Indian J. Phys. **69A** (1), 127 – 132 (1995)

IJP A
— an international journal

AC and pulse susceptibility of Ising ferromagnet: A mean field study

Muktish Acharyya and Bikas K. Chakrabarti
Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Calcutta-700 064, India

Abstract: We have studied the AC susceptibility and dynamic transition in Ising ferromagnet in a periodic field by solving mean field dynamical equation. The imaginary (real) part of AC susceptibility shows a sharp peak (dip) at the temperature at which the dynamic transition occurs. We have also studied the behaviour of Ising system under a pulse magnetic field. The width ratio and pulse susceptibility shows peak near the Ferro-Para transition point.

Keywords: Susceptibility, Ising model

PACS Nos: 05.70.Ln,05.50.+q

I. Introduction:

We have studied, by solving numerically the mean field (MF) equation of motion, the nature of response magnetisation $m(t)$ of an Ising system in presence of a periodically varying external field ($h(t) = h_0 cos(\omega t)$). From these studies, we determine the $m-h$ loop or hysteresis loop area $A(= \oint mdh)$ and the dynamic order parameter $Q(= \oint mdt)$ and investigate their variations with the frequency (ω) and amplitude (h_0) of the applied external magnetic field and the temperature (T) of the system [1]. The dynamic phase boundary (in the $h_0 - T$ plane) is found to be frequency dependent and the transition (from $Q \neq 0$ for low T and h_0 to $Q = 0$ for high T and h_0) accross the boundary crosses over to a continuous from a discontinuous one at a tricritical point [2,1]. These boundaries are determined in various cases. We find that the response can be generally expressed as $m(t) = P(\omega(t - \tau_{eff}))$ where P denotes a periodic function (with amplitude m_0) having same frequency ω of the perturbing field and $\tau_{eff}(h_0, \omega, T)$ denotes the effective delay. We established that this effective delay τ_{eff} of the response is the crucial term and it practically determines all the above observations for A, Q, etc. Investigating the nature of the in-phase (χ') and the out-of-phase (χ") susceptibility, defined as $\chi' = (m_0/h_0)cos(\phi)$ and $\chi" = (m_0/h_0)sin(\phi)$; $\phi = \omega\tau_{eff}$ (and m_0 is the amplitude of $m(t)$), we find that the loop area A is directly given by χ" and also the temperature variation of χ" (χ'), at fixed ω and h_0, gives a prominent peak (dip) at the dynamic transition point [3].

We have also studied the behaviour of response magnetisation by the application of a short-duration (compared with the relaxation time) pulsed magnetic field. Here, we observed that the width ratio (of the half width and the width of the response magnetisation and of the pulsed field respectively) and the susceptibility (the ratio of excess magnetisation over its equilibrium value and the height of the pulsed field) both shows sharp peaks at the order-disorder (ferro-para) transition point.

II. Mean Field Dynamics:

The equation for the dynamics of the magnetisation (m) of a magnet in presence of a sinusoidally time varying magnetic field ($h(t) = h_0cos(\omega t)$), may be written in the mean field approximation as

$$\tau\frac{dm}{dt} = -m + tanh[\frac{m + h_0cos(\omega t)}{T}]. \qquad (1)$$

Here m represents the magnetisation h_0 and ω represent the amplitude and the frequency respectively of the sinusoidally varying magnetic field and T is the temperature of the system (the Boltzmann constant and the spin-spin interaction strength are taken to be unity). We have solved the equation by using fourth order Runge-Kutta method (in single precision; taking the time differential equal to 10^{-6}) to get $m(t)$ from equation (2). Plotting $m(t)$ as a function of $h(t)$ we got the $m-h$ loop.

III. AC Susceptibility:

Let us define, the (linear) AC susceptibility as

$$\chi = (m_0/h_0)e^{-i\phi}, \quad \phi = \omega\tau_{eff},$$

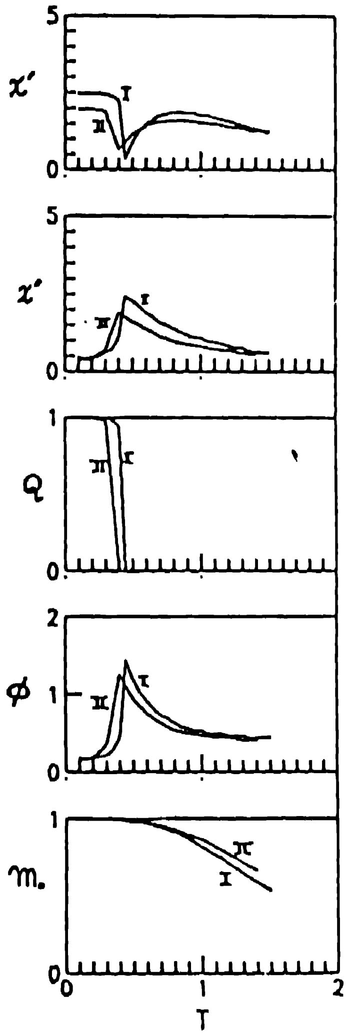

Fig.1. Temperature variations of χ', χ'', Q, ϕ and m_0 for two different field amplitudes (h_0). (I) $h_0 = 0.4$, $\omega = 0.031$, (II) $h_0 = 0.5$, $\omega = 0.031$.

transition. We have identified here the high temperature smeared peak in χ' with the high temperature decay of magnetisation amplitude (m_0) and the second (low temperature) prominent peak in χ" (and the dip in χ') with the dynamic transition (Q changing from zero to nonzero value).

