena are fracture, dielectric breakdown, avalanche or earthquake. Several simple models (at a semi-microscopic level) have been introduced for mechanical [2] and electrical [3] failure of such randomly disordered media; the media are represented by disordered lattices, failure is modelled by individual bonds breaking irreversibly. These theoretical results about the fracture or breakdown strength distribution have also been checked in several experiments [4]. Failure processes *playing intrinsic roles in many systems of industrial importance and in many natural disaster phenomena, the above mentioned statistical studies establishing the nature of fracture-strength distribution, its fluctuations and consequent power laws for the average strength of such solids [1], are of extreme importance.* Although a significant amount of literature has been developed for these studies of failure strength distribution, not much has been done on the dynamics of microscopic failures. Since the microscopic failures are irreversible and therefore require intermediate redistributions of the forces, the equations for the dynamics of failure are intrinsically nonlinear and dissipative. The formulation and study of such equations are neccessary for the search of any precursor effect of the macroscopic failure, specially if the macroscopic failure is at most a critical and not a fully chaotic phenomenon In some recent experiments [5] on the dynamics of the cracks in thin glass plates with thermal stresses, the dynamics seems to undergo a sequence of numerous but reproducible instabilities, not sensitive to every detail of the fluctuations in initial conditions. The dynamics of fracture is thus observed to be mostly critical, on the verge of chaos but not quite chaotic [6], the situation depends somewhat on the velocity of the crack tip. Similar is the case for the dynamical (Burridge-Knopoff [7] type) models of earthquake [8], where also one gets the Guttenberg-Richter type power law for the magnitude variation of the density of quakes, the failure distribution being critical.

These studies establish the very nonlinear, yet nonchaotic, nature of the dynamics of breakdown and give the distribution of the breakdown strength or magnitudes. Recently, we have shown [9] that an appropriately defined pulse susceptibility can be studied for the Burridge-Knopoff model [7] of earthquake and by looking at the approach to divergence of the susceptibility, before the earthquake, we can predict the earthquake point in time in the same model. Here, we apply the same method of study of the growth of the pulse or breakdown susceptibility in a metal-insulator composite dielectric (a dielectric with random conductors before the percolation threshold) to predict the dielectric breakdown point of the random sample. This is done numerically by solving the Laplace equation of a random nonpercolating network of conductors [10]. This investigation is also extended for predicting the self-organised critical (or breakdown) point in the Bak-Tang-Wiensenfeld (BTW) model [11].

We consider now a LxL spuare lattice [10], with $L = 25$ here. A fraction p of its sites are randomly occupied; they represent conducting sites. The $(1-p)$ fraction of unoccupied sites represent the dielectric sites. For each configuration with average conductor concentration p ($< p_c$, p_c representing the percolation threshold [12], with $p_c \equiv 0.593$ here), we first check whether or not there exists any percolating path through the conducting sites Percolation connections are taken here through the nearest-neighbour conductor sites. If no such path exists, giving the macroscopic connection across the lattice, we apply a voltage V across the sample in the horizontal direction, with all sites at the leftmost column at voltage $V = 0$ and those at the rightmost column at voltage $V = V$. All the other sites are at first given a randon distribution of potential. All the nearest neighbour conducting sites are then updated to the same (arithmetic mean) potential. This makes clusters of conductors at different voltages, separated by dielectrics. The discretised version of the Laplace equation is solved and site voltages are updated at each time iteration until the sum of the differences in the successive updates everywhere goes below a small value (10^{-4} here). For breakdown, if any nearest neighbour dielectric bond gets a voltage difference of magnitude more than a fixed value v_c (= 1.0 here), the bond is broken in the sense that both sites on its ends are changed to conductors.

It is clear that the average breakdown voltage for a fixed sample dimension L decreases with increasing disorder, that is conductor concentration p, until it vanishes at the percolation threshold p_c [12]. For any particular initial concentration p_o of random conductors, the sample dielectric has an average breakdown voltage $V_b(p_o)$, above which the sample starts conducting *via* the conductors (original and broken). We intend to predict the $V_b(p_o)$ value by looking at the response of the sample to electrical pulses much before its breakdown occurs.

We thus make an increase dV, over V, in the applied voltage across the sample and look for number (n) of dielctric bond breaking locally as the voltage across any bond increases beyond v_C = 1.0. This helps us to define the susceptibility $\chi_d = \frac{dn}{dV}$. It is shown in Figure 1

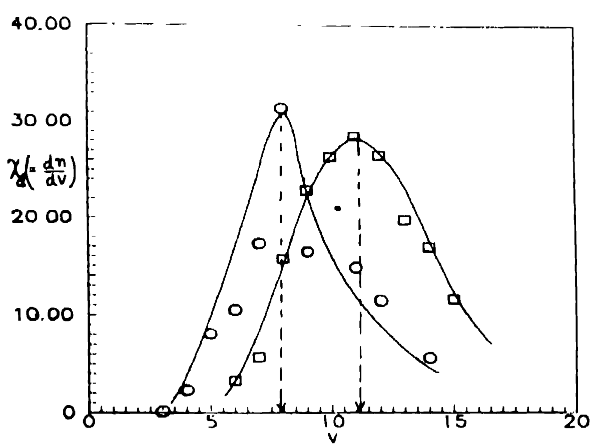

Figure 1. Variation of breakdown susceptibility $\chi_d = dn/dV$ with voltage V. The circles and the squares represent the data for composites with p = 0.4 and 0.3, respectively. The continuous lines are guides to the eye.

