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Excitation of electromagnetic proton cyclotron instability by parallel electric field in the equatorial magnetosphere

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Abstract : The characteristics of the growth rate of electromagnetic ion cyclotron (EMIC) instability is investigated in a mixture of cold species of ions and warm proton in the presence of weak parallel static electric field. An attempt has been made to explain the excitation of EMIC waves through linear wave-particle (W-P) interaction in the equatorial magnetospheric region The proton cyclotron instability is modified in presence of weak parallel electric field and the growth rate is computed for equatorial magnetospheric plasma parameters The results of theoretical investigations of the growth rate are used to explain the excitation mechanism of ELF/VLF waves as observed by satellites

Keywords : Growth rate, equatorial magnetosphere, resonant particles, EMIC

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1. Introduction

The electron and ion cyclotron instabilities are responsible for the pitch angle diffusion and precipitation of electrons and ions in the magnetosphere. These processes give rise to wave-particle interaction and thereby the resonant particles participate in energy exchange process with wave and leads to growth/damping of the instabilities. Experimental evidences for naturally occurring EMIC instability have been summarized by Cornwall [1] and he pointed out that the particle precipitation and wave generation are observed in the density of cold plasma. Cornwall and Schultz [2] have discussed the electromagnetic ion cyclotron (EMIC) instability in the presence of cold plasma and its effect on the growth of the instability. Cuperman [3] and his coworkers [4] have studied the presence of warm and cold species of plasma in equatorial magnetosphere. The evidence of the cold plasma effect on the EMIC instability has been shown by Gomberoff and Cuperman [5] on the basis of the satellite observations. These observations have motivated several studies on EMIC instabilities [6–8].

The first satellite to measure electric fields at the equatorial regions was GEOS-1, later on followed by GEOS-2 [9]. The electric fields are closely related to the bulk motion of the magnetospheric plasma and to the acceleration of the charged particles. The existence of the parallel electric fields at altitudes above 3000 km. above the equator in the magnetosphere are responsible for the acceleration of charged particles which produce the discrete aurora [10] and upward flowing ion beams [10,11]. The existence of small parallel electric fields in the magnetosphere is due to the thermoelectric effect, differential pitch angle anisotropy *etc* [13]. Temerin and Mozer [14] have given a detailed observation of electric fields on auroral field lines and their interaction with charged particles, both electrons and ions. EMIC waves are also generated by auroral electron precipitation [1–5]. The existence of the extremely low frequency (ELF) waves observed by EXOS-D satellite [7,8] and ISEE-1 and ISEE-2 satellites [16] have also confirmed the generation of EMIC waves in the region of equatorial magnetosphere.

To explain a few of the magnetospheric phenomena such as precipitation, acceleration and waves generation in presence of parallel electric field, many efforts have been made to study EMIC waves. Role of parallel electric field on the generation of EMIC waves for magnetospheric plasma, has been recently discussed by Ahmad [17]. The present work deals with a linear theory of electromagnetic proton cyclotron instability (EMIC) in the presence of weak parallel electric field which is introduced due to the modification of the thermal velocity of the particles [17–19]. The modified growth rate of EMIC waves for proton has been computed using magnetospheric plasma parameters which are observed in the equatorial region.

2. Theoretical consideration

The magnetospheric plasma has been considered to contain a uniform static magnetic field B_0 and electric field E_0 along the Z-axis of the coordinate system. In the present paper the following assumptions have been made :

(i) Small amplitude waves have been assumed so that the terms involving the product of time-dependent quantities are negligible.

(ii) One dimensional analysis is applicable in which first order quantities are small and have harmonic spatial and temporal variations of the form exp $[i(kz - wt)]$.

(iii) The effective collision frequency ν is independent of the particle velocity and is very weak for magnetospheric plasma.

(iv) The electric field E_0 is sufficiently weak so that the drift velocity of the particles is smaller than the phase velocity of the EMIC wave.

