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A physical model from the mass empirics of two-particle Baryon resonance states and a postulation of medium and low strong interactions

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Empirical mass formula $M^* = A+B\sqrt{L(L+1)}+CL(L+1)$ with L being the relative orbital angular momentum is found to hold good for the two particle N, Λ, Δ systems of (πN) and $N\kappa^-$ resonances. The estimated parameters from experimental mass data are suggested to interpret a physical model of the two particle resonant states and that each state to possess a predominantly single particle character admixed with a low possibility of rigid rotator or Bohr type structures Utilizing the universal fundamental length concept of Anastassov, Sinha and Sivaram further it has been established that the different N, Λ, Δ states involve an additive medium strong and a subtractive lowstrong interactions giving rise to a resonance charge" or the usual resonance coupling constant for each resonant state.

1. INTRODUCTION

Attempts at deriving empirical relations from known experimental data, for obtaining observed masses of stable particles and resonances are reported earlier by a number of investigators.

Two approaches to this end are, one those which make use of the (inherent) symmetry models and the other which consists in fitting certain linear or non-linear relations by computational procedures As examples of the first approach mention may be made of the works by Oneda *et al* (1974), Budh Ram (1966), Gerald Rosen (1972), Carruthers *et al* (1967), Iowa (1973), Lee *et al* (1974*a*, 1974*b*) But this approach, in the words of Carruthers *et al*, should not be constructed as evidence that the later approach of conventional methods as inferior In the second category we mention specifically of the investigations by Sarma (1963), Regge (1960), Sternheimer (1968), Agarwal (1971), Maglic (1966), French *et al* (1967), Battacharjee (1970, 1971, 1974), Narayana *et al* (1976)

In reference to the resonance systems considered in the present work, following Sarma (1963) it is convenient to classify the resonance κ as belong to specific pair of particles L and N, if the dominant mode of decay of κ is given by

$$\kappa \rightarrow L+N \qquad \dots \quad (1)$$

The empirical regularity of such two particle resonance systems has been suggested by him to satisfy the relation

$$b = \frac{\sqrt{L(L+1)}}{q} \qquad \ldots \quad (2)$$

where q is the centre of mass momentum and L is the relative angular momentum.

The mass of resonance state according to him is given by the relation

$$M^* = \sqrt{q^2+m_1^2}+\sqrt{q^2+m_2^2} \qquad \ldots \quad (3)$$

which on differentiation with respect $\beta = -(L+\frac{1}{2})^2$ leads to the Regge's formula viz

$$\frac{dE}{d\beta} = -\frac{1}{R^2}. \qquad \ldots \quad (4)$$

Here R is interpreted by Regge (1960), Chew (1962) to be the effective size of the compound system. But the presumption the parameter b is constant, we find by utilising the recent mass data as not correct and hence justifying a fresh look at this hypothesis.

Semi-theoretical formulae given by Gerald Rosen (1972) as extensions of Schwinger's formulae (Schwinger 1963) for baryon octuplet and baryon resonance decuplet, (which adopt the fractional hyper charge and iso-spin etc quantum numbers) are found to be of considerable success in explaining the mass splittings, but they make no emphasis or reference to the nature of decay modes of the different resonant states.

French *et al* have classified about 12 sequences each characterising a particular decay mode, but they include *both* the stange and non-strange particles and consider only a *linear* mass relation of the type $M = s+tn(n+1)$, with s, t as constants and n an integral quantum number. However, they advocate a rigid rotator or Bohr-type model, consistent with the earlier findings of Maglic (1966) for baryon masses, and do not mention the possibility of a dependence of M directly on n

Linear mass relations of several types have also been examined by Sternheimer (1968) adopting the general formula $pm+rm = M$ and generated by various values of p and r Agarwal (1971) who adopts this formula for just three sequences of particles and resonances draws an analogy with the Thomson's plum-pudding model of the atom to interpret the p and r parameters. He points out however the possible non-linear character of the graphs of mass values and fails to comment anything about the significance of different decay modes of the masses. On the other hand, though Battacharjee (1974) adopts a polynomial equation to interpret the masses, he mixes up all the elementary particles with no classification at all.

The object of present work is therefore to seek an inherent physical model of the two-particle baryon resonance systems and to discuss the non-linear character of their mass relations. We also indicate a possible connectivity of the work with the unitary symmetry models of baryon resonances. Implication of our mass formula, utilizing the universal fundamental length concept of Anastassov (1974) Sinha & Sivaram (1974) has been given, leading to the postulation of medium-strong and low-strong interactions involved in the formation of resonant states.

