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Characteristics Of *c* Axis And *ab* Axis Tunneling In S-Wave Superconductors.

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Abstract: The peculiarities in tunneling characteristics have been studied in the light of the controversy between s-wave and d-wave character of High T_c superconductivity. We show that anisotropic s-wave gap has the same low voltage power law conductance and two peak structure in the density of states as d-wave superconductors. The assymetric tunneling conductance and zero bias conductance for the c-axis tunneling is shown to occur because of finite band splitting coming from the interlayer hopping parameter. The charecteristics features which have been so far studied in this area have been attributed to d-wave superconductors. Here we find that these features are not unique to the d-wave theory and can also be realised in the context of the s-wave theory as proposed recently by S.Chakraborty et al.

Keywords: *d*-wave superconductivity,tunneling conductance, zero bias conductance

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1 Introduction.

Anomolous single particle tunneling characteristics in NIS and SIS junctions of the high T_c superconductors have remained a subject of great interest. The single particle tunneling conductance in both normal and superconducting states is a measure of the density of states in the normal and superconducting states, and so in principle one gets a lot of informations about the details of the superconducting gap parameter. This is a matter of great current interest, in view of the recent controversy about d-wave or anisotropic s-wave symmetry of the gap function. Whereas some experimental results [1],[2],[3] are in favour of a d-wave symmetry gap function, the recent experiments [4],[5],[6] are in favour of an anisotropic s-wave gap function.

It has been argued that[7], the tunneling characteristics, specifically the quadratic rise of current with voltage at low enough voltages and the two peak structure seen in the conductance voltage characteristics is an evidence for a d-wave superconductor In view of this, we investigated the single particle tunneling characteristics for the anisotropic s-wave superconductors , recently proposed by Chakraborty et. al.[8]

Before we come to the specific problem we address, we highlight the main puzzling features seen in the tunneling spectroscopy of the high T_c superconductors[9].

(1) At low temperature, $V = 0$ tunneling current is zero for tunneling along the ab axis, but nonzero along c axis tunneling. (2) The ab plane conductance becomes smooth at larger temperatures but the c axis conductance goes on incresing with temperature (3) At low temperature and large bias, ab plane tunneling conductance decreases or saturates whereas the c axis conductance goes on increasing roughly linearly with voltage. (4) The ab plane tunneling shows conventional gap like structures but the c axis tunneling shows a much broadened shoulder at the gap edge. (5) Both NIS and SIS junction shows assymetric I-V characteristics with respect to the sign of the bias voltage. (6) Both direction tunneling shows finite density of states for $V < \Delta$ even at the lowest temperatures. For very low voltage current has a quadratic rise. (7) There is large broadening of gap in voltage , and a conductance overshoot for c axis tunneling. (8) For ab axis tunneling, zero bias conductance is zero. So there is no density of states at the Fermi energy, but for very small $V < \Delta$ there is finite current, showing that there is no fully developed gap, or the gap is highly anisotropic with Δ_k being very small in a substantial region of the Brillouin zone. For the c axis tunneling the most common explanation for the ubiquitous zero bias conductance and the characteristic V shaped conductance versus voltage characteristics is explained , as due to either because of tunneling through localized states in the barrier or due to scattering by magnetic impurity inside the junctions. In this paper we emphasize on the distinction between the ab plane and c axis single particle tunneling channels

