# Plasma $\beta$ -parameter, plasma sheet wave temperature and AE index

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Abstract : The wave temperature  $T_w$  of ULF Alfvén wave propagating in the z-direction of the resonance layer of the plasma sheet is computed for twenty-two isolated substorms and their variation with plasma  $\beta$  at the substorm onset are studied. The energy dissipation rates in the resonance layer in terms of  $T_w$  and magnetic flux  $\Phi$  are computed and the dependence with  $\beta$  are verified. The maximum energy dissipation rate when the plasma sheet temperature equals the wave temperature are computed and the dependence with plasma  $\beta$  is found as linear. The thermal energy developed in the resonance layer due to the propagation of the wave is computed for 22 substorm events and their correlation with AE index and  $\beta$  are made.

**Keywords** : Plasma  $\beta$ , wave temperature, AE index

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# 1. Introduction

The studies on the non-adiabatic motion of charged particles in the plasma sheet of the geomagnetotail has been fully appreciated only recently. The auroral electrojet index AE correlates well with the temperature on the plasma sheet [1]. During geomagnetic activity, the thermal energy in the plasma sheet increases [2]. Obviously detailed information on plasma  $\beta$ , which indicates the relative importance of thermal energy density of particles as compared to magnetic field energy density at substorm onset, provide insight into substorm physics. In order to obtain a further understanding of the behaviour of the geotail plasma sheet during substorms, we need more detailed information on plasma  $\beta$ , energy dissipation rate in the plasma sheet and Alfvén wave temperature [3].

The plasma sheet parameters including plasma  $\beta$  at substorm onset derived from GEOS 2 observations have been derived [4]. The expression for the wave temperature for

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ULF wave propagating in the z-direction with an Alfvén speed in the plasma sheet have been derived [5]. They also derived expression for the energy dissipation rate integrated over the resonance layer of the plasma sheet. The expression for the energy dissipation rate during magnetospheric substorms were given by [6]. The magnetic flux  $\boldsymbol{\Phi}$ , the power transferred P and the energy developed E in the plasma sheet during substroms during the solar cycle 1965–1986 were computed recently [7].

The purpose of this work is to investigate the nature of ULF wave temperature  $T_w$  propagating in the z-direction through the resonance layer of the plasma sheet at substorm onsets and also its variation with plasma  $\beta$ . The dependence of the energy dissipation rates in the resonance layer in terms of wave temperature  $T_w$  and magnetic flux  $\Phi$  are investigated. The relationship between AE index, thermal energy  $E_n$  and plasma  $\beta$  are studied. The values of the energy dissipation rate when  $T = T_w$  are also computed.

#### 2. Chaotic motion of a proton

Consider the perturbation displacement of a proton in the z-direction along a closed field line with antisunward propagating ULF waves with wave vector  $k = (k_x, k_y, \theta)$  and frequency  $\omega$ . These ULF waves, perturb a field line with a parallel wave number k and excite oscillations of this field line at its own resonance frequency  $\omega_A$ . If  $\omega = \omega_A$ , oscillations will grow to a very high amplitude and a resonance layer develops between the lobe and neutral sheet.

The energy dissipation rate Q integrated over the resonance layer and the wave temperature  $T_w$  are given by [5]

$$Q = 4\pi^2 \Delta z P/\mu_0 \left[ \left( T/T_w - 1 \right)^2 + \pi^2 k^2 \Delta z^2 \right],$$
 (1)

and

$$T_{w} = \frac{1}{2} m_{\mu} \omega^{2} / k_{\parallel}^{2} (k_{\perp} / k) / 1 + \frac{2}{\beta}, \qquad (2)$$

where  $\Delta z$  is the scale length of the gradient of the Alfvén speed in the resonance layer where the absorption of ULF waves occur. *P* is the power of the wave with frequency *f* and amplitude *z* and is related to the amplitude of the magnetic fluctuations in the lobe (~ 10<sup>-4</sup>-10<sup>-2</sup>, nT<sup>2</sup>-Hz) [8], *T* is the temperature on the plasma sheet side of the resonance layer, *m*<sub>i</sub> is the mass of the propagating ion. The energy dissipation rate in the magnetotail plasma sheet is given by [6]

$$Q' = 2\Phi L^3 / \pi^2 V_s^2 R_T^2$$
(3)

where  $\Phi = B/2 \pi R_T^2 V_s/L$  is the magnetic flux in the plasma sheet at substorm onset, L is the length of the plasma sheet,  $R_T$  is its thickness and  $V_s$  is the solar wind speed.

