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Electronic Transport: Ballistic to Diffusive to Localized

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Abstract: Metal-Insulator transition in disordered (non-interacting) systems is briefly reviewed. With special emphasis to low-dimensions, the transitions in other non-random (quasi-periodic and inhomogeneous) systems are also discussed. Inhomogeneous systems tend to show sub- or super-(exponential) localization. Relevance to quantum chaos is reviewed.

Keywords: Quantum interference, backscattering, non-ergodic, mesoscopic, exponential localization, (sub/ super) localization, power-law localization, mobility edge, quantum chaos.

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1.1 Introduction

In this short review, we will be concerned only with non-interacting electrons and the effects on the fermionic ('spinless') transport properties of the broken translational symmetry of the lattice either due to disorder (randomness) or incommensuration or inhomogeneity of some property. Because of the limited space, I will introduce the essential concepts without much details. The bibliography will also be representative rather than exhaustive. Since quite a few excellent reviews[1,2] already exists for higher dimensions, I will confine myself mainly to one-dimensional systems, while making occasional excursions into two or three dimensions. This confinement to lower dimensions is not just a purely academic exercise any more because of the tremendous advancement in technology (see, e.g., ref. 3) in making

very low (even almost zero-dimensional) dimensional systems. It must also be remembered that many interesting physics still remains to be explored even in 1D as the sequel will hopefully demonstrate. The choice of topics even in this limited sense was partly dictated by the type of papers contributed for the Workshop, and partly tinted by my own prejudice (as happens with almost any author) regarding which topics are of current interest.

2.1 Periodic Systems

In this case the eigenstates are Bloch states, and transport is ***ballistic*** since the mean free path (mfp) for elastic scattering is infinitely large in this case. Since there is no scattering, (four-probe) resistance is zero and the transport is ***superdiffusive*** in the sense that the mean squared displacement (MSD) ~ t^2, where t is the elapsed time. This situation is routinely realized nowadays in ***nanostructures***[3] which are ultra-clean, nano-sized (~ 10^{-9} m) systems where mfp is of the order of 50 μm, which is about four times that in a good metal. The interesting thing that happens in these systems is that as a function of the gate-voltage, the number of channels in the (electronic) flow path increases and the (two-probe) conductance in units of its quantum (e^2/h) jumps in units of two (for electrons of two spin varieties). By confining the gate voltage (or boundaries, etc.), one may further confine these systems to less than one dimensions, i.e., essentially to *Quantum Dots* (zero-dimensional systems) There is particular interest in T-shaped structures for possible use in future transistors.

2.2 Impure Systems: Random Disorder

In this case, electrons scatter from the impurities every so often. The scatterings are still elastic (since we donot consider any phonons etc. at T = 0), and hence the elastic mfp (ℓ_e) < ∞ with the result that the resistance > 0. The question is: Can the electron still diffuse (note that *diffusion*, i.e., MSD ~ t, here implies a metal)? Boltzmann transport theory (which considers electrons as classical particles) says, "Yes". But Anderson[4] for the first time had said, "Not necessarily", and introduced the idea of a ***critical disorder*** above which the electrons cannot diffuse anymore through the system (MSD < ∞ for all t however large, i.e.,

the system becomes *localized* (an insulator). The underlying reason is now known to be **the quantum interference of electron waves** and the process of ***coherent backscattering***.

To understand how quantum transport is more restrictive than its classical counterpart, we consider two paths 1 and 2 for the electron wave from point P to Q, as shown in Fig.1(a). If the amplitudes and phases are A_1 and ϕ_1 for path 1 and A_2 and ϕ_2 for path 2 respectively when they arrive at Q, then the combined probability of finding the electron at Q is

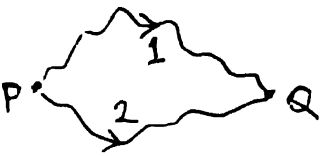

Fig.1(a)

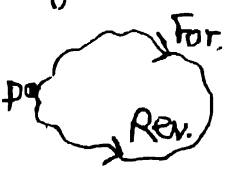

Fig.1(b)