for both supeconductor to normal and superconductor to superconductor (NIS and SIS) junctions. Specifically we shall consider a layer material like YBCO or Bi-2212 material. We model such superconductors by two planar BCS superconductors coupled by a single particle hopping term along the c axis. We consider also the case, when over and above the single particle hopping, there is a Josephson coupling between the planes. We propose that the observed assymetry of the normal state in plane tunneling conductance, with respect to the sign of the bias voltage in MIC(metal-insulator-cuprate) junction is a consequence of the existence of nonbonding and bonding band with finite splitting between them, in the cuprates. In the split band picture, when the metal is held at positive bias with respect to the cuprate, then there are two channels of elastic tunneling into the nonbonding and the bonding bands. In the reverse bias situation, only one of the bands takes part in tunneling. So the conductance will be assymetric, for MIC and NIS junctions. On the other hand for SIS and CIC junctions the conductance voltage characteristics will be symmetric. Now for a CIC , NIS or for a SIS junction, when both sides of the junctions are cuprates, there is an important difference between tunneling along c and ab axis. In the ab axis tunneling geometry electrons tunnel only from antibonding to antibonding and bonding to bonding bands. Whereas for the c axis tunneling there is another additional channel for conduction, i.e from antibonding to bonding band. This tunneling will be present even in absence of a finite bias voltage either way. The chemical potentials for the two bands differ by $2t_{\perp}$, where $t_{\perp}$ is the c axis hopping amplitude. So the tunneling along c axis will show a zero bias conductance, but the ab axis tunneling will have zero conductance at $V = 0$ and $T = 0$.

We find that the zero bias tunneling conductance observed along the c axis tunneling increases with temperature and do not show any sign of saturation at all. This is our main result.

We shall also discuss, the reason why for $T > T_c$ the ab axis tunneling characteristics becomes smooth, while the c axis tunneling continues to be temperature dependent and rises with temperature. Lastly we predict that for MIC geometry tunneling (below T_c) the assymetry (or alternatively the background conductance) will be more for lesser value of the gap in the superconductor.

2 Tunneling Charecteristics in s-wave superconductors

To start we take the effective hamiltonian proposed by Chakraborty et. al.[8]

$$\sum_k(\epsilon_k^1 - \mu)c_{k\sigma}^{1\dagger}c_{k\sigma}^1 \quad + \quad (1 \to 2) \quad + \quad V_{bcs}\sum_{kk'} c_{k\uparrow}^{1\dagger}c_{-k\downarrow}^{1\dagger}c_{-k'\downarrow}^1 c_{k'\uparrow}^1 \quad + \quad (1 \to 2)$$
$$+ \quad \sum_k \frac{t_\perp^2(k)}{t} c_{k\uparrow}^{1\dagger}c_{-k\downarrow}^{1\dagger}c_{-k\downarrow}^2 c_{k\uparrow}^2 . \qquad (1)$$

In this model, there is no hopping term along the c axis from plane to plane, even though the band theory estimates for the c axis hopping amplitude $t_\perp$ is about $\frac{1}{3}$ to $\frac{1}{5}$ of the inplane hopping parameter. The reason is supposed to be that, due to strong correlation in the plane itself, the single particle band motion is absent in the c direction. The conduction along c axis is purely due to incoherent processes. On the other hand coherent propagation of "singlet objects"(pairs of elctrons)is possible That is the origin of the last term (Josephson coupling of a very unusual kind) It should be emphasized that, it has not been proved within a realistic model for hig T_c superconductors.

We prefer to keep the band term in the hamiltonian. The origin of subbands can be understood as follows. We consider a two layer material like YBCO. The individual layers can be modelled by a 2-d tight binding band with dispersion,

$$\epsilon_k \quad = \quad -2t(\cos(kx) + \cos(ky)) \quad + \quad 4t'\cos(kx)\cos(ky)$$

where t and t' are nearest and next to nearest neighbour hopping in the planes of some effective site. We take, $t = 0.3$eV and $4t_\perp = 0.45$eV. For two closely spaced planes, in interlayer matrix element $t_\perp(k) \; = \; t_\perp(\cos(kx) - \cos(ky))^2$ results in formation of subbands, $E_{\phi,\psi}(k) \; = \; \epsilon(k) \pm t_\perp(k)$ where the ϕ and ψ are antibonding and bonding band fermions defined as, $\phi(k), \psi(k) \; = \; \frac{(c_k^1 \pm c_k^2)}{2}$ The location of chemical potentials will be detrmined by the doping.