for two different concentrations p = 0.4 and 0.3, respectively (with averages over about 500 initial configurations). The maximum of χ_d gives the possible location of breakdown voltage. The peak occurs at the voltage $V_b' \cong 11.0$ for $p = 0.3$ and $V_b' \cong 8.0$ for p = 0.4, very near to those obtained from the direct breakdown measurement in this model ($V_b \cong 14.13$ for p = 0.3 and $V_b \cong 9.65$ for p = 0.4, when conductors percolate through broken dielectric bonds). It may be noted that peak in χ_d increases in height with system size L. Thus, an extrapolated point where χ_d^{-1} vanishes gives the predicted location of the breakdown voltage.

We consider now the self-organised criticality (SOC) of the critical 'slope' BTW model [11], considered to be a generic SOC model for sandpile and earthquake. We consider a lattice size of 100×100. At each lattice point the 'slopes' of 'particles' are randomly added in discrete integer addition and avalanches take place if the 'slope' Z_i at any point i exceeds the value 3 (the cut-off value $Z_0 = 4$ here). In such cases, the $Z_{i+\delta}$ of the nearest neighbours δ of the site i gets one unit of 'slope' each and Z_i becomes zero at i. The dynamics continues, until all the sites have $Z < 4$. The simulation studies give the value of the average critical 'slope' Z_c to be around 2.124 [13] for such a model, beyond which the global avalanches take place.

We have studied the effect of addition of a fixed number of particles (or 'slopes') at any central point for a time unit δt, when the system has got the average 'slope' $\bar{Z}(< Z_c)$ and the dynamics has stopped. Immediately after the particles are added, the local dynamics starts and it continues, for a time period Δt ($\geq \delta t$) We measure the ratio $R_p = \Delta t/\delta t$. Figure 2 shows

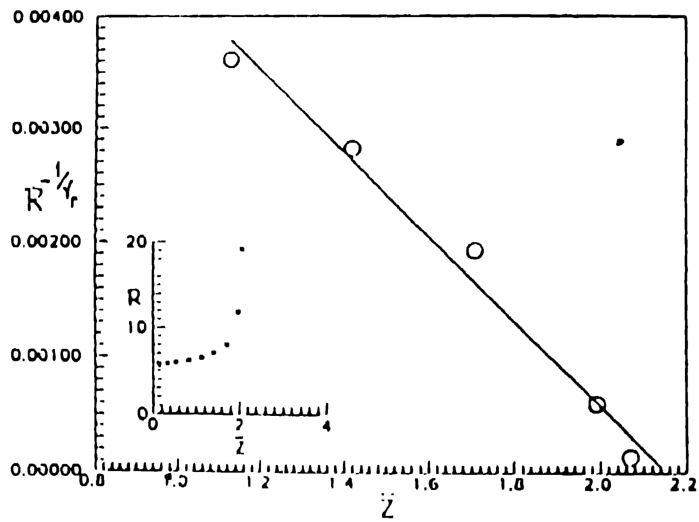

Figure 2. The BTW results R_p^{-1/γ_p} vs $\bar{Z}$. Inset shows the variation of R_p with respect to $\bar{Z}$

the variation of R_p with $\bar{Z}$. We find that $R_p \sim (Z_c - \bar{Z})^{\gamma_p}$, where $\gamma_p \cong 1/3$ (see inset of Figure 2). One can thus clearly locate the self-organised critical point (or critical 'slope') Z_c by plotting R_p^{1/γ_p} with $\bar{Z}$ and by locating its vanishing point, and this gives $Z_c \cong 2.16$. This is very close to the previous straightforward numerical estimate $Z_c \cong 2.124$ [13] in the model

In the Burridge-Knopoff model [7,8] of earthquake, a linear array of N-blocks (earth crust areas) each coupled to its nearest neighbours by elastic springs and each connected to a rigid support at the top by elastic springs, is put on a uniformly moving rough platform (tectonic plate). The rescaled equation of motion of any block can be written as

$$\frac{d^2 u_i}{dt^2} = l^2\left(u_{i+1} - 2u_i + u_{i-1}\right) - u_i - \phi\left[2\alpha v + 2\alpha\left(du_i/dt\right)\right], \qquad (1)$$

where u_j denotes the displacement of j-th block, v the platform velocity with nonlinear friction function $\phi(y) = \text{sign}(y)/(1 + |y|)$. l and α are constants depending on the spring constants. We have already shown [9] that the response to a pulse stress on any block grows with time