2. Method and Results

In the table 1 listed are the various two particle resonance states, their masses, spin, parity assignments and predominant modes of decay

Table 1 Particle data used, in the present work

Resonances	System	Orbital Angular momentum	Spin	Parity	Centre of mass momentum	Mass (Mev) (exp)
πN Resonances	N	1	1/2	$3/2^-$	456	1520
		2	1/2	$5/2^+$	572	1688
		3	1/2	$7/2^-$	888	2190
		4	1/2	$9/2^+$	905	2220
		6	1/2	$13/2^+$	1154	2650
		8	1/2	$17/2^+$	1366	3030
πN Resonances	Δ	2	3/2	$7/2^+$	741	1950
		4	3/2	$11/2^+$	1023	2420
		6	3/2	$15/2^+$	1266	2850
		8	3/2	$19/2^+$	1475	3230
$N\kappa^-$ Resonances	Λ	1	2/2	$3/2^-$	429	1650
		2	1/2	$5/2^+$	542	1814
		3	1/2	$7/2^-$	913	2350
		4	1/2	$9/2^+$	—	2312*
		6	1/2	$13/2^+$	—	2712*
		8	1/2	$17/2^+$	—	3071*

Since no definite exp. values are available, we used values given by Gerald Rosen (1972).

First we did a study of the dependence of centre of mass momentum on $\sqrt{L(L+1)}$ with L being the relative orbital angular momentum, for the following two curves.

Case I. πN–resonance, N–system of spin. $S = \frac{1}{2}$ with $L = 1, 2, 3, 4, 6, 8$.

Case II · πN–resonance, Δ–system of spins, $S = \frac{3}{2}$ with $L = 2$ 4, 6, 8

Two curves result in the case I and only one for the Case II and these are shown in figure 1. Since these graphs are not linear we are to take that the relation given

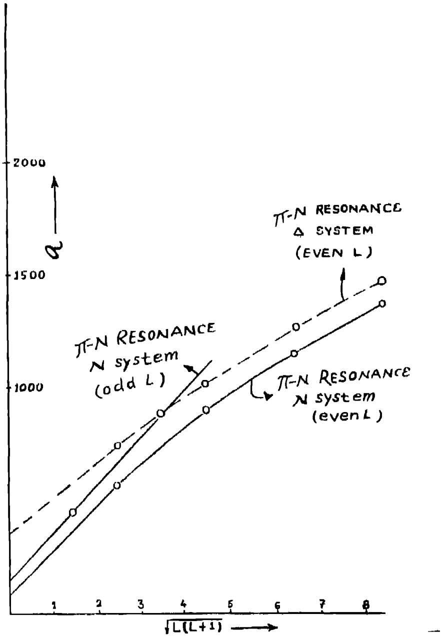

Fig 1. Graph of the centre of mass momentum q versus the factor $\sqrt{L(1+1)}$ for the N-system (odd L), Δ-system (even L) and N-system (even L).

by Sarma viz eq (2) as inadequate to describe the centre of mass-momentum dependence on the relative orbital angular momentum. Our modified relation is,

$$q = f+g\sqrt{L(L+1)}+hL(L+1) \qquad \dots \quad (5)$$

where f, g, h constants are determined by a statistical fit. The obtained values for these constants are listed in table 2.

Table 2. Calculated constants for the q-relation

Relation	$q = f+g\sqrt{L(L+1)}+hL(L+1)$		
Constants	N system (even L)	Δ-system (even L)	N-system (L odd)
f	93·3562	352·6714	150·00
g	213·9668	169·5075	210·60
h	−7·56894	−4·3673	0·00

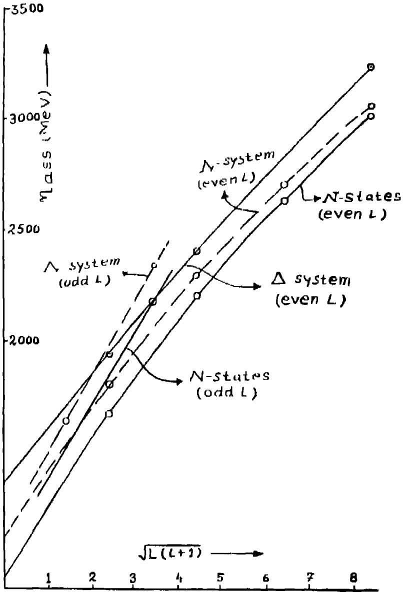

Fig 2. Graph of the mass values M versus the factor $\sqrt{L(L+1)}$ for the N-system (odd L), N-system (even L), Δ-system (even L), Λ-system (odd L) and Λ-system (even L)

Figure 2 gives the curves representing the relation

$$M^* = A+B\sqrt{L(L+1)}+CL(L+1)$$

for both the above cases I and II, and in addition for the experimentally observed masses the $N\kappa^-$ resonance Λ-systems, with both the possibilities L odd and L even The evaluated values for constants A, B, C are listed in table 3.