To illustrate the difference between tunneling along ab axis and along c axis we write down the tunneling hamiltonian without any explicit dependence of the tunneling amplitude on momenta or energy. For tunneling along ab axis, the hamiltonoan will be

$$\sum_{kp} T_{kp}c_{p\sigma}^{\alpha\dagger}c_{k\sigma}^\alpha \equiv \sum_{kp} T_{kp}(\phi_{k\sigma}^\dagger\phi_{k\sigma} \quad + \quad \phi \to \psi)$$

where α denotes layer index (1 and 2). The c axis tunneling hamiltonian on the other hand will be

$$\sum_{kp} T_{kp}c_{p\sigma}^{1\dagger}c_{k\sigma}^2 \; + \; 1 \to 2 \equiv \sum_{kp} T_{kp}(\phi_{k\sigma}^\dagger\phi_{k\sigma} \; + \; \phi \to \psi \; + \; \sum_{kp} T_{kp}(\phi_{k\sigma}^\dagger\psi_{k\sigma} \; + \; h.c)$$

It is clear, that for c axis tunneling there is an extra channel for conduction , i.e from the nonbonding to bonding band which is absent for the ab axis tunneling. At this point, we compare our model with that of Levin and Quader[10], who also consider a split band picture. We insist that there is a major difference between our viewpoints as regards the role of the split bands. Levin et. al.[10] assume that the bonding band(ψ band) is almost submerged below the Fermi surface. For the underdoped case, the ψ hole band is completely filled and frozen much below the Fermi surface, and do not take part in tunneling to the metal on the other side of the junction. Consequently there will not be any conductivity assymetry for underdoped case. For larger doping case, both the bands will be partially filled and take part in tunneling. Moreover one needs additional assumptions, that the ψ band is actually a band of nondegenerate band of fermions since their number is so small. One needs to have, in an adhoc fashion, different dispersion for ϕ and ψ fermions(linear and quadratic in momenta) to reproduce some normal state properties. This picture is approximately right when $t_\perp$ is large, giving rise to large band splitting. We assume , on the other hand that the band splitting is small (small $t_\perp$). So, even at small doping concetrations , both bands will be partially filled.

Within the interlayer tunneling mechanism of superconductivity, even though the intralayer BCS coupling gives a small $T_c \equiv 5K$ on its own, a very small $t_\perp$ is enough to raise the T_c to large values $\equiv 90K$ through the Josephson coupling term. For CIC junctions when both the electrodes are high T_c materails (break junctions), a look at the tunneling hamiltonians for the ab and c axis tunneling shows that, for ab tunneling, the elctrons tunnel from ϕ to ϕ and from ψ to ψ bands only. For the c axis tunneling, cross tunneling also takes place. If $T_{\phi\phi}$ $T_{\psi\psi}$ and $T_{\phi\psi}$ are the tuneling matrix elements between the respective subbands of both electrodes, we get

$$G_{ab}(V) = |T_{\phi\phi}|^2 N_\phi^2[(1+\alpha_1^2) - \alpha_1^2\, f(\frac{-2t_\perp - V}{k_B T})]$$

and

$$G_c(V) = |T_{\phi\phi}|^2 N_\phi^2[(1+\alpha_1^2+2\alpha_2) - \alpha_2\, f(\frac{-2t_\perp + V}{k_B T}) - (\alpha_2+\alpha_1^2)\, f(\frac{-2t_\perp + V}{k_B T})]$$

where $\alpha_1^2 = N_\psi^2 T_{\psi\psi}^2 / N_\phi^2 T_{\phi\phi}^2$ and $\alpha_2 = N_\psi T_{\phi\psi}^2 / N_\psi T_{\psi\psi}^2$. The main features of this expression are: (1) The conductance voltage characteristics is symmetric with respect to bias for both ab and c axis tunneling. (2) For c axis tunneling, there is a zero bias current coming from cross tunneling, which is operative even at zero bias because of finite band splitting. For ab axis tunneling there is no zero bias current. (3) There is a zero bias conductance for both ab and c axis tunneling At $T = 0$ and $V = 0$,