A uniform plasma consisting of warm and cold protons and cold ion components has been considered. The warm protons are described by a bi-Maxwellian distribution function which is given by

$$f_p = \frac{1}{(2\pi)^{3/2}\alpha_\perp^2\,\alpha_\parallel'} \exp \quad \frac{V_\perp^2}{2\alpha_{\perp_p}^2} - \frac{V_\parallel^2}{2\alpha_{\parallel_p}'^2} \tag{1}$$

where

$$\alpha'_{\parallel_p} = \alpha_{\parallel_p}\left[1 - \frac{ieE_0}{m_p \alpha^2_{\parallel_p} k}\right]^{1/2}, \quad \text{modified parallel thermal velocity,}$$

$$\alpha'_{\parallel_p} = \left(\frac{KT_{\parallel}}{m_p}\right)^{1/2}, \quad \alpha_{\perp_p} = \left(\frac{KT_{\perp}}{m_p}\right)^{1/2}.$$

Here i is an imaginary term and $\left(eE_0/m_p\, \alpha^2_{\parallel_p} k\right) \ll 1$. Replacement of temperature by complex temperature has also been discussed by [17–19]. The symbols used are as follows :

K = Boltzmann constant,

k = Propagation constant,

$T_{\parallel}, V_{\parallel}$ = Parallel components of temperature and velocity with respect to B_0,

$T_{\perp}, V_{\perp}$ = Perpendicular components of temperature and velocity with respect to B_0,

m_p = Mass of proton,

E_0 = Parallel electric field with respect to B_0.

Using Boltzmann equation alongwith the above distribution function, the dispersion relation of EMIC wave propagating along a static magnetic field B_0 in the presence of weak parallel electric field E_0 can be written as follows :

$$D(\omega, k) = 1 + \sum_{PW} \frac{\omega^2_{PPW}}{\omega^2}\left[A_p - \frac{\omega}{\alpha_{\parallel} k} Z(\xi_p)\left\{\frac{A_p}{\omega}\left(\Omega_p - \omega\right) - 1\right\}\right]$$

$$- \sum_{CJ} \frac{\omega^2_{PJC}}{\omega^2} \frac{\omega}{\omega - \Omega_J}\left[1 + \frac{K^2 \alpha'^2_{\parallel_p} A_p}{\omega\left(\omega - \Omega_J\right)}\right], \tag{2}$$

where

$$\Omega_{P,J} = \frac{eB_0}{m_p} \quad \text{cyclotron frequency of proton and ion,}$$

$$\omega_{PPW} = \left(\frac{ne^2}{m_p\, \epsilon_0}\right)^{1/2} \quad \text{plasma frequency of warm proton,}$$

$$\omega_{PJC} = \left(\frac{ne^2}{m_J\, \epsilon_0}\right)^{1/2} \quad \text{plasma frequency of cold ion and proton,}$$

$$A_p = \left(\frac{T_\perp}{T_\parallel}\right)_p - 1 \qquad \text{anisotropy factor,}$$

$$Z(\xi_p) = i\sqrt{\pi}\,\exp\left(-\xi_p^2\right) - 1/\xi_p \qquad \text{plasma dispersion function}$$

and

$$\xi_p = \frac{\omega - \Omega_p + i\nu}{k\,\alpha_\parallel},$$

ν being the ion-neutral collision frequency.