#### 3. Data

The plasma  $\beta$ , B and  $R_T$  values of twenty-two isolated substorm events (designated as storm 1-22) derived from GEOS 2 observations at plasma sheet are collected [4]. The wave number of the propagating ULF wave at Alfvén velocity  $V_A$  is calculated. P is taken as

0.015 nT<sup>2</sup>-Hz. The AE indices for the 22 events are collected from the Data Book supplied by WDC  $C_2$  for Geomagnetism.

# 4. Results and discussions

Using eqs. (1), (2) and (3), the wave temperature  $T_w$  of the propagating ULF wave, the energy dissipation rates Q and Q' are computed. The thermal energy  $E_n$  in the resonance layer at the substorm onsets and the energy dissipation rates in the resonance layer when the temperature of the plasma sheet T equals the wave temperature  $T_w$  for the twenty-two isolated substorms are also calculated.

Table 1 shows the correlation between the magnetic field strength in the resonance layer at substorm onset, the thermal energy  $E_n$ , AE index and the plasma  $\beta$ . From Table 1, it

Storm		AE	B nT	β	E <sub>n</sub> keV
Aug. 03,	1978	269	61	3.9	1 738
Oct. 17,	1978	78	48	2.2	0 607
Nov 11,	1978	199	70	16	0 939
Nov. 21,	1978	202	82	0.9	0.725
Nov. 26,	1978	382	77	3.0	2 131
Dec. 04,	1978	160	53	3.1	1.042
Jan. 22,	1979	217	86	2.1	1.860
Jan. 25,	1979	379	69	2.1	1.197
Jan. 26,	1979	410	69	4.9	2 793
Ĵan. 27,	1979	281	75	3 5	2.357
Feb. 06,	1979	300	76	3.9	2.697
Feb. 18,	1979	273	84	2.0	1.690
Feb. 26,	1979	431	76	1.9	1.314
Mar. 04,	1979	314	90	1.0	0.970
Mar. 24,	1979	226	54	2.6	0.907
Mar. 25,	1979	439	60	2.8	1.207
May 23,	1979	267	63	1.0	0.475
May 27,	1979	426	69	10	0.570
June 20,	1979	213	69	1.8	1.026
July 03,	1979	229	56	0.8	3.004
July 16.	1979	225	62	1.3	0.598
July 27,	1979	248	113	4.2	6.442

Table 1. Substorm events and plasma sheet parameters.

is clear that the maximum value of  $E_n$  is obtained corresponding to  $\beta$  of 4.2 and AE index 248. The minimum value of  $E_n$  corresponds to  $\beta$  of 1.0 and AE index 267. The values of

 $E_n$  corresponding to maximum and minimum values of  $\beta$  are respectively 2.793 keV and 3.004 keV.

Figure 1 is the graphical representation of the variation of the wave temperature of ULF Alfvén wave  $T_w$  during the 22 substorm events. From  $\beta$ -values of the corresponding storms, it is found that as  $\beta$  increases, the wave temperature increases and at a particular value

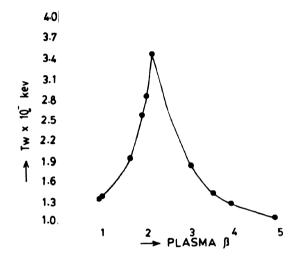


Figure 1. Variation of wave temperature  $T_w$  with  $\beta$  during 22 substorm events.

of  $\beta$ , the wave temperature begins to decrease. We have obtained a peak value of  $T_w$  as 3.492 keV corresponding to  $\beta$  of 2.1 (storm 8). The minimum value of  $T_w$  obtained is 1.070 keV corresponding to maximum  $\beta = 4.9$  (storm 9). Hence for maximum value of  $\beta$ ,  $T_{w}$  is minimum and the resonance layer heating is minimum. The resonance layer shows a slight variation of  $T_w$  between storm 15 and 22. As the heating rate is never zero, the wave acquires a maximum temperature when the plasma sheet heating is balanced by the loss through convection or through heat conduction into the ionosphere [5]. This convection of plasma is through the resonance layer. In thermal catastrophe model of substorm triggered plasma sheet, temperature is correlated positively with AE index and suggests that under the appropriate conditions, Alfvén waves are absorbed in the resonance layer near the edge of the plasma sheet. This gives rise to an increase of thermal energy within the RL. The degree and duration of heating process in the resonance layer depend on the convection speeds within the PS. This model allows the resonance layer to extend in the x-direction and energization at several locations [9]. As  $\beta$  further increases, the convection or conduction rate also increases and wave temperature decreases and thus plasma sheet heating decreases. So the plasma  $\beta$ plays an important role in thermal catastrophe.