Our study on the temperature variation of the complex susceptibility of an Ising model in a periodically varying external field shows that a low temperature prominent peak in χ" (and dip in χ') occurs (at T_d) as one crosses the dynamic phase boundary ($Q \neq 0$ for $T < T_d(h_0, \omega)$ and $Q = 0$ for $T \geq T_d$). In fact, this observation of sharp peak (dip) in the AC susceptibility across the dynamic transition line indicates the dynamic transition to be a truely thermodynamic transition.

VII. Behaviour of the response due to a pulsed field:

We have studied the response of a pulsed magnetic field by solving the dynamical equation of motion for the response magnetisation in the mean field approximation. Here, the time variation of the external field has been taken as

$$h(t) = h_p \quad \text{for } t_0 < t < t_0 + \delta t \qquad (4)$$
$$= 0 \text{ elsewhere.}$$

In this case, we have first allowed the system to relax from a nonequilibrium state ($m = 1, T \neq 0$) to its equilibrium state at any nonzero temperature (T), and then applied the pulse for a short duration δt (compared with the relaxation time). As the pulse is applied, the response magnetisation gets sharply peaked over its equilibrium value (Fig. 2a). This response is characterized by two quantities: the height m_p and its half-width Δt of the pulsed response magnetisation $m(t)$ (over its equilibrium value). We thus measure two important quantitites: (i) width ratio $R = \Delta t/\delta t$ and the pulse susceptibility $\chi_p = m_p/h_p$. We have plotted the temperature variation of R and χ_p in Fig.2b. In this case, the corresponding sharp peaks have been observed at $T = 1.001$ (for $h_0 = 0.01$, $\delta t = 50$ times the time differential for solving the MF equation ; T_c(MF) = 1). In all these cases, the effective T_c obtained from the peak position is slightly over-estimated, and this over-estimation disappears in the limit $h_0 \to 0$ and $\delta t \to 0$. It may also be mentioned that $\chi_p \to \chi$, the static susceptibility, in the limit $h_0 \to 0$, $\delta t \to \infty$.

IV. Concluding Remarks:

In this paper, we mainly studied the dynamic phase transition in Ising ferromagnet. The AC susceptibility components shows peak (dip)

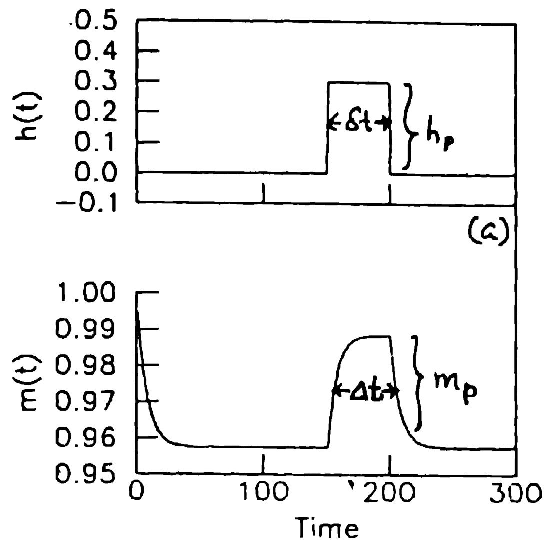

Fig.2a.Time variation of the resonse magnetisation $m(t)$ and the pulse field $h(t)$.

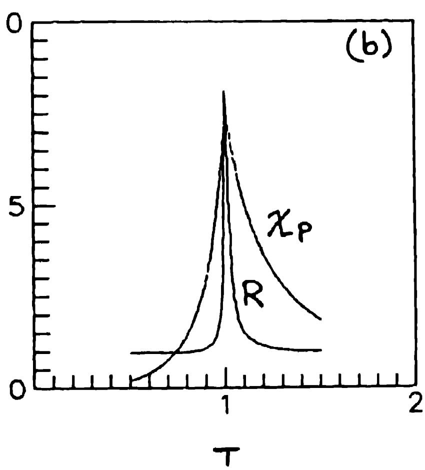

Fig.2b. Temperature (T) variation of width ratio R and χ. The Horizontal arrow indicates the previous results for the critical temperature (T_c).

at the dynamic transition point. This indicates that the dynamic transition is a true thermodynamic phase transition. Another important study that we have done here is the study of response of Ising system due to a pulsed magnetic field of very short duration. The width ratio and the pulse susceptibility show peak near the order-disorder transition point.

One can consider, for example, the study of acoustic pulse response (susceptibility and width ratio) in the system with propagating or spreading rupture/fracture. An increasing tendency of the width ratio here can indeed give the prior indication of the catastrophy.

References

[1]. M. Acharyya and B. K. Chakrabarti, Physica A **192** (1993) 471; Physica A **202** (1994) 467.

[2]. T. Tome and M. J. de Oliviera, Phys. Rev. A **41** (1990) 4251.

[3]. M. Acharyya and B. K. Chakrabarti; J. Mag. Mag. Mat. 136 (1994) L29