The subscripts W and C denote the warm and cold components to the plasma, respectively. The summation $\sum\limits_{CJ}$ is over all cold components. By using the asymptotic expansion of $Z(\xi_p)$ the above dispersion relation can be written in the following form.

$$D(\omega,k) = 1 + \frac{\omega_{PPW}^2}{\omega^2}\left[A_p - \frac{\omega}{\alpha_\parallel k}\left\{i\sqrt{\pi}\,\exp\left(-\xi_p^2\right) - \frac{1}{\xi_p}\right\}\right.$$

$$\left.\times\left\{\frac{A_p}{\omega}\left(\Omega_p - \omega\right) - 1\right\}\right] - \frac{\omega_{PPC}^2}{\omega^2}\,\frac{\omega}{\omega - \Omega_p}\left[1 + \frac{k^2 A_p\,\alpha_{\parallel_p}^2}{\omega\left(\omega - \Omega_p\right)}\right.$$

$$\left.\times\left\{1 - \frac{ie\,E_0}{m_p\alpha_\parallel^2 k}\right\}\right] - \frac{\omega_{PIC}^2}{\omega^2}\,\frac{\omega}{\omega - \Omega_i/M_i}\left[1 + \frac{k^2 A_p \alpha_\parallel^2}{\omega\left(\omega - \Omega_i/M_i\right)}\right.$$

$$\left.\times\left\{1 - \frac{ie\,E_0}{m_p\alpha_\parallel^2 k}\right\}\right]. \tag{3}$$

Since for the case of EMIC waves propagating parallel to the magnetic field, the weak electric field does not change the real part of the dispersion relation or the stability criterion in the low frequency region [18]. So separating real and imaginary parts, we get the following expressions :

$$\mathrm{Re}\,D(\omega,k) = 1 + \frac{\omega_{PPW}^2}{\omega^2}\left[A_p + \frac{\omega}{\alpha_{\parallel_p} k}\,\frac{1}{\xi_p}\left\{\frac{A_p}{\omega}\left(\Omega_p - \omega\right) - 1\right\}\right]$$

$$- \frac{\omega_{PPC}^2}{\omega^2}\,\frac{\omega}{\omega - \Omega_p}\left[1 + \frac{k^2 A_p \alpha_{\parallel_p}^2}{\omega\left(\omega - \Omega_p\right)}\right]$$

$$- \frac{\omega_{PIC}^2}{\omega^2}\,\frac{\omega}{\omega - \left(\Omega_i/M_i\right)}\left[1 + \frac{k^2 A_p \alpha_{\parallel_p}^2}{\omega\left(\omega - \Omega_i/M_i\right)}\right] \tag{4}$$

and
$$\operatorname{Im} D(\omega,k) = -\frac{\omega_{PPW}^2}{\omega^2}\left[\frac{\sqrt{\pi}}{\tilde{k}} \exp\left\{-\frac{(1-x)^2}{\tilde{k}^2} + \frac{y^2}{\tilde{k}^2}\right\}\right.$$
$$\times\left\{A_p(1-x) - x\right\} - \tilde{K}\tilde{k}A_p\left\{\frac{\delta}{(1-x)^2} + \frac{\eta M_i^2}{\left(1-xM_i\right)^2}\right\}$$
$$\left. + \frac{y\left\{A_p(1-x) - x\right\}}{(1-x)^2 - y^2}\right], \tag{5}$$

where
$$x = \frac{\omega}{\Omega_p}, \quad y = \frac{\nu}{\Omega_p}, \quad \delta = \left[\frac{\omega_{PPC}}{\omega_{PPW}}\right]^2, \quad \eta = \left[\frac{\omega_{PIC}}{\omega_{PPW}}\right]^2,$$
$$\tilde{k} = \frac{k\alpha_{\parallel p}}{\Omega_p}, \quad \tilde{K} = \frac{m_p E_0}{kT_{\parallel}B_0}, \quad M_i = \frac{m_i}{Z_i m_p},$$

with m_i and Z_i being the mass of ion and charged state of ion respectively.