$G(V) \equiv T^2_{\phi\phi} N^2_{\phi}(1 + \alpha^2_1)$. The mean field hamiltonian in the superconducting phase is,

$$\sum_k (\epsilon_k + t_\perp)\phi^\dagger_{k\sigma}\phi_{k\sigma} + \sum_k (\epsilon_k - t_\perp)\psi^\dagger_{k\sigma}\psi_{k\sigma} + (V + \frac{t_\perp^2}{t})\sum_k [(\Delta^*\phi_{-k\downarrow}\phi_{k\uparrow} + \Delta\phi^\dagger_{k\uparrow}\phi^\dagger_{-k\downarrow}) + \phi \rightarrow \psi] \quad (2)$$

The hamiltonian looks like a sum of two BCS reduced hamiltonins for the bonding and antibonding electron systems. The generalised gap equation will be

$$\frac{1}{(V + \frac{t_\perp^2}{t})} = \frac{1}{2}\sum_k \frac{tanh(\beta E^\phi_k/2)}{2E^\phi_k} + \frac{1}{2}\sum_k \frac{tanh(\beta E^\psi_k/2)}{2E^\psi_k} \quad (3)$$

where,

$$E^{\phi,\psi}_k = \sqrt{(\epsilon_k \pm t_\perp)^2 + \Delta^2}$$

We have solved the gap equation numerically for different temperatures. In NIS junctions, the tunneling current is given by,

$$I_{NIS} = \equiv \sum_{kp} |T|^2 [u^2_k \delta(eV + E_k - \xi_p)[f(E_k) - f(\xi_p)] + v^2_k \delta(eV - E_k - \xi_p)[1 - f(E_k) - f(\xi_p)]] \quad (4)$$

For the SIS junction the corresponding expression is,

$$I_{SIS} = \sum_{kp} |T|^2 [(1 - f(E_k) - f(E_p))(v^2_k u^2_p \delta(eV - E_p - E_k) - u^2_k v^2_p \delta(eV + E_p + E_k)) + (f(E_k) - f(E_p))(u^2_k u^2_p \delta(eV + E_k - E_p) - v^2_k v^2_p \delta(eV + E_p - E_k))] \quad (5)$$

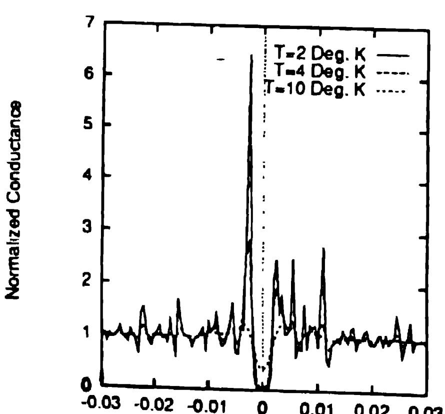

Fig.1 : Parellel (a-b axis) Conductance Charectenstics

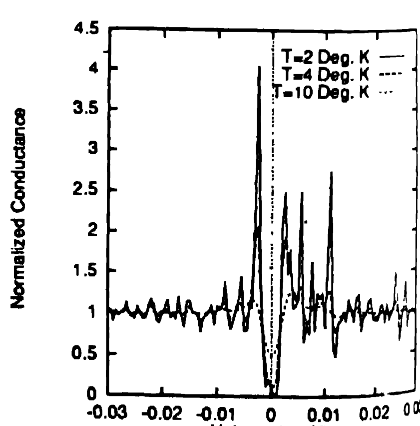

Fig.2 : Perpendicular (c axis) Conductance Charect

The normalised conuctance versus voltage for the *ab* and *c* axis tunneling are plotted in Fig.1 and Fig.2 respectively. The parameter values used in all the calculations are: V_{bcs} = 0.125 eV, $t_{\perp}$ = 0.1 eV, ω_d = 0.02 eV, chemical potential (μ) = 0.45 eV. This gives a $T_c \approx 84^o$ Kelvin.