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2\,|A_1|.|A_2| \cos(\phi_1 - \phi_2),$$

where the last term is the interference term. Now if one looks at the probability of the electron coming back to the same point P as in Fig. 1(b), then for the same loop there could be two paths one along the so-called forward path and another along its exact "time-reversed" path. They interfere constructively and the return probability $p = 4\,|A|^2$ instead of $2\,|A|^2$ as obtained classically (Boltzmann) where one sums over probabilities straight away. Now at T = 0, the inelastic mean free path, ℓ_ϕ (i.e., length upto which the electron does not does not scatter off phonons etc. and lose phase coherence) is infinite. Thus the backscattering is due to the sum of forward and its time-reversed Feynman path (as considered above) for all possible sized loops (upto infinity). In reality/ experiment, there is always a finite temperature (i.e., phonons with finite a ℓ_ϕ) and the loops with sizes greater than ℓ_ϕ cannot contribute. Thus phonons normally tend to reduce the localization effects, i.e., increase the conductance in a disordered system at a low temperature! In any case, it is clear that *localization implies long-range phase coherence*. In one dimension, (almost) all states are exponentially localized[5] and this has been rigorously proved[6]. The 'average' resistance may be shown to increase[7] with system-size as $\langle R(L)\rangle = 1/2\,[\exp(2\gamma L) - 1]$, where $\gamma^{-1} = \xi$ is the *'localization length'*. Clearly this is non-Ohmic behaviour since resistance *does not* increase linearly with length. Indeed, it was shown[7] that the resistance is not a self-averaging quantity but the logarithm of the resistance is. Further, this

demonstrates *exponential* localization in 1D. We know that in a classically disordered (percolative) system, metal-insulator transition is possible in 2D at a percolation threshold, i.e., at a certain volume fraction of conductors in an insulating host. But consistent with the fact that quantum interference makes transport more difficult, the scaling theory of localization[8] predicts that all states are localized even in 2D. It is only above 2D that there is a finite non-trivial value of a *critical disorder* (W_c). For a disorder, $0 < W < W_c$, there is a *mobility edge* E_c separating the spectrum into metallic and insulating regions. For $W > W_c$, a metallic state is not possible at all (all states in the spectrum are localized). Typically one works with site-diagonal disorder, i e , disorder in the site-energy or potential. The 'folklore' is that there are no qualitative changes in the above statements in the presence of off-diagonal (hopping term; related to the overlap between nearby site states) disorder.

2.2.1. Topological Disorder: In this case, the disorder is due to the absence of even an underlying lattice, i.e., one has an amorphous or fluid system. There is no rigorous theorem in this case even in 1D, but it seems that 'all' states are localized in 1D. It may be noted that since distance between nearby sites varies here, sometimes this may be treated as a lattice case with random hopping terms. There is a qualitatively different result for this model with fixed site energy but random nearest-neighbour hopping. The state at the band-centre (E = 0) is *'sublocalized'* [9] (weaker than exponential but stronger than power-law localization) in the sense that it asymptotically decays as $\exp(-\gamma L^{1/2})$ for d = 1 and 2.

2.2.2. Transport in a random fractal medium: Applying the ideas of random walks on fractals, and doing functional integration on random Feynman paths it was theoretically found as well as experimentally observed recently (in a carbon-black-polymer composite) that electrons are *'superlocalized'* [10] (i.e., stronger than exponential localization) in the sense that the wave function asymptotically decays as $\exp(-\gamma L^{\zeta})$, where $\zeta > 1$.

2.2.3. Conductance Fluctuations and its Universality: Fluctuations (standard deviation; sd) in macroscopic systems usually decay as square-root of the system-size (volume) since observing different parts (for a static property) of a large system is equivalent to observing many realizations of a small-sized system, i.e., since *ergodicity* property holds. A disordered system at low temperature is however strongly non-ergodic

mainly because of the breakdown (an outcome of quantum interference) of the Central limit theorem for the distribution of conductances (g). Fluctuations show anomalously large values More interestingly, however, sd g assumes universal values in the mesoscopic regime ($\ell_e < L < \ell_\phi$ and/ or ξ) independent of the Fermi energy, strength of disorder, size of the system etc. and dependent only on the dimensionality. It was found[11] that for the two-probe conductance (in units of $2e^2/h$, for electrons of two spin-variety): sd g = 0.544 in 3D, sd g = 0.431 in 2D and sd g = 0.365 in quasi-1D. Because ξ is completely dependent on ℓ_e ($\xi \sim 4\ell_e$) in 1D, it was believed for a long time that this universality cannot exist in exact 1D until recently when we discovered[12] that there is still a quasi-diffusive regime in 1D where the probability distribution of g is unusually broad and the universal sd g = 0.3