Using basic definition of growth rate γ as

$$\gamma = \frac{-\operatorname{Im} D(\omega,k)}{(\partial/\partial\omega)\operatorname{Re} D(\omega,k)}, \tag{6}$$

which can be determined from the imaginary part of the dispersion relation from the eq. (3) by approximating

$\omega_r \gg \omega_i$, $\left(\omega_r - \Omega_p\right)/k\alpha_{\parallel} \gg 1$ and $\left(\frac{k^2C^2}{\omega^2}\right) \gg 1$, we obtain the modified growth rate of proton cyclotron wave as

$$\frac{\gamma}{\Omega_p} = \frac{\frac{\sqrt{\pi}}{\tilde{k}}\exp\left\{\frac{-(1-x)^2}{\tilde{k}^2}+\frac{y^2}{\tilde{k}^2}\right\}\{A_p(1-x)-x\} - \tilde{K}\tilde{k}A_p\left\{\frac{\delta}{(1-x)^2}+\frac{\eta M_i^2}{(1-xM_i)^2}\right\} + \frac{y[A_p(1-x)-x]}{(1-x)^2-y^2}}{ex\left[\frac{(2-x)(1+\delta)}{(1-x)^2}+\frac{\eta M_i(2-M_i x)}{(1-xM_i)^2}-\frac{2\tilde{k}^2A_p}{x^2}\left\{\frac{(2x-1)\delta}{(1-x)^3}+\frac{(2xM_i-1)\eta M_i^2}{(1-M_i x)^3}\right\}\right]}. \tag{7}$$

When $E_0 = 0$, and $\nu = 0$, the expression for growth rate γ/Ω_p reduces to that of Gomberoff and Cuperman [5].

Eq. (7) is the generalized expression for the growth rate of EMIC wave. With the help of observed satellite parameters for equatorial magnetosphere, we have estimated the growth rate and explain the excitation of proton cyclotron instability by considering $M_i = 1$.

3. Results and discussion

Wave-particle interactions in the magnetosphere are believed to be the basic processes of ELF/VLF wave emissions [20]. Cornwall [1] pointed out that naturally occurring EMIC waves are generated due to high density of cold electrons in the magnetospheric region. Experimental data [22,23] observed on board GEOS 1-2 and ATS-6 spacecrafts, have shown the importance of cold ions in the generation of EMIC waves. Gomberoff and Rogan [24] have given evidence for the generation of EMIC waves due to the presence of cold ions. Recently, Kasahara *et al* [8] have suggested that EMIC waves are generated by the ion cyclotron resonant instability due to the temperature anisotropy of warm and cold ions. For the computation of the growth rate, the following plasma parameters have been considered which suited the magnetospheric conditions [6], $B_0 = 100\ \gamma$, $KT_\parallel = 5$ keV, $E_0 \leq 20$ mV/m, $\delta = 10$, $\eta = 1$. In magnetospheric region, weak collision frequency v/Ω_p of the order of $10^{-2} - 10^{-3}$ has been considered which does not affect the growth rate significantly [21,25]. The presence of weak parallel electric field which has been considered in the present analysis, does not lead to the phenomenon of runaway [18,19].

In the present work, we consider the linear wave-particle interaction where the interaction of warm proton and cold ions with the waves which are present in the magnetospheric region can lead to the amplification of proton cyclotron instability. Since the parallel electric field has been introduced through thermal velocity of particles, the charged particles experience additional thermal energy, thus giving rise to the enhancement of their