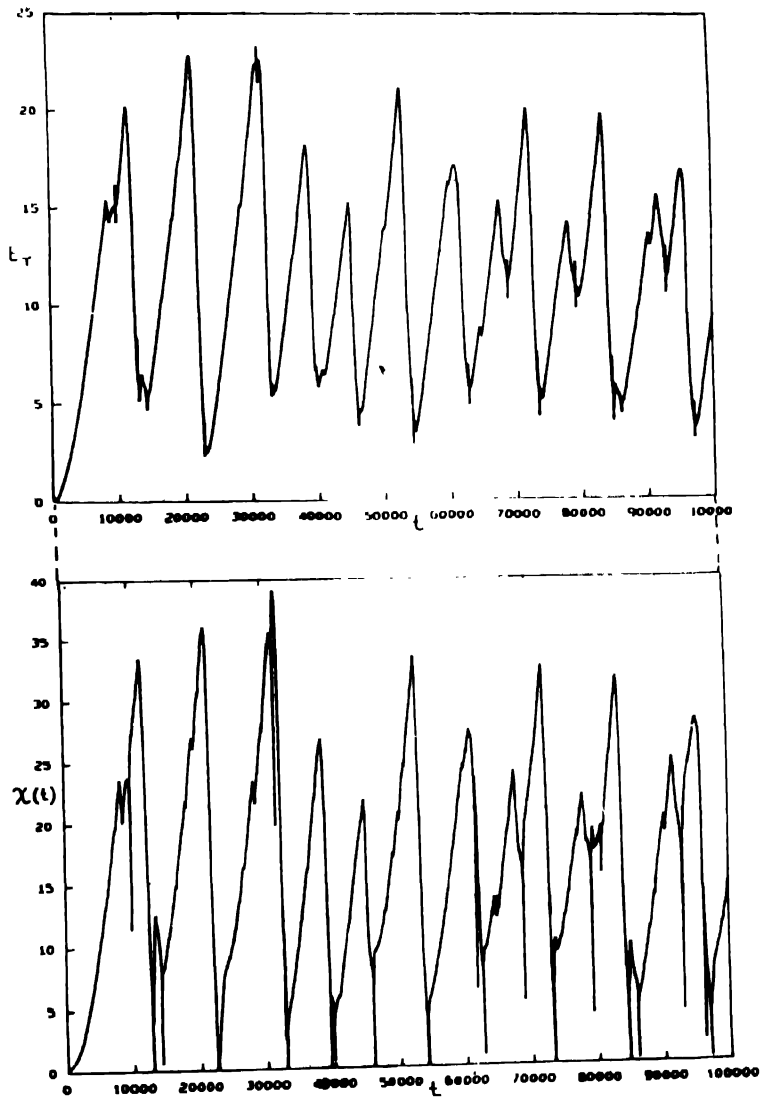

Figure 3. The time variation of total elastic energy E_T of the Burridge-Knopoff system. The corresponding time variation of χ (obtained from stress correlation) is also shown

and the response (pulse susceptibility) χ_p grows with time t as $\chi_p \sim \exp\,[\text{const.}/(t_{c_n} - t)]$ for $t < t_{c_n}$, where t_{c_n} denote the onset time of the n-th earthquake.

We show here that the growth of strain correlation in the same model [7] can be summed up to give a direct susceptibility $\chi(t)$ (with $\chi = \Sigma g(r)$, where $g(r) = \langle \Delta U_0 \Delta U_r \rangle$ with $\Delta U_j \left(= l^2 (u_{j+1} - 2u_j + u_{j-1}) - u_j\right)$ denoting the stress on the j-th block in the model [7,8]). The behaviour of this susceptibility exactly matches with our previously studied results for χ_p [9] in that it also grows exponentially with time and diverges at the 'earthquake' points or times. By studying the susceptibility χ for system sizes in the range $N = 100$ to $N = 1000$, the finite height of the peak of χ is observed to be a finite size effect. The Figure 3 shows the numerical results for the same model (with 100 blocks and the same dynamical parameter values as used in [9]) for this $\chi(t)$ variation with time t obtained from the stress correlations as discussed above and its varation with time has been compared with the time variation of the total elastic energy

$$E_T \left(= \sum_j \left[\left(\frac{l^2}{2} \right) \left\{ \left(u_{j+1} - u_j\right)^2 + \left(u_{j-1} - u_j\right)^2 \right\} + \frac{u_j^2}{2} \right] \right)$$

of the entire system of blocks and springs. This again confirms that the growth of such susceptibilities can indeed give prior information about the point in time of the imminent catastrophe.

Is summary, we find that one can define appropriate susceptibilities for systems having macroscopic breakdown properties. As the breakdown point approaches (for example by increasing the external voltage across random dielectrics, or with the increase of time as in the BTW model of sand-pile or in the Burridge-Knopoff model of earthquake), the appropriate correlations grow and the corresponding susceptibility tends to diverge at the disaster point. By investigating therefore the nature of such susceptibility and by locating the extrapolated point where its inverse vanishes, one can make predictions about the imminent breakdown point.

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