The notable features are:

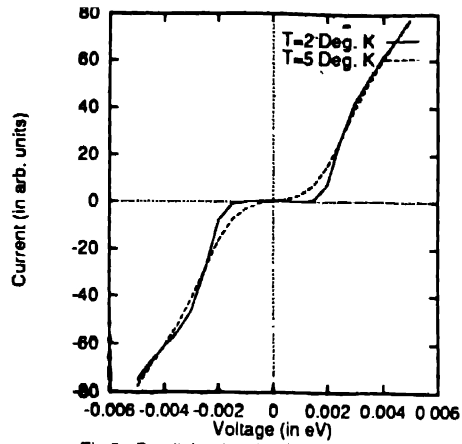

Fig.3 : Parellel (a-b axis) Current Charectenstics

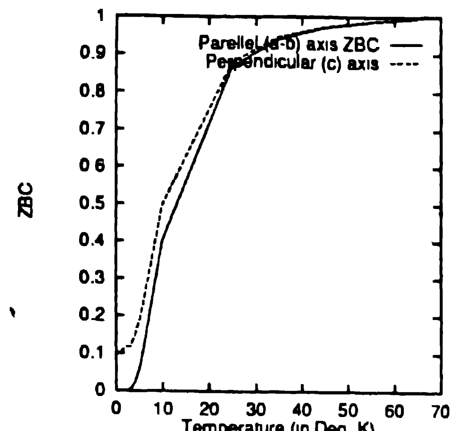

Fig.4 : Zero Bias Conductance (ZBC) Charecteristics

(1) At $T = 0$ there is a sharp voltage threshold for conductivity for the *ab* axis tunneling, whereas there is a finite zero bias conductance for the *c* axis tunneling. (2) The sharp voltage threshold for *ab* axis tunneling gets washed out at a very small temperature ($4K$). (3) The plots for conductance at $T = 4, 10$ clearly shows the characteristic two peak structures seen in experiments. For d-wave superconductors also one gets similar two peak structures.

In Fig.3 we plotted the current versus temperature for *ab* axis tunneling for 2 and 5 degree Kelvin. We emphasize that, even at very low temperatures (5 K) the current rises quadratically with voltage at very low voltages. Fig.4 shows the temperature dependence of the normalised zero bias conductance for both *ab* and *c* axis tunneling.

3 Conclusion

One extraordinary feature of the interlayer tunneling gap function is that the gap along $\Gamma - M$ direction is large and almost temperture independent upto about 90% of the T_c. On the other hand gap in any other direction falls very fast compared to the usual BCS gap. This agrees with the recent photoemission experiment. This is true when $T_J > V_{bcs}$. For weaker T_J or with larger in plane V_{bcs}, the averaged gap falls faster with temperatures and slowly approach the usual BCS temperature dependence. All these pecularities are only because of the *one* · *k* summation in the interlayer josephson coupling term, as emphasized by Anderson. Two things follow automatically from above discussion. One is that, in the interlayer mechanism, the gap magnitude in most part of the BZ is very low (1-3 meV) and also very fragile as far as thermal fluctuation is concerned. The gap in these regions falls faster than in the usual BCS gap. This would mean that we shall not get any sharp gap features at all at any finite temperatures in tunneling experiments. This is what is observed in our numerical calculations at finite temperatures. In Fig. 3. we show the $I-V$ characteristics at $T=2$ and $T=5$ degrees for tunneling along the *ab* plane. We see clearly that already at $T=5$ degrees there is finite current at very low voltages. In other words, indeed it will be very difficult to distinguish between, the situation where the gap function has gap nodes on the Fermi surface like in d-wave superconductors, and the interlayer case. For tunneling along, the *c* axis there will be a finite current for arbitrarily small voltages. So in ceramic materials, where we measure some average current along both directions, we shall always get an I-V characteristics looking just like a superconductors with gap nodes on the Fermi surface even at $T=0$. For single crystal measurements , and for *ab* plane tunneling there will be a sharp voltage threshold, but no sharp threshold for small but finite temperatures. It is worth emphasizing that, we do not really know how impurities and inhomogeneities suppress the interlayer tunneling gap.

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