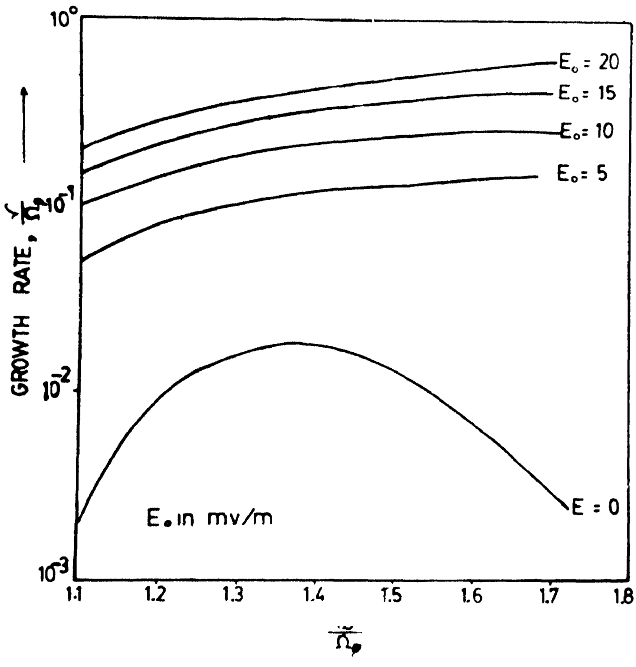

Figure 1. Growth rate of electromagnetic proton cyclotron wave *versus* normalised frequency (ω/Ω_i) for different values of E_0.

thermal velocities. Thus, the thermal anisotropy of ions as well as electric field work like the free energy sources for EMIC wave instability. Figure 1 exhibits the variation of normalized growth rate with normalized frequency for different values of electric field (E_0). At higher frequency range, the growth rate increases gradually with the increase in the magnitude of the electric field because as the thermal velocity of particle along the magnetic field increases, there may be a greater number of particles having velocities higher than the phase velocity of the wave. Thus, the EMIC wave amplitude amplifies in the presence of the electric field. In the absence of the electric field, the growth rate of the proton cyclotron waves enhances upto a critical values of the ω/Ω_p and is reduced at higher frequencies.

The effect of temperature anisotropy has been shown in Figure 2. The growth rate increases with the increase of anisotropy and this may be one of the reasons for the generation

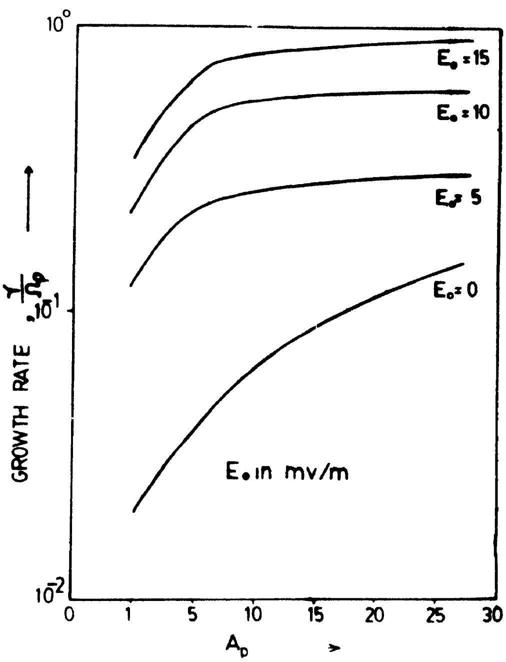

Figure 2. Growth rate of electromagnetic proton cyclotron wave *versus* temperature anisotropy $A_p = (T_\perp/T_\parallel)_p - 1$ for different values of E_0.

of EMIC wave near the equatorial magnetospheric region where instability is caused by the temperature anisotropy *i.e.* $T_\perp > T_\parallel$ [8]. In the absence of the electric field, temperature anisotropy will be the source for instability [26]. Figure 3 shows the variation of growth rate of proton cyclotron wave with $\tilde{k}$ ($k\alpha_\parallel/\Omega_p$) for various values of electric field, $A_p = 0.25$ and $\delta = 10$. Enhancement in growth rate takes place appreciably for smaller values of k (*i.e.* low frequency range). On the basis of this, the generation of ELF/VLF emissions may be explained.