2.3 Moving Away from Random Disorder

2.3.1 Randomness with short-range correlation: random n-mers: Here I discuss the effect of short-range correlation in the randomness (disorder). One of the simplest such models is a Random Dimer Model (RDM), which seems to have some relevance in the study of highly enhanced conductivity (by the order of about 11) of polyaniline upon increasing levels of protonation[13]. In the RDM, there are two species of atoms, say, A and B (with site-energies ε_A and ε_B respectively) and they are thrown randomly only as a pair AA or BB on nearest neighbor lattice sites (1D). It was claimed by Dunlap, Wu and Phillips[13] that there was ***absence of localization*** in this 1D disordered chain. Detailed study on this model by us[14] showed that qualitatively nothing unusual happens here and that almost all states are exponentially localized Note that in a purely (uncorrelated) random system, there is a set of discrete, ***configuration-dependent*** (and hence generally unpredictable) ***stochastic resonance (e.g., Azbel resonances[15])*** energies, whose total measure, or contribution to the integrated density of states, in the infinite volume limit is zero because their widths decay exponentially with length. In contrast, a RDM has correlated (shortest range) disorder and two interesting things happen. (i) Apart from the stochastic resonances, there are two ***configuration-independent***, deterministic energies $E = \varepsilon_A$ and ε_B where very broad resonances (extended states) occur; and (ii) The peak structure around each of these very broad resonances at ε_A or ε_B is found to be quite complicated with external fragmentation

regions where the resonances are Azbel-like and an oscillating (conductance) core whose width in energy decays algebraically $N^{-2/3}$, but the number of extended or nearly extended states within which increases as $N^{1/3}$. Indeed, these increasing number of algebraically decaying (without any particular length scale) resonances are at the heart of the 'apparently superdiffusive' behaviour (Dunlap, Wu and Phillips[13]) of an electron released at some point in the chain at time t = 0 But our scaling analysis[14] indicates that localization effects finally win and that the localization length (implying exponential localization for very large systems studied by us) diverges as $\xi \sim (E - \varepsilon_S)^{-2}$, where S = A or B. That there are only these two extended states may be seen by using the following general idea[16]. Take the transfer matrix[12] product of one of the dimers of the RDM, and find for which energy does it become $\pm$ I (identity matrix). If such an energy exists for one type, say A, then the chain behaves like a perfect chain of B atoms at that energy (and hence gives rise to an extended state) and vice versa. For the RDM, one gets only two such extended states $E = \varepsilon_A$ or ε_B

Similarly in the case of a random trimer model (RTM) made out of random AAA or BBB clusters on three neighbouring sites in a chain, the condition of unity ($\pm$ I) imposed on the product of three random matrices of A or B atoms gives rise to exactly four extended states ($E = \varepsilon_A \pm V$ or $\varepsilon_B \pm V$) which behave like very broad resonances with algebraically decaying widths for finite size chains For the general case of a random n-mer made out of A or B atoms, where n is of the form $n = 2^p$, one gets exactly (n - 1) extended states for each species It turns out that there are (2n - 2) real roots[16]:

$$E = \varepsilon_S \pm V\sqrt{2 \pm \sqrt{2 \pm \sqrt{2 \pm} \ldots \text{m times}}},$$

where V is the hopping energy between nearest neighbours only (irrespective of AA, BB or AB), m is the set of all integers from 0 to (p - 1), and S = A or B. It may be noted that in a random n-mer, the range of correlation (in the *biased* binomial probability distribution) has increased to n-lattice sites and the effect shows up by increasing the number of extended states to 2n - 2 (inside the spectrum). One further notices that if n tends to infinity, not only the number of extended states become infinite, but also the spectrum of these extended states range from (as found using the above formula) $\varepsilon_S - 2V$ to $\varepsilon_S + 2V$, which also happens to be the extent of the band states in this case. Even though it has not been shown

whether these infinite number of extended (resonance) states are dense (in a mathematical sense) or not, the method at least indicates qualitatively the right type of *crossover* from exponentially localized states, for purely random distribution, to extended (band) states, for a random distribution with infinite range of correlation.

2.3.2. Strength/ Width of disorder (randomness) decreasing towards zero as the system size increases: Let us consider a 1D lattice with site energies chosen from a uniformly distributed disorder whose width decay algebraically as: $W(L) = \lambda L^{-\alpha}$. For $\alpha = 0$, one knows that almost all states are *purely* exponentially localized. Intuitively, for $\alpha > 0$, one expects the effects of disorder to become smaller (e.g., localization length becomes larger) as the length L increases. Indeed, localization other than pure exponential [e.g., sub-localization with resistance $R(L) \sim \exp(\gamma L^{\mu(\alpha)})$; $0 < \mu(\alpha) < 1$] may appear here. The following is a summary of results (See Sec. 2.7 of Ref. 2 and references therein):

(i) For $\alpha > 1/2$, the spectrum is absolutely continuous, and has extended states.

(ii) For $0 < \alpha < 1/2$, the spectrum is pure-point, and one has $\mu(\alpha) = (1 - 2\alpha)$ and thus is clearly stretched exponential localization.

(iii) For $\alpha = 1/2$, for large enough disorder strength ($\lambda >$ some λ_1), one has power-law localization with $R(L) \sim L^{\nu}$, where $\nu = \nu(\lambda,E)$.