The variation of energy dissipation rates Q (curve *a*) and Q' (curve *b*) over the resonance layer of the plasma sheet during 22 substorm events are shown in Figure 2. From curve *a*, it is found that as  $\beta$  increases Q also increases, reaches a maximum and then begins to decrease.

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to decrease. The maximum value of Q obtained is  $101.92 \times 10^6$  J/sec corresponding to  $\beta$  of 2.1. Curve b also gives the same nature as curve a. Maximum Q' obtained is  $104.92 \times 10^6$  J/sec for a  $\beta$  of 2.1. The minimum value of Q' is  $60.84 \times 10^6$  J/sec for a  $\beta$  of 0.8 (storm 20)

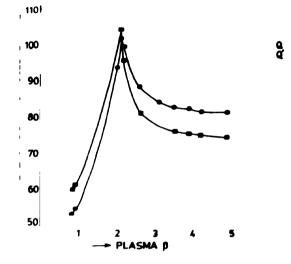


Figure 2. Variation of Q and Q' with  $\beta$  during 22 substorm events.

and minimum of Q also corresponds to a  $\beta$  of 0.8. This result is in agreement with the analytically intractable form of the energy dissipation [10]. From Figure 2, it is found that the energy dissipation rate Q is slightly less than Q'. This can be interpreted as that the absorption of energy waves occurs in a discontinuous fashion along z-direction at the resonance layer and in a continuous manner in the presence of gradients in the quantity  $n_0 B_o^2 / \varepsilon$  [10]. Also the nature of absorption due to Alfvén mode propagation has been discussed by [11]. This continuous absorption serves to increase the enthalpy flux in the resonance layer which in turn, reduces the rate of dissipation of energy flux. The energy dissipation rate in the plasma sheet is a major sink for substorm energy comparable with the ionosphere. The statistical studies of Huang *et al* [12] reveal that the plasma temperature is correlated with geomagnetic activity level. So the energy dissipation depends on the temperature of the plasma sheet which in turn, depends on plasma  $\beta$  parameter. The works of Huang *et al* [12] reveal that the energy dissipation rate at the time of substorms, support for the reconnection at the magnetotail.

When the temperature on the plasma sheet of the resonance layer T is equal to the temperature of the propagating Alfvén wave  $T_w$ , the energy dissipation rate is maximum (Figure 3). When  $T = T_w$ , there is a continuous increase of Q with  $\beta$ . When  $T < T_w$ , the heating rate in the resonance layer increases and when  $T > T_w$ , the heating rate decreases. At  $T = T_w$ , the heating rate has the maximum value. This result agrees well with [5]. The maximum value of Q obtained is  $136 \times 10^6$  J/sec corresponding to maximum  $\beta$  of 4.9. Also Q has minimum value  $64.84 \times 10^6$  J/sec when  $\beta = 0.8$ , the minimum value. The entropy change in the resonance layer in terms of Q is given as

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$$\Delta S = \left[ \frac{2}{3} (SQ) / PV_z \right],$$

where S is the specific entropy of the resonance layer,  $V_z$  is the velocity of proton in the z-direction. This entropy-change invokes the heating mechanism in the thermal catastrophe

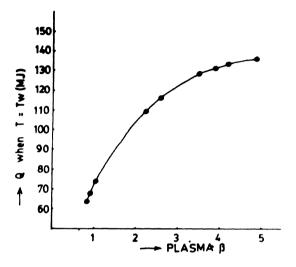


Figure 3. Maximum energy dissipation rate variation (when  $T = T_w$ ) with  $\beta$  during 22 substorm events.

model of the substorm-triggered plasma sheet. Since Q is a function of plasma  $\beta$  parameter, the entropy changes are in accordance with the variation of  $\beta$ . This entropy-change is active in the plasma sheet which leads to the loss of plasma through the formation of a neutral line and ejection of plasmoids.

# 5. Conclusion

The response of the plasma sheet resonance layer to the wave temperature of ULF Alfven wave and the energy dissipation rate are studied. It has been possible to correlate the energy dissipation rates at the time of substorms in terms of  $T_w$  and  $\Phi$  and to understand the energy absorption rate in the resonance layer. The nature of wave temperature of ULF wave propagating through the resonance layer in accordance with plasma  $\beta$  and when  $T = T_w$ , the maximum energy dissipation rate variation with  $\beta$  are also studied. The wave temperaturechange and the energy dissipation rate-variation studies invoke the heating mechanism in the resonance layer and thermal catastrophe model of the substorm-triggered plasma sheet. The current work will also help to account for the entropy change and to understand why chaos would occur in the resonance layer of the plasma sheet.

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