The effect of cold proton concentration on the growth rate is depicted in Figure 4. It is clearly seen from this figure that in the absence of electric field, the growth is maximum when the cold proton density is nearly comparable to the warm proton density $n_w \sim n_p$ [27]. The growth of wave decreasing slowly with increase of the cold proton concentration because cold plasma decreases the phase velocity of growing mode and therefore more particles in the warm plasma resonate with the wave. This effect evidently saturates when the resonant wave velocity is of the order of the thermal velocity of the warm plasma. In the presence of electric

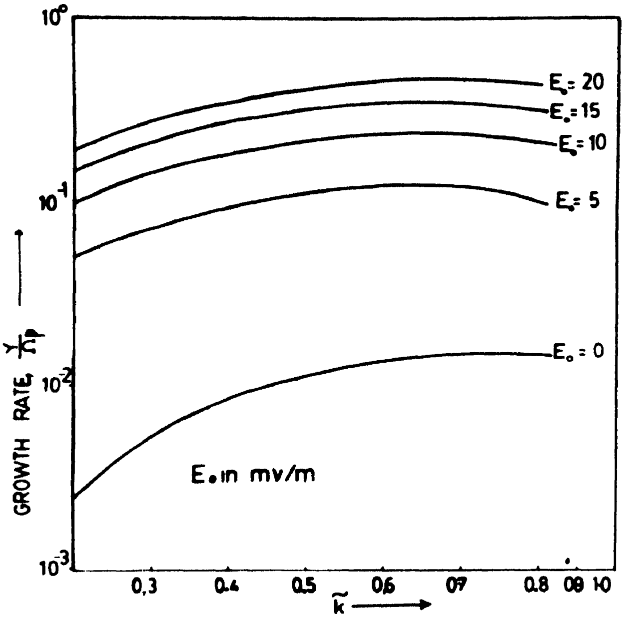

Figure 3. Growth rate of electromagnetic proton cyclotron wave *versus* $\tilde{k}\left(k\alpha_1/\Omega_p\right)$ for different values of E_0.

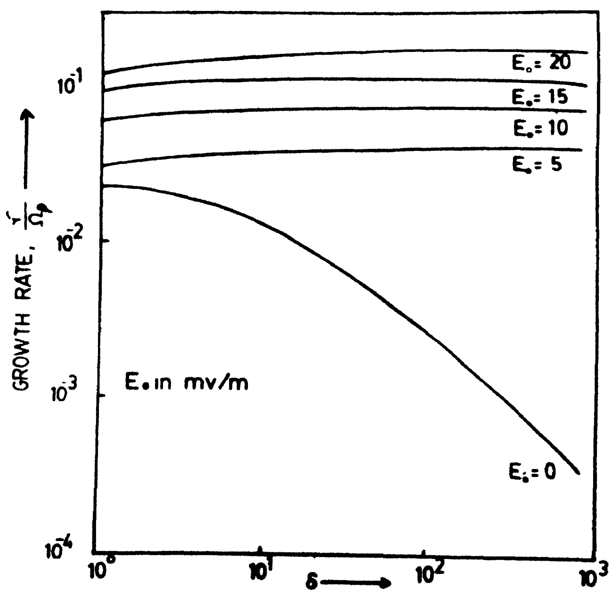

Figure 4. Growth rate of electromagnetic proton cyclotron wave *versus* $\delta\left(n_c/n_W\right)$ for different values of E_0.

field, as the magnitude of the electric field increases the growth rate increases at the lower values of $\delta(n_c/n_w)$ but further increase in the cold protons concentration do not play more significant role in the enhancement of growth, because the cold protons concentration increase the dielectric constant of the plasma and hence weaken the effect of the electric fields [28].

4. Conclusion

In this paper, we have presented a theoretical study of linear EMIC wave and reported an explanation for excitation of proton cyclotron instability in the equatorial magnetospheric region. The addition of cold protons increases the instability range in k-space. Application of parallel electric field also increases the range of instability in k-space, but this increase is in the range towards the lower values of k *i.e.*, the generation of low frequency waves. The effect of electric field is pronounced for lower values of k, $\delta(n_c/n_w)$ and A_p while it is small for higher values of k. Generation of electromagnetic proton cyclotron waves may explain some of the ELF/VLF waves as observed by satellite [8,29] at equatorial magnetosphere and this study is also important for magnetospheric-ionospheric coupling, *via* convection of electric fields.

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