2.4 No Impurities (i.e., No Disorder): but Loss of Crystalline Symmetry

These are typically incommensurate (or, quasi-periodic) systems, or systems which have inhomogeneity in some characteristics at different lengths. Indeed, we did already consider inhomogeneous systems above (Sec.2.3.2), but the system was disordered in that case.

2.4.1. Fibonacci Sequences: Let the sites on a 1D lattice be generated from the left end using two species of atoms A and B, and a pair of rules, e.g., (i) an A is replaced by B, and (ii) a B is replaced by AB. Solution of Schrodinger equation on such quasi-periodic Fibonacci sequences was first attempted upon by two groups[17] It was found that the spectrum is singularly continuous, i.e., all the states are critical. In the case of a tight binding hamiltonian, one may apply the rules individually to the site-energies or the hopping terms.

But, if the hopping energies are correlated to the nearest neighbour site energies[18], or if the site-energies tend to cluster[16], extended states may appear.

2.4.2. Incommensurate Systems (Harper model) The quasi-periodicity in this 1D model appears in the site-energy $\varepsilon_n = \lambda \cos(2\pi nQ + \varphi_0)$, where Q is an irrational number The nearest neighbour hopping energy V is held constant. Here λ plays the role of 'disorder'. If Q is not too closely approximated by a rational (e.g., Liouville numbers) there is a metal-insulator transition in this model at $\lambda = 2V$. Aubry and Andre[19] derived a duality property and showed that all states are extended for $\lambda < 2V$, exponentially localized for $\lambda > 2V$, and critical for $\lambda_c = 2V$ A multifractal analysis[20] showed that for λ close to λ_c , the extended states start with a sub-diffusive (critical-like) behaviour for finite-size chains and has a finite resistance which could be quite large, but as the length increases towards infinity the subdiffusive behaviour slowly crosses over to an extended behaviour but with a large resistance This is consistent with the behaviour of many experimental quasiperiodic metals

2.4.3 Systems with periods varying (increasing) with distance. If the site energies are given by $\varepsilon_n = \lambda \cos(2\pi n^{\nu}Q + \varphi_0)$, where $0 < \nu < 1$ and Q is any real number, then the period keeps increasing and the potential becomes locally flat at very large lengths. The case $\nu > 1$ where the period keeps decreasing is not very interesting since intuitively it is clear that all states become localized For $\nu < 1$, it was shown by Das Sarma *et al* [21] that there is a metal-insulator transition with mobility edges at $\pm E_c$ ($E_c = |2t - \lambda|$) separating extended states for $|E| < E_c$ and exponentially localized states for $|E| > E_c$. Since this is an inhomogeneous system, we[22] undertook a transfer matrix study of the conductance alongwith finite-size scaling analysis (numerical) and our study indicates presence of sublocalized states in the vicinity of $|2t - \lambda|$ More work needs to be done in this area.

3.1 Random Matrices, Level Spacing Distribution and Quantum Chaos

Wigner and Dyson[23] had originally applied the theory of random matrices in the study of the statistics of level spectra (considering them as random) of complex nuclei. Because of the universality of the Wigner-Dyson (W-D) statistics, it is relevant for various quantum systems In the case of a quantum particle in a random potential, the energy levels are

random and so are the energy-level spacings (s) between two consecutive levels. Obviously for the band states, the energy levels are perfectly correlated in the entire spectrum. In the other extreme of localized states (insulating regime), they are totally uncorrelated and one expects the probability p(s) to be Poissonian. In the case of weakly disordered metallic regime, the effective hamiltonian for the ensemble of energy levels has a Coulomb gas form (plus some attractive interaction) and thus there is a finite range of correlation (level repulsion) in the energy domain and $p(s) = s^{\beta}\exp[-\alpha(\beta)s^2]$, the W-D statistics, where $\beta = 1, 2$ or 4 for orthogonal, unitary or symplectic symmetries respectively. It may be mentioned here that while chaos in classical systems is reasonably well-understood, it is not so for quantum systems[25] People believe that quantum chaotic systems are those which are classically chaotic, if such a classical limit exists. As one would intuitively expect, quantal effects tend to suppress classical chaos (because of the uncertainty relation in phase space) They suppress classical chaos and diffusion by a mechanism similar to Anderson localization. The W-D statistics plays a central role in the classification of quantum chaotic systems. Currently it serves as one definition of quantum chaos. But very recently strong deviation from both the Poisson statistics in the insulating regime and the W-D statistics in the metallic regime has been found[26] close to the mobility edge for $d > 2$. Similarly, for the Harper model (see above) at criticality, level attraction[25] instead of repulsion was observed for ***small values*** of s, i.e., $p(s) \sim s^{\beta}$, where $-2 < \beta < -1$. This field is developing very fast.

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