Table 3. Calculated constants of the M^* relation

	Relation : $M^* = A + B\sqrt{L(L+1)} + CL(L+1)$				
Constants	(for even L)			(for odd L)	
	N-system	Δ-system	Λ-system	N-system	Λ-system
A	957·9939	1338·6805	1125·0000	1200	1380
B	321·43503	259·8656	303·0129	335	350
C	−9·13316	−4·0757	−8·65282	0	0

3. Discussion and Conclusions

At first it must be pointed out that the simple relation given Sarma (1963) viz eq. (2) (with $n/1 = c = 1$), has an analogy with DeBroglis's relation $\lambda = h/p$, for the wave length λ associated with a particle of momentum p. The analogy however fails in that the relation of Sarma involves in the numerator a multiple of the relative orbital angular momentum of the state, though that both the relations agree as far the dimensions are examined. Again from the curves of figure 1 for the πN-Resonance, Δ-system and N-system (L even) states, it is clear that a simple linear relation may not be correct. The L odd states which imply the linear relation, are inadequate as only two non-ambiguous experimental values could be used, in each sequence.

The physical significance of the deviations from a linear relation while in view of the analogy with the DeBroglie's relation may be mentioned, that instead of a single parameter, such as the Quantum wavelength associated with a particle, the resonance state requires a description in terms of more number of characteristic parameters.

The set of parameters from the suggested relation in our work being (f, g, h) or (A, B, C) respectively as one uses either the centre of mass momentum or the experimentally observed mass of the any chosen resonance state.

A constant term, as it occurs in the relation

$$M^* = \sqrt{\frac{L(L+1)}{b^2} + m_1^2} + \sqrt{\frac{L(L+1)}{b^2} + m_2^2} \quad \ldots \quad (6)$$

obviously refers to the rest mass energy of the resonance system. While in simple relation

$$M^* = A + B\sqrt{L(L+1)} + CL(L+1) \quad \ldots \quad (7)$$

the constant term A though may be taken to represent the rest mass energy of the system, it is to be recognized as being the same for a number of distinct states differing in their total angular momentum. It may also be tempting to identify the constant term 'f' of the M, with the mass of $L = 0$ particles of the system, as has been done by Gerald Rosen (1972) in his formulation following Schwinger (1963) of a fraction hypercharge and isospin etc semi-theoretical mass formula for Baryon mass spectra. But we prefer to retain the $L = 0$ masses inferred from the values of f (or alternatively from the intercept on the ordinate of the graphs) as a characteristic mass of the resonance system. These mass values denoted by M_{int} are compared with those of $L = 0$ particle mass values denoted by M_{exp} in table 4 The M_{int}–M_{exp} for the Δ-system of πN resonances appears

Table 4 Comparison of the M_{int} with M_{exp}

Resonances	System	M_{int} Intercept (Mev)	M_{exp} Expt mass for $L = 0$ (Mev)	$M_{int}-M_{exp}$ (Mev)
πN Resonance	N	957·9939	939·6	18·3939
	Δ	1338 6805	1230–1236	(108·6705–103 6805)
$N\kappa^-$ Resonance	Λ	1125	1115 6	9·4

to be anarmously large. This suggests that $L = 0$, Δ-resonant state must be distinguished from the $L = 0$, Δ-particle state (Vasavada *et al* 1966, Kim *et al* 1972). The reason for distinguishing between M_{int} and M_{exp} values (or the particle $L = 0$ and the $L = 0$ resonant states) in general therefore is that we presume essentially different strengths of strong interaction as may be responsible for the formation of $L = 0$ resonant states or the $L = 0$ particles This would become clearly established at subsequent stage of this paper

As regards the second parameter of our work it is convenient to consider $q = f + g\sqrt{L(L+1)} + L(L+1)$ Here if f and h vanish, we note that $g = 1/b$ given by Sarma (1963) and thus simply corresponds to a single parameter of *a fundamental length*—associated with the state as defined by him This parameter also gives the direct dependence on the relative orbital angular momentum of either q or M^1 in our relations. From the values given in table 2 it is noteworthy that f and h are small compared to g excepting in the case of Δ system. The g values for the different systems considered approximately are of the same order of magnitude, which is perhaps the reason why Sarma could obtain a characteristic b parameter

The occurrence of $L(L+1)$ in the empirical mass formulae has been previously interpreted by French *et al* (1967) and Maglic (1966) as indicative of a rigid-rotator or Bohr-type physical model for baryon resonances of both strange and non-strange character. But in view that the data by that procedure does not reproduce the accepted values of angular momenta, French *et al* (1967) interpret in terms of a classical picture of the pion field coupled to the nucleon source through a non-relativistic limit of the relativistic γ_5 interaction, $g(\sigma \Delta\phi)$. Thus they suggest one can consider an induced moment of inertia in the nucleon to explain the observed baryon masses reasonably and with the accepted values of their angular momenta.

Contrary to this proposition of Bohr-type or rigid rotator model we get a negative value for the coefficient h or C of $L(L+1)$ term in our relations, excepting in the cases of L odd but where these Vanish. The negative sign we suggest to interpret as an opposed nature of the resonance system to exist as a rigid rotator. The subtractive tendency in the energy cannot be argued as the possibility of minimization in energy and hence towards a stable rigid rotator model of resonance system because the order of negative energies involved (within the mass limits considered) are very small. The positive contribution is relatively compared to the contributions from a rigid rotator type structure (i.e., from the third term involving the factor $L(L+1)$)

Next by extending the concept of fundamental length introduced by Anastassov (1974), Shivaram & Sinha (1974) as one of the universal constants, to hold good also for the resonance systems, we derive the respective coupling constants associated with each mass value of the systems. For example, if the mass of the N-resonance state of orbital angular momentum $L = 2$, with the predominant πN mode of decay is chosen then,

$$\frac{g_r^2}{\hbar c} = \frac{m_r l_0 c}{\hbar} = \frac{1690 l_0 c}{\hbar} \qquad \ldots (8)$$

m_r is the resonance mass obtained from eq. (6) l_0 is the fundamental length and $g_r^2/\hbar c$ is coupling constant

Utilising the value of l_0 given by Sivaram & Sinha (1974) $l_0 = 2{\cdot}8\times10^{-13}$ cms. we get,

$$\frac{g_r^2}{\hbar c} = 23{\cdot}971 \qquad \ldots (9)$$

for this case It is note-worthy that this value is about twice that for the strong interaction of elementary particles We define therefore g_r as some kind of a *resonance charge*, similar to the baryonic or nuclear charge. Interesting further, is we get

$$\frac{g_r^2}{\hbar c} = A^1 + B^1\sqrt{L(L+1)} + C^1 L(L+1) \qquad \ldots (10)$$

with $A^1 = \frac{A l_0 C}{\hbar}$ etc. and similar expression for B^1 and C^1. The evaluated coupling constants associated with the different resonance masses are listed in table 5 and table 6 shows the values A^1, B^1, C^1 The lists of coupling constants are useful for studies of the unitary symmtery models. Further the relation given above implies that every few basic coupling constants, A^1, B^1, C^1 are enough to generate the resonant states of a given resonance system and that these various states arises only as a consequence of their relative orbital angular momentum values. We state that the constants A^1, B^1, C^1 represent physically the strong interaction, the medium-strong interaction and low-strong interactions respectively of a two particle resonance system The low-strong interactions however is opposed to both the medium-strong and strong interactions. With increasing resonance mass this postulation results in a dominant low-strong, interaction contribution. Beyond the limit of mass values considered here, recent experimental finding has been found to have an exponential increase This shows that for that range the low-strong interactions may become additive and further involve higher order terms. Extension of our relation may be attempted as and when unambiguous higher experimental resonance mass data becomes available

Table 5. The calculated interaction strengths using the relation

$$\frac{g_r^2}{\hbar c} = \frac{m_r + l_0 c}{\hbar}$$

Orbital Angular Momentum	πN-Resonances N-system	Δ-system	$n\kappa^-$Resonances Λ-system
1	21·55*	--	23·96*
2	23·971	27·66141	25·74103
3	31·05*	—	32·66*
4	31·37758	35 4035	32·71427
6	37 68358	40·43551	38·64566
8	42·93503	46·08822	43·57005

* Experimental mass value is used to calculate the interaction strength

Table 6. Calculated constants A^1, B^1 and C^1 adopting the relations

$$A^1 = \frac{A l_0 C}{\hbar} \text{ etc.}$$

Constants	$\frac{g_r^2}{\hbar C} = A^1 + B^1\sqrt{L(L+1)} + C^1 L(L+1)$ N-system	Δ-system	Λ-system
A^1	13·58415	18·98221	15·95227
B^1	4·55788	3·68484	4·29679
C^1	−0·12950	−0·05779	−0·12269

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